**Microscopic Description**

- A real gas consists of a vast number of molecules, each moving randomly and undergoing millions of collisions every second.
  - Despite the apparent chaos, averages, such as the average number of molecules in the speed range 600 to 700 m/s, have precise, predictable values.
  - The “micro/macro” connection is built on the idea that the macroscopic properties of a system, such as temperature or pressure, are related to the average behavior of the atoms and molecules.
AN IDEAL GAS

- The distribution of molecular speeds in a sample of N₂ gas at 20°C

![Histogram of molecular speeds in N₂ gas at 20°C](image)

- Most likely speed:
  - 16% of the molecules have speeds between 600 m/s and 700 m/s.
- N₂ molecules at 20°C

![Bar chart showing percentage of molecules in different speed ranges](image)
AN IDEAL GAS

- Consider a container with volume $V$ containing $N$ particles of mass $m$
  - The particles bounce \textit{elastically} off the sides of the wall
  - Consider the 1 dimensional case first

- The force acting on the wall when \textit{one particle bounces} is:
  \[
  F = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{\Delta t}
  \]

- To determine what the \textit{total force from all of the collisions} is, we need to figure out how many such collisions take place
AN IDEAL GAS

• Consider a small part of the container surface, with area A, against which the particles are bouncing
  • If a particle is going to collide with the wall in a time $\Delta t$, then it must be at least a distance of $v_x \Delta t$ from the wall
• How many particles are in this volume?
  \[
  \frac{N}{V} = \frac{N_{\text{vol}}}{v_x \Delta t A} \rightarrow N_{\text{vol}} = \frac{N}{V} v_x \Delta t A
  \]
  • Let’s assume that half of them are moving towards the wall, and half away from the wall
  • Therefore the number of collisions against the wall is:
  \[
  N_{\text{col}} = \frac{1}{2} N_{\text{vol}} = \frac{1}{2} \frac{N}{V} v_x \Delta t A
  \]
AN IDEAL GAS

- Let’s put these two pieces together
  - The total force against the wall is the force from one collision times the number of collisions

\[
F_{\text{tot}} = F N_{\text{col}}
\]

\[
= \left( \frac{2m v_x}{\Delta t} \right) \left( \frac{1}{2} \frac{N}{V} v_x \Delta t A \right)
\]

\[
= m v_x^2 \frac{N}{V} A
\]

- Since the pressure is the total force per unit area, we find that

\[
p = \frac{F_{\text{tot}}}{A} = m v_x^2 \frac{N}{V}
\]

- Is this starting to look like the ideal gas law…?
An Ideal Gas

- The next step in deriving the ideal gas law is to extrapolate from 1 dimension to 3 dimensions.
  - To do this, remember that the particles aren’t all traveling with velocity $v_x$.
    - If half are traveling with $v_x$ and half with $-v_x$, then the average $v_x$ is 0!
    - Better to think about the average of $|v_x|$ or $v_x^2$, which is non-zero.
- So now, let’s assume that $v_x$, $v_y$, and $v_z$ should be treated on equal footing.
  - That is, the average value of $(v_x)^2$, $(v_y)^2$, and $(v_z)^2$ are all the same.
    \[
    (v^2)_{\text{avg}} = (v_x^2)_{\text{avg}} + (v_y^2)_{\text{avg}} + (v_z^2)_{\text{avg}} = 3(v_x^2)_{\text{avg}}
    \]
  - Similarly, the average translational kinetic energy is
    \[
    K_{\text{avg}} = \frac{1}{2}m(v^2)_{\text{avg}} = \frac{3}{2}m(v_x^2)_{\text{avg}}
    \]
**An Ideal Gas**

- Therefore we have:
  
  \[ p = m \left(v_x^2\right)_{\text{avg}} \frac{N}{V} = \frac{2}{3} K_{\text{avg}} \frac{N}{V} \rightarrow pV = \frac{2}{3} NK_{\text{avg}} \]

- The final step in “deriving” the ideal gas law is to relate the average kinetic energy to the temperature

  - It turns out that:

  \[ \frac{1}{2} m (v^2)_{\text{avg}} = \frac{3}{2} k_B T \]

  - where \( T \) is the temperature and \( k_B \) is Boltzmann’s constant
    \( (k_B=1.381\times10^{-23} \text{ J} \cdot \text{K/molecule}) \)

  - Boltzmann’s constant is the constant of proportionality relating the average kinetic energy to the temperature
AN IDEAL GAS

• Finally, this means that

\[ pV = Nk_B T \]

• For those used to the “chemistry version” of this equation

\[ R = k_B N_A \]

• where \( R \) is the “gas constant” 8.3145 J·K/mol
• and \( N_A \) is Avogadro’s number 6.022x10^{23} molecules/mol

• Therefore,

\[ pV = Nk_B T = \frac{N}{N_A} RT = nRT \]

• where \( n \) is the number of moles of the molecule
Macro-Micro Connection

- Assumptions for ideal gas:
  - # of molecules $N$ is large
  - they obey Newton’s laws
  - thermal equilibrium
  - elastic collisions with walls and each other

- What we call temperature $T$ is a direct measure of the average translational kinetic energy

- What we call pressure is a direct measure of the number density of particles and how fast they are moving

\[
T = \frac{2}{3k_B} K_{\text{avg}}
\]

\[
p = \frac{2}{3} \frac{N}{V} K_{\text{avg}}
\]

\[
u_{\text{rms}} = \sqrt{\langle v^2 \rangle_{\text{avg}}} = \sqrt{\frac{3k_B T}{m}}
\]
**Ideal Gas Simulation**

Two cylinder are filled to the same height $H$ with ideal gas. The gases are different, and the cross-sectional areas of the cylinders are different. Both cylinders have pistons that are free to move without friction.

