Midterm 1

• Sunday October 14th 12:00-1:20pm
• Physics Lecture Hall
• Lectures 1-4 (one dim motion – 2d and 3d motion, relative motion)
• No forces!
• Practice exam solutions posted online at http://www.physics.rutgers.edu/ugrad/123H/Exams/
5. Forces and Motion I

- **Force:** ~ physical effect which changes the velocity of an object

- **Newton’s 1st law**

  If no net force acts on a body,
  \[ \vec{F}_{\text{net}} = 0, \]
  the body’s velocity cannot change;
  the body cannot accelerate.
What is the main effect of a nonzero net force?

- An applied force can set a static object in motion or can stop a moving object.

\[ \downarrow \]

change in velocity, hence acceleration.
What is the relation between the net force acting on an object and the resulting acceleration?

Newton’s second law

\[ \vec{F}_{\text{net}} = m\vec{a}, \]

- \( m \)=Mass: intrinsic characteristic of a body relating a force on the body to the resulting acceleration.
Newton’s Third Law

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

- **Inertial System**: reference system where Newton’s laws are valid
Tension, cords, pulleys

Ideal cord: massless, unstretchable

Ideal pulley: massless, frictionless

The forces at the two ends are always equal in magnitude.
Problem 58 Ch. 5: What magnitude of the pull force is needed in order to climb with constant acceleration $a$?
Combined mass of man and cage:

$$m = 95 \text{ kg}$$
Suppose \( \vec{T} \) is the tension force in the rope acting on the man and \( \vec{F} \) is the pull force acting on the rope. What is the relation between them?

A) \( \vec{T} - \vec{F} = 0 \)
B) \( \vec{T} + \vec{F} = 0 \)
C) \( 2\vec{T} + \vec{F} = 0 \)
D) \( \vec{T} - 2\vec{F} = 0 \)
Answer

Suppose \( \vec{T} \) is the tension force in the rope acting on the man and \( \vec{F} \) is the pull force acting on the rope. What is the relation between them?

A) \( \vec{T} - \vec{F} = 0 \)

B) \( \vec{T} + \vec{F} = 0 \)

C) \( 2\vec{T} + \vec{F} = 0 \)

D) \( \vec{T} - 2\vec{F} = 0 \)
Suppose the gravitational force acting on the combined system man + cage is $\vec{G}$. What is the net force acting on this system?

A) $\vec{T} + \vec{G}$

B) $\vec{T}$

C) $\vec{G}$

D) $2\vec{T} + \vec{G}$

E) $\vec{T} + \vec{G} + \vec{F}$
Answer

Suppose the gravitational force acting on the combined system man + cage is $\vec{G}$. What is the net force acting on this system?

A) $\vec{T} + \vec{G}$
B) $\vec{T}$
C) $\vec{G}$
D) $2\vec{T} + \vec{G}$
E) $\vec{T} + \vec{G} + \vec{F}$
Newton’s second law

\[ 2\vec{T} + \vec{G} = m\vec{a} \]

Newton’s third law

\[ \vec{T} + \vec{F} = 0 \]
Projection onto $y$-axis

$$2T - mg = ma_y \Rightarrow F = \frac{m(a_y + g)}{2}$$

$$a_y = 0 \Rightarrow F = mg/2 = 47.5 \times 9.8 = 465.5 \text{ N}$$

In order to climb with $a_y = 1 \text{ m/s}^2$

$$F = 47.5 \times 10.8 = 513 \text{ N}.$$ 

What if the man climbed the rope without the aid of a pulley?

$$\vec{T} + \vec{G} = m\vec{a}, \quad \vec{F} + \vec{T} = 0$$

$$T - mg = ma_y \Rightarrow F = m(a_y + g)$$

Hence

$$a_y = 0 \Rightarrow F = 95 \times 9.8 = 931 \text{ N}$$

$$a_y = 1 \text{ m/s}^2 \Rightarrow F = 95 \times 9.8 = 1026 \text{ N}$$
• Example:

- The object of mass $m_2$ hangs at the end of an ideal cord tied to the object of mass $m_1$.
- The object of mass $m_1$ is placed on a frictionless inclined plane.
- The cord is wrapped around an ideal pulley attached to the plane.
- Find the acceleration $\vec{a}_2$. 

