You are standing on the edge of a rotating merry-go-round. You feel an acceleration pointing towards the center of the merry-go-round. This means:

The rate of rotation of the merry-go-round is constant. Since you are moving in a circle, there is a component of acceleration towards the center of the circle (centripetal acceleration).

If it was speeding up or slowing down, there would be another component of acceleration tangential to the circle. In that case the acceleration you feel would be a vector sum of the centripetal acceleration and this tangential acceleration.

Since you feel an acceleration towards the center, it means there is no tangential acceleration, and the merry-go-round is not speeding up or slowing down, it is rotating at a constant rate.

A hoop and a disk with uniform mass distribution have the same radius but the total masses are not known. They both roll down a ramp without slipping, reaching the bottom in the same time. What can you deduce about the relative masses?

This is obviously impossible. No matter the mass or radius of the hoop and disk. Question was dropped from exam (all students were given points for this one)

A ball is released from rest on a no-slip surface, as shown in the figure. After reaching its lowest point, the ball begins to rise again, this time on a frictionless surface as shown in the figure. When the ball reaches its maximum height on the frictionless surface, it is:

At a lesser height than when it was released. This is because at the bottom the potential energy has turned into kinetic energy of motion and kinetic energy of rotation. However once you start climbing the
frictionless side, only the kinetic energy of motion is converted to potential energy (height). The kinetic energy of rotation does nothing.

A phonograph record of Mozart's N.41 Symphony is whirling around at 103 rpm. Two balls, each of mass \(1 \times 10^{-1}\) kg, are sitting on the disk, and are at rest with respect to the disk. The first one, ball 1 sits the end of the paper label, 5 cm from the center of the record. The second one, ball 2, sits 10 cm out from the center, on the very edge of the record, as shown in the figure. What is the ratio of the friction forces acting on them?

\[
\text{Ratio} = \frac{\text{friction on ball 2}}{\text{friction on ball 1}}
\]

2. The balls are moving in a circle, so there is a centripetal force \(F_c = m \cdot v^2 / r\). Since there is no slipping, \(\omega = v / r\)

\[
F_c = m \cdot \omega^2 \cdot r
\]

This centripetal force is provided by friction. Since the balls have the same mass, and \(\omega\) is obviously the same, the only difference is radius. 10/5 = 2.

A wheel is spun up from rest by application of a force applied to the rim of the wheel. Consider the wheel to be in the plane of the paper, rotating clockwise. This force must point in what direction?

It is impossible to tell with the information given. The force could be in any direction, depending on where on the rim it is applied.

A force \(\mathbf{F} = i\text{-hat} + 2 \ j\text{-hat} - 4 \ j\text{-hat} \ \text{N}\) is acting on a point with position vector \(\mathbf{r} = 2 \ i\text{-hat} - 4 \ j\text{-hat} \ \text{m}\). What is the corresponding torque (in Nm) around the origin?
+8 \hat{k}

torque = r \times F = \begin{vmatrix} i & j & k \\ 2 & -4 & 0 \\ 1 & 2 & 0 \end{vmatrix} = +8 \hat{k}

A slender uniform rod 100 cm long is used as a meter stick. Consider two parallel axes that are perpendicular to the rod. The first axis passes through the 50-cm mark and the second axis passes through the 30-cm mark. What is the ratio of the moment of inertia through the second axis to the moment of inertia through the first axis?

\[ \frac{I_2}{I_1} = 1.5 \]

\[ I_1 = \frac{1}{12} M(100)^2 \]
\[ I_2 = \frac{1}{12} M(100)^2 + M(20)^2 \]

The mass (M) cancels in the ratio.

Starting from rest, a disk rotates with constant angular acceleration. If it takes 10 revolutions to reach an angular velocity omega, then how many additional revolutions are required to reach an angular velocity 2omega?

30 rev

Recall the kinematic equation \( v_{\text{final}}^2 = v_{\text{initial}}^2 + 2a \cdot x \) (you can derive this from the time dependent equations)

The rotational analog is:
\[ \omega_{\text{final}}^2 = \omega_{\text{initial}}^2 + 2 \alpha \cdot \theta \]

To find alpha, we use the first part of the question:
omega^2 = 0 + 2 alpha * (2pi * 10)

this gives us: alpha = omega^2/40pi

Now we use this alpha in the second part

(2omega)^2 = omega^2 + 2*alpha*theta

3 omega^2 = 2 * alpha * theta

plug in what we know for alpha

3 omega^2 = omega^2/20 pi * theta

theta = 60 pi

Since 1 revolution is 2pi, this is 30 revolutions.

