Analytical Physics 1B Lecture 3: Gravity Continued

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Friday, February 5th, 2016
• Common Exam 1: Monday, February 15, 9:50-11:10 PM
  • Location by first letter of your last name:
    • Aa to Jz: ARC 103
    • Ka to Nz: Hill 114
    • Oa to Sh: SEC 111
    • Si to Zz: PHY LH

• Content
  • 15 multiple choice questions covering material from the first three lectures from Jan. 22 through Feb. 5, corresponding to chapters 10, 11, and 13
  • Bring pencils, one 8.5x11 sheet of paper with hand written formulae on both sides, and a scientific calculator
  • No cell phone calculators, smart watches, or sharing of calculators permitted
**SUPERPOSITION OF FORCES**

- The gravitational force between any two massive bodies is given by

\[ F = \frac{G m_1 m_2}{r^2} \]

- In the presence of multiple bodies, the forces add **vectorially**

\[ \vec{F} = \vec{F}_{10} + \vec{F}_{20} + \vec{F}_{30} + \cdots \]
Shown below are snapshots of a meteor of mass $m$ next to massive objects. How does the net force on the meteor in Case A compare to the net force on the meteor in Case B?

A. The net force on the meteor is greater in Case A.
B. The net force on the meteor is the same in both cases.
C. The net force on the meteor is greater in Case B.
**METEOR NEAR MASSIVE OBJECTS**

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The weight of a body is the total gravitational force exerted on the body by all other bodies in the universe.

13.8 An astronaut who weighs 700 N at the earth’s surface experiences less gravitational attraction when above the surface. The relevant distance \( r \) is from the astronaut to the center of the earth (not from the astronaut to the earth’s surface).
WEIGHTLESSNESS

• Apparent weightlessness can occur when in freefall
  • Consider when Stephen Hawking and the vomit comet fall with an acceleration of $g$
2001: A Space Odyssey

- https://www.youtube.com/watch?v=UqOOZux5sPE
By my estimate, the period of the space station’s rotation was about 30 seconds. What is the radius of the space station, assuming that the effective gravitation acceleration is $g$?

$$g = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$$

solving for “$r$” gives…

$$r = \frac{gT^2}{4\pi^2} = \frac{9.8 \text{ m/s}^2 (30 \text{ s})^2}{4\pi^2} \approx 220 \text{ m}$$
COMPARISON TO ISS

• The International Space Station (ISS) has a width of 108.5 m compared to diameter of ~440 m for the space station in 2001.
The potential energy due to gravity is given by (the derivation is in the book)

\[ U = -\frac{GMm}{r} \]

The difference between this equation and the more familiar equation for gravitational potential energy \((U=mg\,h)\) is that this accounts for the change in the strength of gravity over large distances.

Something to notice:

- \(U=0\) when the objects are infinitely separated, and \(U<0\) otherwise
- This definition is a (convenient) choice. What is physical is not the absolute value of \(U\), but the difference in \(U\) between two points
**ESCAPE VELOCITY**

• What is the minimum required velocity to escape the Earth’s gravitational field?
  
  • We can calculate this by computing using conservation of energy

\[
K_1 + U_1 = K_2 + U_2
\]

\[
\frac{1}{2}mv^2 + \left(-\frac{GM_E m}{R_E}\right) = 0 + 0
\]

• In the second term, we need to calculate the kinetic and potential energy when the object is **infinitely far away**. \(K_2 = 0\) because we want to know the minimum energy to escape, and \(U_2 = 0\) because we are at \(r = \infty\)

Solving for \(v\):

\[
v = \sqrt{\frac{2GM_E}{R_E}} \approx 11 \text{ km/s}
\]
BLACK HOLES

• Imagine you had a planet so massive that the escape velocity exceeded the speed of light. What size is the planet?
  • let’s do something a bit silly and just set the escape velocity to the speed of light, and see what we get.

\[ c = \sqrt{\frac{2GM}{R}} \quad \Rightarrow \quad R_S = \frac{2GM}{c^2} \]

• Amazingly, this gives us the Schwarzschild radius, which was calculated using Einstein’s theory of general relativity
  • It turns out that we made a lucky mistake

• If you could squeeze the entire mass of the sun into a sphere with a radius of 3 km, you would have a black hole!
  • (the actual radius of the sun is ~700,000 km)
Motion of Satellites

• A projectile is launched from A to B
  • If the velocity is great enough, it enters an **elliptical** orbit
  • If the velocity exceeds ~11 km/s, it escapes completely

A projectile is launched from A toward B. Trajectories 1 through 7 show the effect of increasing initial speed.
Let’s assume that an object is moving in a **circular orbit**

In this case, the centripetal acceleration is being provided by the gravitational force.

\[
\frac{mv^2}{r} = \frac{GM_E m}{r^2}
\]

Solving for \(v\)

\[
v(r) = \sqrt{\frac{GM_E}{r}}
\]
A planet X has a radius $R_X = \frac{R_{\text{earth}}}{2}$ and a mass $M_X = \frac{M_{\text{earth}}}{3}$. Find the free fall acceleration at the surface of planet X in terms of the free fall acceleration at the surface of the earth, $g_{\text{earth}}$.