![Diagram of objects and pulley](image-url)
**Step 1:** free body diagram for object 1

\[ \vec{F}_{\text{net1}} = m_1 \vec{a}_1 \]

\[ \vec{F}_{\text{net1}} = \vec{T}_1 + \vec{F}_{g1} + \vec{F}_N \]

\[ (F_N)_x = 0, \quad (F_{g1})_x = -m_1gsin\alpha \]

\[ T_1 - m_1gsin\alpha = m_1a_{1x} \]

\[ a_{1y} = 0 \text{ (no motion \ perpendicular to plane)} \]

\[ m_1a_{1x} = T_1 - m_1gsin\alpha \]
**Step 2:** free body diagram for object 2

\[
\vec{F}_{\text{net}2} = m_2 \vec{a}_2
\]

\[
\vec{F}_{\text{net}2} = \vec{T}_2 + \vec{F}_{g2}
\]

\[
(F_{\text{net}2})_x = 0 \quad (F_{\text{net}2})_y = T_2 - m_2g
\]

\[
m_2 a_{2x} = 0 \quad m_2 a_{2y} = T_2 - m_2g
\]
Step 3: put equations together

\[ m_1 a_{1x} = T_1 - m_1 g \sin \alpha \]  
Can we solve?

\[ m_2 a_{2y} = T_2 - m_2 g \]  
Need extra conditions!
i-Clicker

What is the relation between the magnitudes of the two tension forces?

A) $T_1 = T_2$

B) $T_2 = T_1 \sin(\alpha)$

C) $T_2 = T_1 \cos(\alpha)$

D) No relation.
Answer

What is the relation between the magnitudes of the two tension forces?

A) $T_1 = T_2$

B) $T_2 = T_1 \sin(\alpha)$

C) $T_2 = T_1 \cos(\alpha)$

D) No relation.
i-Clicker

What is the relation between the magnitudes of the acceleration vectors?

A) No relation.
B) $a_2 = a_1 \sin(\alpha)$
C) $a_2 = a_1 \cos(\alpha)$
D) $a_2 = a_1$
Answer

What is the relation between the magnitudes of the acceleration vectors?

A) No relation.

B) \( a_2 = a_1 \sin(\alpha) \)

C) \( a_2 = a_1 \cos(\alpha) \)

D) \( a_2 = a_1 \)

Kinematic constraint:

\[
(\Delta x)_1 = -(\Delta y)_2 \text{ at all times}
\]

\[
\downarrow
\]

\[
a_{1x} = -a_{2y}
\]
\[ m_1 a_{1x} = T_1 - m_1 g \sin \alpha \quad \text{Can we solve?} \]

\[ m_2 a_{2y} = T_2 - m_2 g \quad \text{Yes!} \]

\[ T_1 = T_2 \]

\[ a_{1x} = -a_{2y} \]

\[ (m_1 + m_2) a_{2y} = m_1 g \sin \alpha - m_2 g \]

\[ a_{2y} = \frac{m_1 g \sin \alpha - m_2 g}{m_1 + m_2} \]
i-Clicker

Which way will box 2 move? Up or down?

A) Up.
B) Down.
C) Depends on the data.
D) Cannot be determined.
Answer

Which way will box 2 move? Up or down?
A) Up.
B) Down.
C) Depends on the data. \[ a_{2y} = \frac{m_1 g \sin \alpha - m_2 g}{m_1 + m_2} \]
D) Cannot be determined.

\[ a_{2y} > 0 \iff m_1 \sin \alpha > m_2 \quad \text{Up} \]

\[ a_{2y} < 0 \iff m_1 \sin \alpha < m_2 \quad \text{Down} \]

\[ a_{2y} = 0 \iff m_1 \sin \alpha = m_2 \quad \text{No motion (if initial velocity is 0)} \]
6. Forces and Motion II

- **Friction**

- Suppose an object is launched with velocity \( \vec{v}_0 \) along a surface.

- Newton’s 1st law: if \( \vec{F}_{\text{net}} = 0 \) it should move forever with constant velocity \( \vec{v}_0 \).

- Does it really happen in practice?
• **Experiment:** a toy truck is quickly accelerated to some velocity $\vec{v}_0$ and then released.

It stops in finite time; $x$ vs time $\sim$ parabola

$v_x$ vs time $\sim$ linear once it starts deccelerating $\Rightarrow$ constant negative acceleration
What force is necessary to set a static object in motion and then maintain constant velocity?

**Static**

**In motion**
• **Static** friction \( \vec{f}_s \) acts when no \textit{relative motion} between object and contact surface.

• The magnitude \( f_s \) increases as the applied force increases until it reaches a maximum value \( (f_s)_{\text{max}} \).

• **Kinetic** friction \( \vec{f}_k \) acts when there is \textit{relative motion} between object and contact surface.

