Midterm 1

• Sunday October 13th 11:30am-1:00pm
• Physics Lecture Hall
• Lectures 1-4.5 (one dim motion – 2d and 3d motion, relative motion)
• No forces! No Newton’s laws!
• Practice exam posted online at http://www.physics.rutgers.edu/ugrad/123H/Exams/
Relative motion in one dimension

- cars travel at 50-60 mph yet they barely move with respect to each other.
passengers stand still on moving walkway yet they move with respect to the ground.

The velocity of a particle depends on the reference frame of the observer.
- What is the velocity of car $P$ measured by the driver of Car $B$?

\[ x_{PA} = x_{PB} + x_{BA} \]

$x_{BA}$ coordinate of $B$ in frame $A$

$x_{PA}$ coordinate of $P$ in frame $A$

$x_{PB}$ coordinate of $P$ in frame $B$

$x_{PA} = x_{PB} + x_{BA}$
\[ \frac{dx_{PA}}{dt} = \frac{dx_{PB}}{dt} + \frac{dx_{BA}}{dt} \Rightarrow v_{PA} = v_{PB} + v_{BA} \]
\[
\frac{dv_{PA}}{dt} = \frac{dv_{PB}}{dt} + \frac{dv_{BA}}{dt} \quad \Rightarrow \quad a_{PA} = a_{PB} + a_{BA}
\]

- **Note:** if \( a_{BA} = 0 \) then \( a_{PA} = a_{PB} \).

Different observers moving at constant velocity relative to each other will measure the same acceleration for a moving particle \( P \).
Relative motion in two dimensions

The motion of particle $P$ is studied by two observers moving at constant velocity relative to each other.

Observer $B$ is moving with constant velocity $\vec{v}_{BA}$ relative to observer $A$.

Fig. 4-19 Frame $B$ has the constant two-dimensional velocity $\vec{v}_{BA}$ relative to frame $A$. The position vector of $B$ relative to $A$ is $\vec{r}_{BA}$. The position vectors of particle $P$ are $\vec{r}_{PA}$ relative to $A$ and $\vec{r}_{PB}$ relative to $B$. 
• $\vec{r}_{PA}$ position vector of $P$ relative to $A$
• $\vec{r}_{PB}$ position vector of $P$ relative to $B$
• $\vec{r}_{BA}$ position vector of $B$ relative to $A$

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

• same notation for velocity and acceleration
\[ \vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA} \]
\[ \downarrow \quad \frac{d}{dt} \]
\[ \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \]
\[ \downarrow \quad \frac{d}{dt} \]
\[ \vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA} \]
\[ \vec{a}_{BA} = 0 \quad \Rightarrow \quad \vec{a}_{PA} = \vec{a}_{PB} \]

**Fig. 4-19** Frame B has the constant two-dimensional velocity \( \vec{v}_{BA} \) relative to frame A. The position vector of B relative to A is \( \vec{r}_{BA} \). The position vectors of particle P are \( \vec{r}_{PA} \) relative to A and \( \vec{r}_{PB} \) relative to B.
Problem 75 Chapter 4

A train travels due south at 30 m/s (relative to the ground) in a rain that is blown toward the south by the wind. The path of each raindrop makes an angle $\theta = 70^\circ$ with the vertical, as measured by an observer stationary on the ground. An observer on the train, however, sees the drops fall perfectly vertically. Determine the speed of the raindrops relative to the ground.
\[ \begin{align*}
\vec{v}_{\text{train}} &= v_{\text{train}} \hat{i} \\
\vec{v}_{\text{rel}} &= -v_{\text{rel}} \hat{j} \\
\vec{v}_{\text{drop}} &= \vec{v}_{\text{train}} + \vec{v}_{\text{rel}} \\
\sin(\theta) &= \frac{v_{\text{train}}}{v_{\text{drop}}} \\
v_{\text{drop}} &= \frac{v_{\text{train}}}{\sin(\theta)} \\
&= 31.92533316 \ldots \text{ m/s} \\
\tan(\theta) &= \frac{v_{\text{train}}}{v_{\text{rel}}} \\
v_{\text{rel}} &= \frac{v_{\text{train}}}{\tan(\theta)} \\
&= 10.91910700 \ldots \text{ m/s}
\end{align*} \]
\[ \vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j} \]  