How does temperature of the gas in cylinder A compare to the temperature of the gas in cylinder B?

1. $T_A = \frac{1}{2} T_B$
2. $\frac{1}{2} T_A = T_B$
3. $\frac{1}{4} T_A = T_B$
4. $T_A = \frac{1}{4} T_B$
Two cylinders are filled to the same height $H$ with ideal gas. The gases are different, and the cross-sectional areas of the cylinders are different. Both cylinders have pistons that are free to move without friction.

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B has twice the area that A has, but it has twice the force, so pressure is the same for both. $n$ is also the same for both. Thus, B has twice the volume, so it has twice the temperature.
**IDEAL GAS IN CYLINDERS (II)**

Cylinders with equal cross-sectional areas contain different volumes of an ideal gas sealed in by pistons. There is a weight sitting on top of each piston. The gas is the same in all four cases and is at the same temperature. The pistons are free to move without friction.

![Diagram of cylinders with different volumes and weights]

**Rank the pressure of the gas in each cylinder.**

A. $B > C > D > A$
B. $D > B > A > C$
C. $D > B = A > C$
D. $B = C > A = D$
E. We need to know how many moles of gas are in each piston to answer this.
Cylinders with equal cross-sectional areas contain different volumes of an ideal gas sealed in by pistons. There is a weight sitting on top of each piston. The gas is the same in all four cases and is at the same temperature. The pistons are free to move without friction.

p = F/A. A is the same everywhere, so the pressure should be ranked by the forces applied. All other information are distractors.

Rank the pressure of the gas in each cylinder.

A. B > C > D > A
B. D > B > A > C
C. D > B = A > C  \[\text{Correct Answer}\]
D. B = C > A = D
E. We need to know how many moles of gas are in each piston to answer this.
The probability density to find a particle in a gas at a given velocity is given by the equation

\[ f(v) = 4\pi \left( \frac{m}{2\pi k_B T} \right)^{3/2} v^2 \exp \left( \frac{-mv^2}{2k_B T} \right) \]

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases
Kelvin Scale

- Since we have a picture of temperature as kinetic energy of particles, we now can appreciate Kelvin as a scale
  - The temperature in kelvin is related to the temperature in Celsius by: \( T_K = T_C + 273.15 \)
    - Hint: don’t forget to measure temperature ratios in kelvin (temperature differences in either kelvin or celsius are fine)

- Zero kelvin (aka “absolute zero”) corresponds to the point where the matter has no motion whatsoever
INTERMOLECULAR FORCES

• We obtained the ideal-gas equation from a simple molecular model
  • This ignored many different factors, among which are the size of the molecules themselves and the attractive forces between them

• van der Waals equation accounts for these effects empirically:
  \[
  \left( p + \frac{an^2}{V^2} \right) (V - nb) = nRT
  \]
  • a and b are empirical constants that differ for different gases
  • “b” represents the volume of a mole of molecules so that V-nb represents how much volume there is left of empty space
  • “a” represents the force of attraction between particle species effectively reducing the force of collision against the walls
INTERNAL ENERGY

- Summing up the energy of the particles is the same as multiplying the average energy per particle times the total number of particles
  - This total energy is called the internal energy of the system

- Energy can be added to/removed from the system either by doing work on/by the system or heating/cooling the system
  - This is the first law of thermodynamics
HEATING AND TEMPERATURE

• Q: What happens to matter when it is heated or cooled?
  • “heating” and “cooling” are processes which add or take away energy from the system
  • so they increase or decrease the kinetic energy of the particles in an ideal gas (at a constant volume)

• Q: What about other matter, like solids and liquids?
  • There is electrical potential energy involved between particles, and so the response to heating is more complicated…
PHASES OF MATTER

Solid
- Holds Shape
- Fixed Volume

Liquid
- Shape of Container
- Free Surface
- Fixed Volume

Gas
- Shape of Container
- Volume of Container

Arrangement of the molecules:
- Solid
- Solid/liquid mixture
- Liquid
- Liquid/gas
- Gas
Heat Transfer

- **Conduction** is the transfer of heat through kinematic interactions in a material
  - Metals can conduct heat in much the same way that they conduct current (electrons that move freely within the atomic lattice)
  - Don’t confuse conductivity with heat capacity

- **Convection** is the transfer of heat by the movement of fluid from one region to another

- **Radiation** is the transfer of heat by electromagnetic waves such as visible light, infrared, and ultraviolet radiation