• Usually \( f_k < (f_s)_{\text{max}} \).
Static

Dynamic

Transition

F > f_k

F = f_s

F = f_k

F = (f_s)_{max} > f_k

F = f_k

f_k is approximately constant

Magnitude of frictional force

Time

Breaking away

Max value of f_s

F = f_s

F = f_k

F_N

F

F_s

F_k

F_g
Properties of friction

1. For a stationary object \( \vec{F} + \vec{f}_s = 0 \) i.e. the applied force and the static friction balance each other out.

2. The magnitude of static friction has a maximum value depending on the magnitude of the normal force \( F_N \):
   \[(f_s)_{\text{max}} = \mu_s F_N\]
   where \( \mu_s = \text{coefficient of static friction} \). When \( F \geq (f_s)_{\text{max}} \) the object begins to slide.

3. When sliding begins, the magnitude of friction rapidly decreases to a value
   \[f_k = \mu_k F_N, \quad \mu_k = \text{coefficient of kinetic friction}\]
An object is placed on an inclined plane of angle $\theta$. The coefficient of static friction between at the contact surface is $\mu_s$. For which values of $\theta$ will the object stay on the plane.

A) For all values of $\theta$.

B) There is no such value of $\theta$.

C) For $\tan \theta \leq \mu_s$.

D) For $\sin \theta \leq \mu_s$.

E) For $\tan \theta > \mu_s$. 
Answer

An object is placed on an inclined plane of angle $\theta$. The coefficient of static friction between at the contact surface is $\mu_s$. For which values of $\theta$ will the object stay on the plane.

A) For all values of $\theta$.
B) There is no such value of $\theta$.
C) For $\tan \theta \leq \mu_s$.
D) For $\sin \theta \leq \mu_s$.
E) For $\tan \theta > \mu_s$. 
The object will stay on the plane if

\[ F_{gx} = f_s \]

\[ F_{gx} = mg\sin\theta \quad f_s \leq (f_s)_{max} = \mu_s F_N \]

\[ F_N = -F_{gy} = mg\cos\theta \]

\[ mg\sin\theta \leq \mu_s mg\cos\theta \quad \Rightarrow \quad \tan\theta \leq \mu_s. \]
• **Example:** emergency braking

A car slides 290 m on pavement with wheels locked. Assuming $\mu_k = 0.6$ and constant acceleration, what is $v_0$?
Newton’s 2nd law:

\[ \vec{F}_{\text{net}} = m\vec{a} = ma_x \hat{i} \]  
\( \text{no vertical motion} \)

\( x \)-axis:  \( ma_x = -\mu_k F_N \)  
\( y \)-axis:  \( 0 = F_N - mg \)
\[ x - \text{axis: } m a_x = -\mu_k F_N \quad y - \text{axis: } 0 = F_N - mg \]

\[ a_x = -\mu_k g \]

**Constant acceleration model:**

\[ v^2 = v_0^2 - 2\mu_k g(x - x_0) \]

\[ v = 0 \implies v_0^2 = 2\mu_k g(x - x_0) = 58 \text{ m/s} = 210 \text{ km/h}. \]
Example:

A horizontal tension force $\vec{T}$ is applied to an object of mass $m$ on an inclined plane.

The object moves upwards along the plane with constant acceleration $\vec{a}$.

Find the kinetic friction coefficient $\mu_k$ between the object and the plane.

For which values of $T, a$ is this motion possible?
Newton’s 2nd law:

\[ \vec{F}_{\text{net}} = m\vec{a} = ma \hat{i} \quad \vec{F}_{\text{net}} = \vec{T} + \vec{F}_g + \vec{F}_N + \vec{f}_k \]

\[ (F_{\text{net}})_x = T\cos\theta - mg\sin\theta - f_k \]

\[ (F_{\text{net}})_y = -T\sin\theta - mg\cos\theta + F_N \]
\[(F_{\text{net}})_x = ma \quad (F_{\text{net}})_y = 0\]

\[(F_{\text{net}})_x = T\cos\theta - mg\sin\theta - f_k\]

\[(F_{\text{net}})_y = -T\sin\theta - mg\cos\theta + F_N\]

\[f_k = T\cos\theta - mg\sin\theta - ma, \quad f_k = \mu_k F_N\]

\[F_N = T\sin\theta + mg\cos\theta\]

\[\mu_k = \frac{T\cos\theta - mg\sin\theta - ma}{T\sin\theta + mg\cos\theta}\]
For which values of $T, a$ is the motion possible?