**relative to sled**

1) Find \( v_{0x} \), \( v_{0y} \) assuming the ball lands at the same height as launching point.
Relative to ground:
\[ \vec{v}_{bg,0} = \vec{v}_0 + \vec{v}_s = (v_{0x} - v_s)\hat{i} + v_{0y}\hat{j} \]

Total flying time
\[ \Delta t = \frac{2v_{0y}}{g} \]
\[ \Delta x_{bg} = (v_{0x} - v_s)\Delta t = \frac{2v_{0x}v_{0y}}{g} - \frac{2v_{0y}v_s}{g} \]

**Magnitude** of slope of the line
\[ \frac{2v_{0y}}{g} = 4 \quad \Rightarrow \quad v_{0y} = 2g = 19.6 \ m/s \]

Intersection with vertical axis
\[ v_s = 0 \quad \Rightarrow \quad \frac{2v_{0x}v_{0y}}{g} = 40 \]
\[ v_{0x} = 10 \ m/s \]
2) Find horizontal displacement relative to sled, assuming the speed of the sled does not change when the ball is shot.

In reference frame of the sled

\[ \vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j} \]

Flying time the same

\[ \Delta t = \frac{2v_{0y}}{g} \]

\[ \Delta x = v_{0x}\Delta t = \frac{2v_{0y}v_{0x}}{g} \]
5. Force and Motion I

- What *causes* motion? What *changes* motion?

- How do objects interact with each other?
• First mathematical theory:

Isaac Newton, *Principia Mathematica*, 1687
Newtonian Mechanics

- Relation between **force** and **acceleration**
  \[ \downarrow \]
  **Three fundamental laws.**

- Does not apply to **all** physical phenomena:
  - Particles travelling close to the speed of light. *(Special relativity)*
  - Atomic and sub-atomic particles. *(Quantum mechanics)*
• How can we define **force**?
  • Basic idea: a physical effect which changes the **velocity** of an object.
  • Mathematically: **force** is a **vector quantity**; it has **direction** and **magnitude**

• **Examples:**
  • **Gravitational attraction**

    All objects near the Earth’s surface are attracted towards the Earth.
• Push and pull forces
• Normal force

Any object placed on a surface is subject to a push force stopping it from falling through.

*Always perpendicular to the surface.*

• Frictional force (friction)

Resistance force opposing motion.

*Always tangent* to the surface.
• **Tension**: pull force exerted by a (*massless unstretchable*) cord

![Diagram showing tension forces](image)

The forces at the two ends of the cord are equal in magnitude.

• **Other forces in nature**: electromagnetic, weak, strong . . .
Newton’s First Law

If no force acts on a body, the body’s velocity cannot change; the body cannot accelerate.

If the body is at rest, it will stay at rest. If it is moving, it will continue to move with the same velocity (same magnitude and direction.)
• **Good approximation:** puck sliding on ice

Friction is negligible for a short ice strip but on a very long ice strip the puck will eventually stop.
• **Forces** are **vectors** (magnitude and direction.)

• The **net force** acting on an object is the **vector sum** of all individual forces acting on that object.

\[ \vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_N + \vec{f} + \vec{T} + \cdots \]
Newton’s 1st Law (reformulation)

If no net force acts on a body, \( \vec{F}_{\text{net}} = 0 \), the body’s velocity cannot change; the body cannot accelerate.
• **Inertial Frame:** a frame in which Newton’s laws hold.

(a) the path of the puck seen by a stationary observer in space (inertial)

(b) the path of the puck seen by a ground observer rotating with the Earth. (noninertial)

Puck sliding on a strip of frictionless ice.