Your professor has put 5 objects near the top of an inclined plane: a uniform density solid sphere, a hollow sphere, a hollow cylinder, a special solid cylinder in which the density is proportional to the radius rho(r) = \alpha r, and a solid cube. The spheres and cylinders have radius R and will roll down the plane, while the cube has sides of length 2R and a frictionless coating, so it will slide down the plane. All objects are released at the same time. Which is the last to get to the bottom of the plane?

hollow cylinder. The object with the most kinetic energy of rotation will have the least kinetic energy of motions, and be the last one down.

Two identical bullets are fired horizontally
with the identical velocities.
One bullet is fired from a "rifled" barrel which makes the bullet spin along the axis of travel. The other bullet is fired from a smooth-bore barrel which imparts no spin. What is the downward drop due to gravity for these two bullets?

Both bullets drop the same amount. Gravity acts on the center of mass, it does not care about rotation.

You are standing in a field and see a 4000 kg airplane flying at horizontally at 250 m/s. The distance from you to the airplane is 2km, along a line 30 degrees from the horizontal. The figure shows the airplane, flying from right to left of the page. What is the magnitude and direction of the angular momentum of the airplane with respect to you, the observer?

\[ L = r \times p \sin(\theta) = 2000 \text{ m} \times (4000 \text{ kg}) \times (250 \text{ m/s}) \times 0.5 \]

\[ 10^9 \text{ kg m}^2 \]

This question was dropped from the exam all students got points.

A museum is building a scale model of the solar system. The planets are masses stuck to the end of long thin rods. These model systems (rod+mass) will be rotated by a motor at their other end of the rod, as shown in the figure. The length of the rod is proportional to the orbital distance of the planet, and the mass at the end is proportional to the mass of the planet. For this problem we focus on the moments of inertia of the Earth and Mars model systems, about the point of attachment to the motor.

- The rod is made of a material with mass density
• of 1.0 kg per meter.

• The Earth is represented by a rock of mass 1 kg,
  • at the end of a 1 meter rod.

• The mass of Mars is 0.1 \times \text{the mass of Earth}.

• The orbital radius of Mars is 1.5 \times \text{the orbital radius of Earth}.

How do the moments of inertia of the Earth ($I_E$) and Mars ($I_M$) model system compare? Pick the closest answer.

$I_E$ is approximately equal to $I_M$. The total I of either model is the I of the rod + I of the planet. The I_planet is simply $m \cdot d^2$ (it is a point mass at the end of the rod).

The body shown in the figure is pivoted at $O$ and two forces act on it as shown. If $r_1 = 1.30$ m and $r_2 = 2.15$ m, $F_1 = 4.20$ N and $F_2 = 4.90$ N, $\theta_1 = 75.0^\circ$ and $\theta_2 = 60.0^\circ$, what is the net torque about the pivot? Torque pointing out of the page is positive. \\

-3.85 N m

The dwarf planet Pluto's orbit around the sun is highly elliptical. Pluto's nearest (perihelion) and farthest (aphelion) distances from the Sun are 30 AU and 50 AU, respectively. (One AU or Astronomical Unit = the mean distance from the center of the Earth to the center of the Sun)
If its orbital speed at aphelion is 4 km/s, how fast is the Pluto moving when at perihelion?

between 5 km/s and 7 km/s. Note that angular momentum is conserved in this case. At perihelion and aphelion, the planet p is perpendicular to r, so L = r * p (since sin(90 degrees) = 1).

Since we are only going to compare, no need to change AU into m.

L_aphelion = m*50*4 = L_perihelion = m*30* v_perihelion.

The mass cancels, and we get v_perihelion = 200/30 = 6.7 m/s

The mass of Tesla Model S is 2000 kg and its engine produces 400 N m of torque. The Mercedes-Benz S Class is 50% heavier, M Benz = 1.5 M_Tesla and the Benz engine produces 50% more torque than the Tesla.

For this problem we consider those cars as solid cylinders of the same radius, rolling down the road.
What is the ratio of power generated by the engines 6 seconds after a start from rest? (ratio = Tesla Model S/Benz S Class)

2/3

Recall the kinematic equation for power: P = F (dot) v

The rotational analog is P = torque (dot) omega.

So we just need to find out what the omega is, 6 seconds after start.

omega = alpha * t = torque/I * t (where t = 6 seconds)

torque_Benz = torque_Tesla (Benz produces 50% more)

I_Benz = I_Tesla * 1.5 (Benz is 50% more massive, but R is same)

So alpha_Tesla = alpha_Benz, and omega (at 6 sec) is the same for both.
So $\frac{P_{\text{Tesla}}}{P_{\text{Benz}}} = \frac{\text{torque}_{\text{Tesla}}}{\text{torque}_{\text{Benz}}} = \frac{1}{1.5} = \frac{2}{3}$