\[ \mu_k = \frac{T \cos \theta - mg \sin \theta - ma}{T \sin \theta + mg \cos \theta} \geq 0 \]

\[ T \cos \theta \geq m(g \sin \theta + a) \]
• **Drag force and terminal speed**

![Image of a person snowboarding](image1.jpg)

![Image of a person fishing](image2.jpg)

**Drag force:**

• force caused by relative motion of an object and a fluid
• opposed the relative motion and points in the direction the fluid flows relative to the object
For a fast blunt object moving through air such that the flow becomes **turbulent**

\[ D = \frac{1}{2} C \rho A v^2 \]

- \( A = \text{effective crosssection}: \text{area of crosssection } \perp \vec{v} \)
- \( C = \text{drag coefficient} \)
- \( \rho = \text{air density} \)
• **Terminal velocity:**

Body falling from rest through air:

The drag force increases gradually until it balances the gravitational force.

The body reaches terminal velocity.

\[
\vec{D} + \vec{F}_g = 0
\]

\[
\frac{1}{2} CA \rho v_y^2 - mg = 0 \quad \Rightarrow \quad v_y = -\sqrt{\frac{2mg}{C\rho A}}
\]
Theoretically, how long does it take to reach terminal velocity?

(A) $\Delta t = \sqrt{\frac{2m}{gC\rho A}}$

(B) $\Delta t = 0$

(C) Infinite amount of time.

(D) None of the above.
Theoretically, how long does it take to reach terminal velocity?

(A) $\Delta t = \sqrt{\frac{2m}{gC\rho A}}$

(B) $\Delta t = 0$

(C) Infinite amount of time.

(D) None of the above.

Why?
\[ \vec{F}_{\text{net}} = m\vec{a} \]
\[ \vec{F}_{\text{net}} = \vec{D} + \vec{F}_g \]

\[ (F_{\text{net}})_y = \frac{1}{2} CA \rho v_y^2 - mg \]

\[ \frac{dv_y}{dt} = \kappa v_y^2 - mg, \quad \kappa = \frac{1}{2} CA \rho \]

\[ \downarrow \]

\[ v_y = -\sqrt{\frac{mg}{\kappa}} \tanh \left( \sqrt{mg \kappa} t \right) = -\sqrt{\frac{mg}{\kappa}} \frac{e^{\sqrt{mg \kappa} t} - e^{-\sqrt{mg \kappa} t}}{e^{\sqrt{mg \kappa} t} + e^{-\sqrt{mg \kappa} t}} \]
Terminal velocity:

\[ \lim_{t \to \infty} v_y = -\sqrt{\frac{mg}{\kappa}} = -\sqrt{\frac{2mg}{C\rho A}} \]

\[ \lim_{t \to \infty} a_y = \lim_{t \to \infty} \frac{dv_y}{dt} = 0 \]
<table>
<thead>
<tr>
<th>Object</th>
<th>Terminal Speed (m/s)</th>
<th>95% Distance&lt;sup&gt;a&lt;/sup&gt; (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shot (from shot put)</td>
<td>145</td>
<td>2500</td>
</tr>
<tr>
<td>Sky diver (typical)</td>
<td>60</td>
<td>430</td>
</tr>
<tr>
<td>Baseball</td>
<td>42</td>
<td>210</td>
</tr>
<tr>
<td>Tennis ball</td>
<td>31</td>
<td>115</td>
</tr>
<tr>
<td>Basketball</td>
<td>20</td>
<td>47</td>
</tr>
<tr>
<td>Ping-Pong ball</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Raindrop (radius = 1.5 mm)</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Parachutist (typical)</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

<sup>a</sup>This is the distance through which the body must fall from rest to reach 95% of its terminal speed.

A sky diver jumps from a flying airplane and falls for several seconds before she reaches terminal velocity. She then opens her parachute, reaches a new terminal velocity, and continues her descent to the ground. Which one of the following graphs of the drag force versus time best represents this situation?
A sky diver jumps from a flying airplane and falls for several seconds before she reaches terminal velocity. She then opens her parachute, reaches a new terminal velocity, and continues her descent to the ground. Which one of the following graphs of the drag force versus time best represents this situation?

\[ D = \frac{1}{2} C \rho A v^2, \]
Before opening the parachute terminal velocity is reached when

\[ \vec{D} + \vec{F}_g = 0 \quad D = F_g \]

After opening the parachute terminal velocity is again reached when

\[ \vec{D} + \vec{F}_g = 0 \quad D = F_g \]

A sufficiently long time after opening the magnitude of the drag force must equal its value before opening. At the same time it should spike upwards during opening.