**Note:** apparent deflection caused by gradient in Earth’s rotation; for a short strip of ice a ground observer is $\sim$ inertial.
What is the main effect of a nonzero net force?

- An applied force can set stationary object in motion or can stop a moving object.
  \[\downarrow\]
  change in velocity, hence acceleration.
What is the relation between the net force acting on an object and the resulting acceleration?

\[
\vec{F}_{\text{net}} = m\vec{a},
\]

- \( m \) = Mass: intrinsic characteristic of a body relating a force on the body to the resulting acceleration.
i-Clicker

The graph below represents the $x$-component of the velocity of an object as a function of time.

Which of the following graphs represents the time dependence of the $x$-component of the net force acting on the object?
**Answer**

The graph below represents the $x$-component of the velocity of an object as a function of time.

Which of the following graphs represents the time dependence of the $x$-component of the net force acting on the object?
• **Force unit:** Newton (N)

1 Newton = the force exerted on a standard mass of 1 kg to produce an acceleration of 1 m/s².
Example: puck on frictionless ice

A puck of mass $m$ is simultaneously pulled by two ideal cords as shown above. What is its acceleration?

Recall: ideal cord = massless, unstretchable cord
Newton’s second law:

\[ \vec{F}_{\text{net}} = ma \]

\[ \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 \]

\[
\begin{align*}
(\vec{F}_{\text{net}})_x &= F_{1x} + F_{2x} \hfill \\
&= F_1 \cos \alpha - F_2 \sin \beta \\
(\vec{F}_{\text{net}})_y &= F_{1y} + F_{2y} \hfill \\
&= F_1 \sin \alpha - F_2 \cos \beta
\end{align*}
\]

\[
\begin{align*}
ma_x &= F_1 \cos \alpha - F_2 \sin \beta \\
ma_y &= F_1 \sin \alpha - F_2 \cos \beta
\end{align*}
\]

Free body diagram.
• **Gravitational force**

• Pull force $\vec{F}_g$ acting on any object near the Earth's surface. How do we compute it?

• Free fall acceleration $\vec{a} = -g\hat{j}$.

• Newton’s second law for freely falling object:

$$\vec{F}_g = -mg\hat{j}$$

• **Note:** approximation of a more general theory (studied later)
- **Weight**: magnitude of gravitational force acting on the object.

\[ W = F_g = mg \]
• **Normal force:** \( \vec{F}_N \) stops the object from falling through; always perpendicular to the surface

\[
\vec{F}_N + \vec{F}_g = m\vec{a} \Rightarrow (F_N)_y + (F_g)_y = ma_y \Rightarrow F_N = m(g + a_y)
\]

\( a_y \) vertical acceleration of table + block
**Example:** box pulled up a frictionless inclined plane by an ideal cord

\[ \vec{F}_{\text{net}} = m\vec{a} \]

\[ \vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_N + \vec{T} \]

\[ T - mg\sin \theta = ma_x \]

\[ F_N - mg\cos \theta = ma_y \]

\[ a_y = 0 \text{ (no motion along y axis)} \Rightarrow F_N = mg\cos \theta. \]

\[ a_x = \frac{(T - mg\sin \theta)}{m} \]
A box of mass $m$ is pulled along a frictionless table as shown below. What is the horizontal component $a_x$ of the acceleration of the box?

\[ A) \ a_x = 0 \]
\[ B) \ a_x = -(T/m)\cos \theta \]
\[ C) \ a_x = T/m \]
\[ D) \ a_x = (T/m)\sin \theta \]
\[ E) \ a_x = -T/m \]
Answer

A box of mass $m$ is pulled along a frictionless table as shown below. What is the horizontal component $a_x$ of the acceleration of the box?

