Analytical Physics 1B Lecture 12: 2nd Law of Thermodynamics

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FUNDAMENTALS OF HEAT ENGINES

• Heat engines cycle
  • absorb heat from a hot reservoir
  • perform mechanical work
  • discard heat at a lower temperature
  • return to the initial state

• A hot reservoir provides a substantial amount of heat without changing its own temperature
  • A cold reservoir can absorb heat without changing temperature
  • The heat provided at temperature $T_H$ is $Q_H$
  • The heat discarded at temperature $T_C$ is $Q_C$

• According to the 1st law, the work performed must be $|Q_H| - |Q_C|$
HEAT ENGINE EFFICIENCY

• It would be ideal if we could convert all of the heat into work done
  • Unfortunately, however, that is impossible
  • The thermal efficiency of a heat engine is determined by the fraction of heat that gets converted into work:

\[ e = \frac{W}{Q_H} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \left| \frac{Q_C}{Q_H} \right| \]

• There is a maximum possible efficiency, the Carnot efficiency (more on that later)
**Refrigerators**

- Refrigerators are like heat engines in reverse
  - takes heat from a cold place and moves it to a warm place using mechanical work
  - Here, $Q_C > 0$, $Q_H < 0$ and $W < 0$

- The **coefficient of performance** for a refrigerator is a measure of how much heat is removed given some amount of work

$$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$$
**The Otto Cycle**

- **Intake stroke**: Piston moves down, causing a partial vacuum in cylinder; gasoline-air mixture enters through intake valve.
- **Compression stroke**: Intake valve closes; mixture is compressed as piston moves up.
- **Ignition**: Spark plug ignites mixture.
- **Power stroke**: Hot burned mixture expands, pushing piston down.
- **Exhaust stroke**: Exhaust valve opens; piston moves up, expelling exhaust and leaving cylinder ready for next intake stroke.
**The Otto Cycle**

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The cycle can be visualized as:

1. **Adiabatic compression (compression stroke)**
2. **Heating at constant volume (fuel combustion)**
3. **Adiabatic expansion (power stroke)**
4. **Cooling at constant volume (cooling of exhaust gases)**
Otto Cycle Efficiency

- The efficiency is related to the heat added and removed, $Q_H$ and $Q_C$
- Let’s assume an ideal gas
  - Processes $b \rightarrow c$ and $d \rightarrow a$ are performed at constant volume, so for an ideal gas
    
    $$Q_H = nC_V(T_c - T_b)$$
    $$Q_C = nC_V(T_a - T_d)$$

  - notice that $Q_C < 0$ and $Q_H > 0$
- Therefore, the efficiency is
  
  $$e = 1 - \left| \frac{Q_C}{Q_H} \right| = 1 - \left| \frac{T_a - T_d}{T_c - T_b} \right|$$
• As we learned from last lecture:
  \[ pV^\gamma = \text{constant} \]
  \[ TV^{\gamma-1} = \text{constant} \]
  \(<\text{just plug } pV=nRT \text{ into prev. equation}\)

• where
  \[ \gamma \equiv \frac{C_P}{C_V} \]

• Therefore, from the \(c\to d\) process
  \[ T_c V^{\gamma-1} = T_d (rV)^{\gamma-1} \rightarrow T_c = T_d r^{\gamma-1} \]

• and from the \(a\to b\) process
  \[ T_b V^{\gamma-1} = T_a (rV)^{\gamma-1} \rightarrow T_b = T_a r^{\gamma-1} \]
Otto Cycle Efficiency

- From before we have

\[ e = 1 - \frac{T_a - T_d}{T_c - T_b} \]

\[ T_c = T_d r^{\gamma-1} \]

\[ T_b = T_a r^{\gamma-1} \]

- Plugging these into each other we get

\[ e = 1 - \frac{T_a - T_d}{T_d r^{\gamma-1} - T_a r^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}} \]

- \( \frac{C_p}{C_v} = 1.4 \) for a typical gas, and \( r = 8 \) for many modern engines
  - therefore \( e = 56\% \)
  - (more like 30\% in real life)
STIRLING ENGINES

1. Power piston (dark grey) has compressed the gas, the displacer piston (light grey) has moved so that most of the gas is adjacent to the hot heat exchanger.

2. The heated gas increases in pressure and pushes the power piston to the farthest limit of the power stroke.

3. The displacer piston now moves, shunting the gas to the cold end of the cylinder.

4. The cooled gas is now compressed by the flywheel momentum. This takes less energy, since its pressure drops when it is cooled.
2nd Law of Thermodynamics

• It is impossible for any system to undergo a process in which it absorbs heat from a reservoir at a single temperature and converts the heat completely into mechanical work, with the system ending in the same state in which it began
  • This is the “engine” version of the 2nd law
  • This tells us that there is a limit to how efficient an engine can be

• Similarly, it is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter body
  • This is the “refrigerator” version of the 2nd law

• Both of these statements have to do with a more fundamental principle known as entropy
**Entropy**

- Qualitatively, entropy is the measure of disorder in a system
  - Quantitatively, it is measure of how many states are available to a system
  - For instance, your socks could be anywhere in your house, but it is only “ordered” when they are in your dresser drawer (or on your feet)

- Because the number of ways a system could be disordered is so much greater than the number of ways a system could be ordered, we observe that systems tend towards disorder
  - For a gas with many particles, the particles interact so that very quickly (once equilibrium is established) the disorder has been maximized
  - In this picture, the 2nd Law of Thermodynamics is that the entropy of a system always increases with time
REVERSIBLE PROCESSES

- Reversible processes are those which can be reversed by making only an infinitesimal change to the system
  - These don’t exist perfectly in reality, but they can be approximated
  - A system that undergoes an idealized reversible process is always very close to being in thermodynamic equilibrium within itself and with its surroundings
- In a reversible process, the total change in entropy for the system is 0

(a) A block of ice melts **irreversibly** when we place it in a hot (70°C) metal box.
(b) A block of ice at 0°C can be melted **reversibly** if we put it in a 0°C metal box.

Heat flows from the box into the ice and water, never the reverse.

By infinitesimally raising or lowering the temperature of the box, we can make heat flow into the ice to melt it or make heat flow out of the water to refreeze it.
Carnot Cycle

- The gas expands isothermally at temperature $T_H$, absorbing heat $Q_H$ (ab).
- It expands adiabatically until its temperature drops to $T_C$ (bc).
- It is compressed isothermally at $T_C$, rejecting heat $|Q_C|$ (cd).
- It is compressed adiabatically back to its initial state at temperature $T_H$ (da).
Carnot Cycle

- A Carnot engine is maximally efficient

\[ e = 1 - \frac{T_C}{T_H} \]

- The efficiency depends only on the temperature of the reservoirs
- The larger the difference in temperature, the more efficient the engine
**Entropy Change**

• If an infinitesimal amount of heat $dQ$ is added to a system in an infinitesimal reversible process,

\[ \Delta S = \int_{1}^{2} \frac{dQ}{T} \]

• Adding heat increases the number of available states
  • If the system is already at a high temperature, adding heat does not increase the randomness of the motion as much as if it were at a lower temperature
  • Didn’t I say a few slides back that the change in entropy is 0 for a reversible process?
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- Didn’t I say a few slides back that the change in entropy is 0 for a reversible process?
  - For the example of the melting of ice in a box, the box loses entropy, while the ice gains entropy (turning into water)
  - The total change in entropy for the entire system must cancel in a reversible process