A. $g_X = \frac{2}{3} g_{\text{earth}}$
B. $g_X = \frac{4}{3} g_{\text{earth}}$
C. $g_X = \frac{1}{6} g_{\text{earth}}$
D. $g_X = \frac{1}{12} g_{\text{earth}}$
E. $g_X = g_{\text{earth}}$. 

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\[
g_X = \frac{GM_X}{R_X^2} = \frac{GM_{\text{Earth}}/3}{(R_{\text{Earth}}/2)^2} = \frac{4}{3} \frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2} = \frac{4}{3} g_{\text{Earth}}
\]
Kepler’s 1st Law

- Each planet moves in an **elliptical orbit**, with the sun at one focus of the ellipse
  - There is nothing at the other focus

- The longest dimension is the major axis, with length 2a
  - a is the **semi-major axis**

- The distance from the foci to the center is ea, where e is the **eccentricity**
  - e is between 0 and 1
  - e for Earth is 0.0167

13.18 Geometry of an ellipse. The sum of the distances SP and S'P is the same for every point on the curve. The sizes of the sun (S) and planet (P) are exaggerated for clarity.

A planet P follows an elliptical orbit.

The sun S is at one focus of the ellipse.

There is nothing at the other focus.
Planetary Orbits

Halley's Comet

Diagram showing the orbits of Neptune, Uranus, Saturn, Jupiter, and Pluto with emphasis on Halley's Comet's trajectory over time.
**Kepler’s 2nd Law**

- A line that connects a planet to the sun sweeps out equal areas in equal times
  - Justification: consider differential area $dA$ as approximately a triangle
    
    $$
    dA \approx \frac{1}{2} r v dt
    $$

- Treat the angular momentum as a point-like particle
  
  $$
  L \approx m r v
  $$

- Plugging in $L$ gives:
  
  $$
  \frac{dA}{dt} \approx \frac{1}{2} \frac{L}{m}
  $$

  Since $L$ and $m$ are constants, the planet sweeps out equal areas in equal times
Kepler’s 3rd Law

- Kepler’s 3rd law is that the period squared is proportional to the semi-major axis cubed

\[ T^2 = \left( \frac{4\pi^2}{GM} \right) a^3 \]

M is the mass of the Sun

- Let’s consider the simpler case of a circular orbit

- from before (1):

\[ v = \sqrt{\frac{GM}{r}} \]

- The period is (2):

\[ T^2 = \left( \frac{2\pi}{\omega} \right)^2 = \left( \frac{2\pi r}{v} \right)^2 \]

- Plugging (1) into (2) gives:

\[ T^2 = \left( \frac{4\pi^2}{GM} \right) r^3 \]
Two planets move around a sun. The first one (P1) is three times as massive and five times further from the sun than the second one (P2).

Which of the following statements accurately compares the two periods?

A. $T_1 = 11T_2$
B. $T_2 = 11T_1$
C. $T_2 = 0.6T_1$
D. $T_2 = 42T_1$
E. $T_1 = 42T_2$
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$$T_1 = \sqrt{\frac{4\pi^2}{GM} r_1^3} = \sqrt{\frac{4\pi^2}{GM} (5r_2)^3} = \sqrt{5^3} \sqrt{\frac{4\pi^2}{GM} r_2^3} \approx 11T_2$$
Tidal Forces

• Strictly speaking, the gravitational force is not uniform over the extent of the body (F depends on r!)
  • In particular, the force of attraction is greater for the closer parts and weaker for the parts further away
  • From the viewpoint of the earth, this results in the waters being force both towards and away from the moon! (This explains why there are two tides per day)
Black Hole Tidal Forces

• For a black hole, the tidal forces would (probably) be lethal, stretching and squeezing a potential visitor apart.
PERIHELION SHIFT OF MERCURY

• Einstein discovered that energy and mass can warp space and time
  • Einstein’s general theory of relativity describes the gravitational force in relativistic terms
    • The force of gravity of conveyed by the warping of space time
    • This also predicts several other distinct effects, including the magnitude of the perihelion shift of Mercury

• Newtonian physics was only able to account for about 90% of the perihelion advance
  • Most of this is due to the gravitational tug of other planets