Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 8
• **Dynamics of uniform circular motion**

**Centripetal acceleration:**

\[ \vec{a} = -\frac{v^2}{r} \hat{r} \]

- \( r \) = radius of the circle
- \( v \) = speed
- \( \hat{r} = \frac{\vec{r}}{r} \)  \textbf{unit} radial vector
Newton’s 2nd law:

\[ \vec{F}_{\text{net}} = m\vec{a} \]

\[ \Downarrow \]

\[ \vec{F}_{\text{net}} = -\frac{mv^2}{r} \]

**Centripetal Force**

A centripetal force accelerates a body by changing the direction of the body’s velocity without changing the body’s speed.
Example

A ball attached to a string rotates in a vertical plane near Earth’s surface such that the string is stretched taut all points on the circular path. What is the minimum value of its speed at the highest point \( A \) of the trajectory.

\[
A) \quad v_{\text{min}} = 0 \\
B) \quad v_{\text{min}} = \sqrt{rg} \\
C) \quad v_{\text{min}} = \sqrt{rg/2} \\
D) \quad v_{\text{min}} = \sqrt{2gr} \\
E) \quad \text{none of the above}
\]
\[ \vec{F}_{\text{net}} = -\frac{mv^2}{r} \hat{r} \quad \vec{F}_{\text{net}} = \vec{F}_g + \vec{T} \]

\( (F_{\text{net}})_y = -(mg + T) = -\frac{mv^2}{r} \)

\[ T = \frac{mv^2}{r} - mg \geq 0 \]

\[ v^2 \geq rg \quad \Rightarrow \quad v_{\text{min}} = \sqrt{rg} \]
What is the tension $T$ at the lowest point?

\[
m\vec{a}_c = \vec{T} + \vec{F}_g
\]

\[
\frac{mv^2}{R} = T - mg
\]

\[
T = mg + \frac{mv^2}{R}
\]
Is the motion **uniform** (constant speed)?

Does the net force have a **tangential** component?
Radial:
\[(F_{\text{net}})_y = -T - mg \sin(\theta)\]

Tangential:
\[(F_{\text{net}})_x = mg \cos(\theta)\]

Newton’s 2nd law
\[T + mg \sin(\theta) = ma_c\]
\[mg \cos(\theta) = ma_t\]
\[\theta \neq 90^\circ \Rightarrow a_t \neq 0\]
General circular motion

\[ \vec{a} = \vec{a}_c + \vec{a}_t \]

\[ \vec{a}_c = -\frac{v^2}{r} \hat{r} \]

Note: \( v \) and \( a_t \) may depend on angular displacement and time
**Example:** A ball attached to a string of length \( l \), which makes an angle \( \theta \) with the vertical, rotates uniformly in a horizontal plane as shown below. Find the speed \( v \).
\[ \vec{F}_{\text{net}} = m\vec{a} \quad \vec{F}_{\text{net}} = \vec{F}_g + \vec{T} \]

\[
(F_{\text{net}})_x = T\sin \theta \\
(F_{\text{net}})_y = -mg + T\cos \theta
\]

\[
a_x = \frac{v^2}{r} \quad a_y = 0
\]

\[
T\sin \theta = \frac{mv^2}{r} = \frac{mv^2}{l\sin \theta}
\]

\[
T\cos \theta = mg
\]

\[
\frac{v^2}{l\sin \theta} = \tan \theta \quad \Rightarrow \quad v = \sin \theta \sqrt{\frac{lg}{\cos \theta}}
\]
**Example:** a car of mass $m$ moving with constant speed makes a turn of radius $r$. Suppose the static friction coefficient between the wheels and the road is $\mu_s$. Find the maximum value of the speed $v$ such that the car will remain on the road.
\[ \vec{F}_{\text{net}} = m\vec{a} \quad \vec{a} = -\frac{mv^2}{r}\hat{i} \]

\[ F_N - mg = ma_y = 0 \]

\[ -f_s = max = -\frac{mv^2}{r} \]

\[ f_s \leq \mu_s F_N \implies \frac{mv^2}{r} \leq \mu_s mg \]

\[ v \leq \sqrt{\mu_s rg} \]
7. Kinetic energy and work

- What is energy?

- **Basic Idea:** scalar quantity with quantifies the state of motion and/or the capacity for motion of a system

- **Conserved** as the state of motion of the system changes.
Speed of arrow in flight?