- $A) \ a_x = 0$
- $B) \ a_x = -(T/m) \cos \theta$
- $C) \ a_x = T/m$
- $D) \ a_x = (T/m) \sin \theta$
- $E) \ a_x = -T/m$
\[ \vec{F}_{\text{net}} = m\vec{a} \]
\[ \vec{F}_{\text{net}} = \vec{T} + \vec{F}_g + \vec{F}_N \]
\[ (F_{\text{net}})_x = T_x = -T\cos\theta \]
\[ m a_x = -T\cos\theta \]
\[ a = -(T/m)\cos\theta \]
A box of mass $m$ is pulled along a frictionless table as shown below. For which values of $T, m, \theta$ will the box remain on the table?

- **A)** For all values of $T, m, \theta$
- **B)** $T \sin \theta > mg$
- **C)** $T \sin \theta \leq mg$
- **D)** $T \cos \theta < mg$
- **E)** None one the above
A box of mass $m$ is pulled along a frictionless table as shown below. For which values of $T, m, \theta$ will the box remain on the table?

A) For all values of $T, m, \theta$
B) $T \sin \theta > mg$
C) $T \sin \theta \leq mg$
D) $T \cos \theta < mg$
E) None one the above
The condition for the box to stay on the table is

\[ a_y = 0 \implies T \sin \theta + F_N - mg = 0. \]

Note that \( F_N \geq 0 \) since it is the magnitude of \( \vec{F}_N \). Hence

\[ T \sin \theta \leq mg \]
Newton’s Third Law

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.
The force on $B$ due to $C$ has the same magnitude as the force on $C$ due to $B$.

\[ \vec{F}_{BC} + \vec{F}_{CB} = 0 \]

\[ F_{BC} = F_{CB} \]
\[ \vec{F}_{CE} + \vec{F}_{EC} = 0 \]
\[ \vec{F}_{CT} + \vec{F}_{TC} = 0 \]
\[
\vec{F}_{app} = (20 \ N)\hat{i}
\]

\(m_A = 4 \ kg, \ m_B = 6 \ kg\)

Find the magnitude and direction of \(\vec{a}_A, \ \vec{a}_B\)
Block $A$ pushes $B$ with a horizontal force

$$\vec{F}_{BA}$$

Newton’s second law for $B$

$$\vec{F}_{BA} = m_B \vec{a}_B$$

**Note:** vertical forces cancel each other

$$\vec{G}_B + \vec{N}_B = 0$$
Block $B$ pushes back with a force $\vec{F}_{AB}$

Newton’s second law for $A$

$$\vec{F}_{app} + \vec{F}_{AB} = m_A \vec{a}_A$$

Note: vertical forces cancel each other

$$\vec{G}_A + \vec{N}_A = 0$$
\[ \vec{F}_{app} + \vec{F}_{AB} = m_A\vec{a}_A, \quad \vec{F}_{BA} = m_B\vec{a}_B \]

Newton’s third law:

\[ \vec{F}_{AB} + \vec{F}_{BA} = 0 \]

Projection onto \( x \)-axis

\[ F_{app} - F_{AB} = m_A a_{Ax} \quad F_{BA} = m_B a_{Bx} \]

where \( F_{AB} = F_{BA} \) (magnitudes).

Can we solve?
Kinematic constraint: the two bodies move together along the $x$ axis. Hence

$$a_{Ax} = a_{Bx} = a_x$$

Then:

$$F_{app} - F_{AB} = m_A a_x \quad F_{AB} = m_B a_x$$

$$F_{app} = (m_A + m_B) a_x \Rightarrow a_x = \frac{F_{app}}{m_A + m_B}$$

Magnitude of interaction force:

$$F_{AB} = F_{BA} = m_B a_x = \frac{m_B F_{app}}{m_A + m_B}$$
- Tension, cords, pulleys

Ideal cord: massless, unstretchable

Ideal pulley: massless, frictionless

The forces at the two ends are always equal in magnitude.
• Example:

• The object of mass $m_2$ hangs at the end of an ideal cord tied to the object of mass $m_1$.

• The object of mass $m_1$ is placed on a frictionless inclined plane.

• The cord is wrapped around an ideal pulley attached to the plane.

