Analytical Physics 1B Lecture 12:
2nd Law of Thermodynamics

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Friday, April 20\textsuperscript{th}, 2018
Final Exam Information

• Final exam, Tuesday, May 8, 4:00 to 7:00 pm on CAC.
  • If you have 3 exams on May 9 or have 3 exams in a row that includes Physics 124, ASAP but not later than 5:00 pm on April 23 send your ENTIRE exam schedule to Professor Lath lath@physics.rutgers.edu.
    • There will be a conflict makeup exam on May 4 at noon

• FINAL Exam location by first letters of your last name:
  • Aa-Jz: CAC Gym Annex
  • Ka-Sz: CAC Gym
  • Ta-Zz: Scott 123

• The exam will have 30 multiple choice questions;
  • The exam will cover the entire semester.

• Bring pencils, two (2) sheets of paper no larger than 8.5x11 with handwritten formulae on both sides, and a scientific calculator. No cell phones, smart watches, or sharing of calculators permitted. You may also want to bring water – it can get hot in the Gym in May.
Fundamentals of Heat Engines

• Heat engines cycle
  • absorb heat from a hot reservoir
  • perform mechanical work
  • discard heat at a lower temperature
  • return to the initial state

• A hot reservoir provides a substantial amount of heat without changing its own temperature
  • A cold reservoir can absorb heat without changing temperature
  • The heat provided at temperature $T_H$ is $Q_H$
  • The heat discarded at temperature $T_C$ is $Q_C$

• According to the 1st law, the work performed must be $|Q_H| - |Q_C|$
Heat Engine Efficiency

• It would be ideal if we could convert all of the heat into work done
  • Unfortunately, however, that is impossible
  • The **thermal efficiency** of a heat engine is determined by the fraction of heat that gets converted into work:

\[
e = \frac{W}{Q_H} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \left| \frac{Q_C}{Q_H} \right|
\]

• There is a **maximum possible** efficiency, the Carnot efficiency (more on that later)
Refrigerators

- Refrigerators are like heat engines in reverse
  - takes heat from a cold place and moves it to a warm place using mechanical work
  - Here, $Q_C > 0$, $Q_H < 0$ and $W < 0$

- The **coefficient of performance** for a refrigerator is a measure of how much heat is removed given some amount of work

$$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$$
The Otto Cycle

**Intake stroke:** Piston moves down, causing a partial vacuum in cylinder; gasoline–air mixture enters through intake valve.

**Compression stroke:** Intake valve closes; mixture is compressed as piston moves up.

**Ignition:** Spark plug ignites mixture.

**Power stroke:** Hot burned mixture expands, pushing piston down.

**Exhaust stroke:** Exhaust valve opens; piston moves up, expelling exhaust and leaving cylinder ready for next intake stroke.
The Otto Cycle

1. Adiabatic compression (compression stroke)
2. Heating at constant volume (fuel combustion)
3. Adiabatic expansion (power stroke)
4. Cooling at constant volume (cooling of exhaust gases)

**Intake stroke:** Piston moves down, causing a partial vacuum in cylinder; gasoline-air mixture enters through intake valve.

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Otto Cycle Efficiency

- The efficiency is related to the heat added and removed, $Q_H$ and $Q_C$
- Let’s assume an ideal gas
  - Processes $b \rightarrow c$ and $d \rightarrow a$ are performed at constant volume, so for an ideal gas
    \[
    Q_H = nC_V(T_c - T_b)
    \]
    \[
    Q_C = nC_V(T_a - T_d)
    \]
  - notice that $Q_C < 0$ and $Q_H > 0$
  - Therefore, the efficiency is
    \[
    e = 1 - \left| \frac{Q_C}{Q_H} \right| = 1 - \left| \frac{T_a - T_d}{T_c - T_b} \right|
    \]
Otto Cycle Efficiency

- It turns out that for an ideal gas during an adiabatic process \((\text{adiabatic} = \text{no heat in/out})\)

\[ TV^{\gamma - 1} = \text{constant} \quad pV^\gamma = \text{constant} \]

- where

\[ \gamma \equiv \frac{C_P}{C_V} \]

- Therefore, from the \(c \rightarrow d\) process

\[ T_c V^{\gamma - 1} = T_d (rV)^{\gamma - 1} \Rightarrow T_c = T_d r^{\gamma - 1} \]

- and from the \(a \rightarrow b\) process

\[ T_b V^{\gamma - 1} = T_a (rV)^{\gamma - 1} \Rightarrow T_b = T_a r^{\gamma - 1} \]
Otto Cycle Efficiency

• From before we have

\[ \frac{C_p}{C_v} = 1.4 \text{ for a typical gas, and } r = 8 \text{ for many modern engines} \]

\[ e = 1 - \left| \frac{T_a - T_d}{T_c - T_b} \right| \]

\[ T_c = T_d r^{\gamma-1} \]

\[ T_b = T_a r^{\gamma-1} \]

• Plugging these into each other we get

\[ e = 1 - \left| \frac{T_a - T_d}{T_d r^{\gamma-1} - T_a r^{\gamma-1}} \right| = 1 - \frac{1}{r^{\gamma-1}} \]

• \( C_p/C_v = 1.4 \) for a typical gas, and \( r = 8 \) for many modern engines
  • therefore \( e = 56\% \)
  • (more like 30\% in real life)
Stirling Engines

1. Power piston (dark grey) has compressed the gas, the displacer piston (light grey) has moved so that most of the gas is adjacent to the hot heat exchanger.