Speed of train at the lowest point?
Energy

\{ 
\begin{align*}
&\text{• Kinetic energy: associated to motion} \\
&\text{• Potential energy: associated to the capacity of generating motion}
\end{align*}
\}

\textbf{Example:} freely falling ball from height \( h \).

\begin{itemize}
\item initial speed: \( v_0 = 0 \)
\item final speed: 
\[ v^2 = v_0^2 + 2gh = 2gh \]
\end{itemize}
• initially: potential energy: $mgh$

• finally: kinetic energy: $\frac{mv^2}{2}$

• Energy conservation:

$$mgh = \frac{mv^2}{2}$$
• **Kinetic Energy**: energy associated to the motion of an object

\[ K = \frac{1}{2}mv^2 \]

**Unit**: Joule

1 J = 1 kg × m²/s²
Non-zero force

⇓

Acceleration: change in velocity

⇓

Change in kinetic energy

How does $K$ change under an applied force?
• **Work**: energy transferred to or from an object by means of a force acting on the object.

Energy transferred to the object is **positive work** while energy transferred from the object is **negative work**.
How do we calculate the work of a force?

- **Example:** bead on frictionless rod subject to constant force $\vec{F}$

Newton's 2nd law:

$$F_x = ma_x$$

Constant acceleration model:

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x$$

$$\frac{mv_x^2}{2} - \frac{mv_{0x}^2}{2} = F_x \Delta x$$
• Work done by the force $\vec{F}$

$$F d \cos \phi = \vec{F} \cdot \vec{d}$$

$$\vec{d} = (\Delta x) \hat{i}$$

displacement vector

$$F d \cos \phi \begin{cases} > 0, \text{ for } 0 \leq \phi < \pi/2 & \text{positive work, } K_f > K_i \\ = 0, \text{ for } \phi = \pi/2 & \text{zero work, } K_f = K_i \\ < 0, \text{ for } \pi/2 < \phi \leq \pi & \text{negative work, } K_f < K_i \end{cases}$$
• **Net work:**

When two or more forces act on an object, the net work done on the object is the sum of the works done by the individual forces.

\[
W_{\text{net}} = \sum W = \sum \vec{F} \cdot \vec{d} = (\sum \vec{F}) \cdot \vec{d} = \vec{F}_{\text{net}} \cdot \vec{d}
\]
• **Work-Kinetic Energy Theorem**

\[
\Delta K = W = K_f - K_i = W
\]
What if the trajectory is curved?

\[ ds = v \, dt \]
\[ d \vec{s} = \vec{v} \, dt \]

\[
\frac{dK}{dt} = \vec{F}_{\text{net}} \cdot \frac{d\vec{s}}{dt}
\]

\[ dK = \vec{F}_{\text{net}} \cdot d\vec{s} \]

infinitesimal work

\[ \Delta K = \int_{\text{trajectory}} \vec{F} \cdot d\vec{s} \]
• **Work done by the gravitational force**

Freely falling object moving downwards:

\[ W = \vec{F}_g \cdot \vec{d} = m g d \cos 0^\circ = m g d > 0 \]

\[ K_1 = \frac{mv_1^2}{2} \quad K_2 = \frac{mv_2^2}{2} \]

\[ K_2 - K_1 = m g d > 0 \]
Freely falling object moving upwards:

\[ W = \vec{F_g} \cdot \vec{d} = mgd \cos 180° = -mgd > 0 \]

\[ K_1 = \frac{mv_1^2}{2} \quad K_2 = \frac{mv_2^2}{2} \]

\[ K_2 - K_1 = -mgd < 0 \]
\[ \vec{F}_{net} = m\vec{a} \]
\[ (F_{net})_x = ma_x \quad (F_{net})_y = ma_y = 0 \]
\[ \vec{F}_{net} = \vec{F}_g + \vec{F}_N + \vec{f}_k \]
\[ (F_{net})_x = -mg\sin\theta - f_k \quad (F_{nety})_y = F_N - mg\cos\theta = 0 \]
\[ F_N = mg\cos\theta \quad f_k = \mu_k mg\cos\theta \]
Work done by kinetic friction:
\[ W_{f_k} = \vec{f}_k \cdot \vec{d} = -f_k d = -\mu_k mgd \cos \theta \]

Work done by gravitational force:
\[ W_{F_g} = \vec{F}_g \cdot \vec{d} = -mgh = -mgds \sin \theta \]

Work done by normal force
\[ \vec{F}_N \cdot \vec{d} = 0 \]
Total work:
$$W = W_f + W_g = -mgd(\sin\theta + \mu_k \cos\theta)$$

Work-Kinetic energy theorem:
$$W = \Delta K = 0 - K_i = -\frac{mv_0^2}{2}$$

Work done by kinetic friction:
$$\frac{W_{f_k}}{|W|} = -\frac{\mu_k \cos\theta}{\sin\theta + \mu_k \cos\theta} = \frac{1}{3}$$
$$W_{f_k} = -\frac{mv_0^2}{6}$$