• Find the acceleration $\vec{a}_2$. 
Step 1: free body diagram for object 1

\[ \vec{F}_{\text{net1}} = m_1 \vec{a}_1 \]

\[ \vec{F}_{\text{net1}} = \vec{T}_1 + \vec{F}_{g1} + \vec{F}_N \]

\[ (F_N)_x = 0, \quad (F_{g1})_x = -m_1 g \sin \alpha \]

\[ T_1 - m_1 g \sin \alpha = m_1 a_{1x} \]

\[ a_{1y} = 0 \text{ (no motion \perp to plane)} \]

\[ m_1 a_{1x} = T_1 - m_1 g \sin \alpha \]
**Step 2:** free body diagram for object 2

\[ \vec{F}_{\text{net}2} = m_2 \vec{a}_2 \]

\[ \vec{F}_{\text{net}2} = \vec{T}_2 + \vec{F}_{g2} \]

\[ (F_{\text{net}2})_x = 0 \quad (F_{\text{net}2})_y = T_2 - m_2 g \]

\[ m_2 a_{2x} = 0 \quad m_2 a_{2y} = T_2 - m_2 g \]
**Step 3:** put equations together

\[ m_1 a_{1x} = T_1 - m_1 g \sin \alpha \]

\[ m_2 a_{2y} = T_2 - m_2 g \]

Can we solve?

Need extra conditions!

\[ T_1 = T_2 \]

\[ (\Delta x)_1 = -(\Delta y)_2 \text{ at all times} \]

\[ a_{1x} = -a_{2y} \]
\[ m_1 a_{1x} = T_1 - m_1 g \sin \alpha \quad \text{Can we solve?} \]

\[ m_2 a_{2y} = T_2 - m_2 g \quad \text{Yes!} \]

\[ T_1 = T_2 \]

\[ a_{1x} = -a_{2y} \]

\[ (m_1 + m_2) a_{2y} = m_1 g \sin \alpha - m_2 g \]

\[ a_{2y} = \frac{m_1 g \sin \alpha - m_2 g}{m_1 + m_2} \]
Which way will box 2 move? Up or down?

\[ a_{2y} = \frac{m_1 g \sin \alpha - m_2 g}{m_1 + m_2} \]

\[ a_{2y} > 0 \iff m_1 \sin \alpha > m_2 \quad \text{Up} \]

\[ a_{2y} < 0 \iff m_1 \sin \alpha < m_2 \quad \text{Down} \]

\[ a_{2y} = 0 \iff m_1 \sin \alpha = m_2 \quad \text{No motion (if initial velocity is 0)} \]
Problem 58 Ch. 5: What magnitude of the pull force is needed in order to climb with constant acceleration \( a \)?

Combined mass of man and cage:

\[
m = 95 \text{ kg}
\]
Newton's second law
\[ 2\vec{T} + \vec{G} = m\vec{a} \]

Newton's third law
\[ \vec{T} + \vec{F} = 0 \]
Projection onto $y$-axis

$$2T - mg = ma_y \quad \Rightarrow \quad F = \frac{m(a_y + g)}{2}$$

$a_y = 0 \quad \Rightarrow \quad F = mg/2 = 47.5 \times 9.8 = 465.5 \text{ N}$

In order to climb with $a_y = 1 \text{ m/s}^2$

$$F = 47.5 \times 10.8 = 513 \text{ N}.$$ 

What if the man climbed the rope without the aid of a pulley?

$$\vec{T} + \vec{G} = m\vec{a}, \quad \vec{F} + \vec{T} = 0$$

$$T - mg = ma_y \quad \Rightarrow \quad F = m(a_y + g)$$

Hence

$$a_y = 0 \quad \Rightarrow \quad F = 95 \times 9.8 = 931 \text{ N}$$

$$a_y = 1 \text{ m/s}^2 \quad \Rightarrow \quad F = 95 \times 9.8 = 1026 \text{ N}$$