2. The heated gas increases in pressure and pushes the power piston to the farthest limit of the power stroke.

3. The displacer piston now moves, shunting the gas to the cold end of the cylinder.

4. The cooled gas is now compressed by the flywheel momentum. This takes less energy, since its pressure drops when it is cooled.
Engine Efficiency

During one cycle, a heat engine exhausts 110 J of thermal energy for every 200 J of thermal energy it absorbs. What is the efficiency of the engine?

1. 45%
2. 55%
3. 35%
4. 40%
5. 65%
Engine Efficiency

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2nd Law of Thermodynamics

• It is impossible for any system to undergo a process in which it absorbs heat from a reservoir at a single temperature and converts the heat \textit{completely} into mechanical work, with the system ending in the same state in which it began
  
  • This is the “engine” version of the 2nd law
  • This tells us that there is a limit to how efficient an engine can be

• Similarly, it is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter body
  
  • This is the “refrigerator” version of the 2nd law

• Both of these statements have to do with a more fundamental principle known as entropy
Entropy

• Qualitatively, entropy is the measure of disorder in a system
  • quantitatively, it is measure of how many states are available to a system
  • For instance, your socks could be anywhere in your house, but it is only “ordered” when they are in your dresser drawer (or on your feet)

• Because the number of ways a system could be disordered is so much greater than the number of ways a system could be ordered, we observe that systems tend towards disorder
  • For a gas with many particles, the particles interact so that very quickly (once equilibrium is established) the disorder has been maximized
    • In this picture, the 2nd Law of Thermodynamics is that the entropy of a system always increases with time
Reversible Processes

• Reversible processes are those which can be reversed by making only an infinitesimal change to the system
  • **These don’t exist** perfectly in reality, but they can be approximated
  • A system that undergoes an idealized reversible process is always very close to being in thermodynamic equilibrium within itself and with its surroundings
  • In a reversible process, the total change in entropy **for the system** is 0

(a) A block of ice melts *irreversibly* when we place it in a hot (70°C) metal box.
(b) A block of ice at 0°C can be melted *reversibly* if we put it in a 0°C metal box.

Heat flows from the box into the ice and water, never the reverse.

By infinitesimally raising or lowering the temperature of the box, we can make heat flow into the ice to melt it or make heat flow out of the water to refreeze it.
Carnot Cycle

- The gas expands isothermally at temperature $T_H$, absorbing heat $Q_H$ (ab).
- It expands adiabatically until its temperature drops to $T_C$ (bc).
- It is compressed isothermally at $T_C$, rejecting heat $|Q_C|$ (cd).
- It is compressed adiabatically back to its initial state at temperature $T_H$ (da).
Carnot Cycle

• A Carnot engine is maximally efficient

\[ e = 1 - \frac{T_C}{T_H} \]

• The efficiency depends only on the temperature of the reservoirs
• The larger the difference in temperature, the more efficient the engine
A real heat engine takes in 1000 J at a 600K hot reservoir and does 250 J of work. Which of the following is true?

1. The temperature of the cold reservoir must be lower than 275 K.
2. The temperature of the cold reservoir must be 450 K.
3. The temperature of the cold reservoir could be lower than 400 K.
4. The temperature of the cold reservoir could be higher than 500 K.
5. There is not enough information to determine whether any of these statements is true.
A Real Heat Engine

A real heat engine takes in 1000 J at a 600K hot reservoir and does 250 J of work. Which of the following is true?

1. The temperature of the cold reservoir must be lower than 275 K.
2. The temperature of the cold reservoir must be 450 K.
3. The temperature of the cold reservoir could be lower than 400 K.
4. The temperature of the cold reservoir could be higher than 500 K.
5. There is not enough information to determine whether any of these statements is true.

The engine’s efficiency is 250 J/1000 J = 25%. If it were running at maximum efficiency that would imply that the cold reservoir is 450 K ($e_{\text{max}}=1-450/600$). Since engines may run below max efficiency but not above, that implies that the cold reservoir is at most 450 K, which is only true of answer 3.
Entropy Change

- If an infinitesimal amount of heat $dQ$ is added to a system in a infinitesimal reversible process,

$$\Delta S = \int_1^2 \frac{dQ}{T}$$

- Adding heat increases the number of available states
  - If the system is already at a high temperature, adding heat does not increase the randomness of the motion as much as if it were at a lower temperature
- Didn’t I say a few slides back that the change in entropy is 0 for a reversible process?
Entropy Change

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$$\Delta S = \int_1^2 \frac{dQ}{T}$$

• Adding heat increases the number of available states
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• Didn’t I say a few slides back that the change in entropy is 0 for a reversible process?
  • For the example of the melting of ice in a box, the box loses entropy, while the ice gains entropy (turning into water)
  • The total change in entropy for the entire system must cancel in a reversible process
In a cyclic process, a real heat engine takes in 400 J at a 900 K reservoir and deposits 200 J into a 300 K reservoir. What is the TOTAL entropy change of the two reservoirs?

1. $\frac{2}{9}$ J/K
2. $\frac{1}{3}$ J/K
3. $\frac{1}{2}$ J/K
4. $\frac{10}{9}$ J/K
5. $\frac{2}{3}$ J/K
Entropy Change

In a cyclic process, a real heat engine takes in 400 J at a 900 K reservoir and deposits 200 J into a 300 K reservoir. What is the TOTAL entropy change of the two reservoirs?

1. 2/9 J/K
2. 1/3 J/K
3. 1/2 J/K
4. 10/9 J/K
5. 2/3 J/K

Since the reservoir’s temperatures are constant under an exchange of heat, \( \Delta S = \frac{Q}{T} \). Heat is leaving the hot reservoir and entering the cold reservoir via the engine. Therefore, \( \Delta S = -400 \text{ J/900 K} \) for the hot reservoir and \( \Delta S = 200 \text{ J/300 K} \). The total entropy change is the sum of the two: 2/9 J/K.