Rutgers University
Department of Physics & Astronomy

01:750:123H Honors Analytical Physics I

Lecture 14
1. A small ball of mass \( m = 0.1 \) kg is shot horizontally with velocity \( \vec{v}_i = (20m/s)\hat{i} \) against a massive block resting on a horizontal table. The ball bounces off the block moving horizontally in opposite direction with velocity \( \vec{v}_f = -0.7\vec{v}_i \) after collision. At the same time the block does not move during the collision. What is the \( x \)-component \( J_x \) of the impulse of the static friction force acting on the block during collision, measured in \( kg \cdot m/s \)? Gravity and air drag on the small ball are negligible.

a) 2.8  
b) -3.4  
c) 4.4  
d) -2.2  
e) -3.8
\[ \vec{v} \]

\[ \cdot \]

\[ \vec{J}_{fs} = \Delta \vec{p}_{ball} = m \vec{v}_f - m \vec{v}_i = -(1.7 \times 0.1 \times 20) \hat{i} = -3.4 \hat{i} \]

2. In the figure below the spring, pulley and cord are ideal. The pulley is fixed to the ceiling while the loose end of the spring is glued to the floor. The spring makes a 45° degree angle with the horizontal direction. The spring constant is \( k = 30 \) N/m and the mass of the block is \( m = 1 \) kg. There is no friction or air drag. The block is initially held in place such that the spring is undeformed and then released with zero
initial velocity. What is the speed of the block when the elongation of the spring is $\Delta l = 0.2$ m. The result should be rounded off to two significant digits.

a) 2.36 m/s
b) 1.87 m/s
c) 1.65 m/s
d) 2.35 m/s
e) 0.98 m/s
\[ mg(\Delta l) = \frac{k(\Delta l)^2}{2} + \frac{mv^2}{2} \]
\[ v = \sqrt{2g(\Delta l) - \frac{k(\Delta l)^2}{m}} = 1.649\ldots \]

3. A block of mass \( m \) slides down from height \( h \) along a frictionless ramp with zero initial velocity. The block then travels a distance \( d \) along a rough horizontal surface and climbs a frictionless curved ramp. The turning point on the curved ramp is at height \( \frac{2h}{5} \) above the ground. Let \( \mu_k \) be the kinetic friction coefficient between the block and the rough horizontal surface. Then the ratio \( \frac{\mu_k d}{h} \)
is
a) $2/5$

b) $3/5$

c) $1/5$

d) $1$

e) $4/5$

$$3mgh/5 = \mu_kmgd \quad \mu_kd/h = 3/5.$$ 

4. A block of mass $m = 0.2$ kg placed on a frictionless horizontal table is attached to an ideal spring of constant $k = 10$ N/m. The other end of the spring is fixed at the pivot point $O$ and the block is set in uniform circular motion on a circle of radius $R = 1$ m about $O$. The length of the relaxed spring is $l = 0.8$
m. What is the total mechanical energy of the system block + spring measured in J?

a) 2.4
b) 3.6
c) 0.8
d) 1.2
3) 1.8

\[ \frac{mv^2}{R} = k(R - l) \]
5. An ideal spring of constant \( k = 60 \) N/m pulls a block of mass \( m = 0.3 \) kg along a rough horizontal surface. The initial elongation of the spring relative to its relaxed configuration is \( \Delta l = 0.25 \) m and the initial velocity of the block is zero. The kinetic friction coefficient between the block and the surface is \( \mu_k = 0.6 \). What is the speed of the block when the spring reaches its relaxed configuration? The result should be rounded off to two significant digits.

a) 6.98 m/s
b) 8.48 m/s  
c) 3.09 m/s  
d) 10.12 m/s  
e) 9.78 m/s  

\[ \frac{k(\Delta l)^2}{2} = \mu_k mg \Delta l + \frac{mv^2}{2} \]

\[ v = \sqrt{\frac{k(\Delta l)^2}{m} - 2\mu_k g \Delta l} = 9.56 \text{ m/s}. \]

6. A very narrow tunnel is drilled through a uniform spherical planet of radius \( R \) and mass \( M \), passing through the center of the planet. A small object is dropped with zero initial velocity from one end of the...
tunell and falls toward the center. Neglecting friction and air drag, let \( a \) be the acceleration of the object when it reaches a distance \( 2R/3 \) from the center. Then the ratio
\[
\frac{a}{GM/R^2}
\]
is
a) \( 1/2 \)
b) \( 3/2 \)
c) \( 1 \)
d) \( 1/3 \)
e) \( 2/3 \)

Using the sphericall shell theorem, the radial component of the force on the object at distance \( r \) from
the center is

\[ F_r = -\frac{4\pi}{3} \rho r^3 \frac{Gm}{r^2} \]

where

\[ \rho = \frac{M}{(4\pi/3)R^3} \]

is the density of the planet. Therefore

\[ F_r = -\frac{GmM}{R^3} r. \]

The acceleration at distance \(2r/3\) from the center is

\[ a = \frac{2GM}{3R^2}. \]

7. In the previous problem, suppose the initial velocity of the object is zero. Then what is its speed
when it reaches the center?

a) \( \sqrt{\frac{GM}{R}} \)

b) \( \sqrt{\frac{GM}{2R}} \)

c) \( \sqrt{\frac{3GM}{2R}} \)

(d) \( \sqrt{\frac{2GM}{R}} \)

e) \( \sqrt{\frac{2GM}{3R}} \)

The force is the same as a spring force with constant

\[
k = \frac{GmM}{R^3}.
\]

Therefore the work done by the gravitational force
until the falling object reaches the center is

\[ W_g = \frac{1}{2} kR^2 = \frac{GmM}{2R} \]

Then

\[ \Delta K = \frac{mv^2}{2} = W_g \]

which yields

\[ v = \sqrt{\frac{GM}{R}}. \]

8. A small block of mass \( m \) is placed on the equator of a uniform spherical planet of mass \( M \) and radius \( R \). The planet is spinning about its north-south axis such that the block is in uniform circular motion with
kinetic energy

\[ K = \frac{3GmM}{20R} \]

Let \( N \) be the magnitude of the normal force acting on the block. Then the ratio

\[ \frac{N}{GmM/R^2} \]

is

a) 0.5  
b) 0.7  
c) 0.3  
d) 1.1  
e) 0.9
\[-\frac{mv^2}{R} = -\frac{GmM}{R^2} + N\]

\[N = \frac{GmM}{R^2} - \frac{mv^2}{R} = (1 - 6/20)\frac{GmM}{R^2} = 0.7\frac{GmM}{R^2}\]
9. A space shuttle moves on a circular orbit of radius $r$ with constant speed $v$ around a planet of mass $M$. At some point the shuttle fires its engines and reduces its speed to $v/\sqrt{2}$. Find the semi-major axis of the resulting elliptic trajectory.

a) $2r/3$

b) $r/2$

c) $3r/4$

d) $3r/2$

e) $4r/3$

The total mechanical energy of the shuttle on an
elliptic trajectory is

\[ E = -\frac{GmM}{2a} \]

where \( m \) is the mass of the shuttle and \( a \) the semi-

major axis.

For the circular trajectory the gravitational force must
be equal to the centripetal force:

\[ \frac{mv^2}{r} = \frac{GmM}{r^2}. \]

Therefore the kinetic energy on the circular trajectory
is

\[ K = \frac{GmM}{2r}. \]

When the speed is reduced to \( v/\sqrt{2} \), the kinetic energy
is reduced by half. At the same time the gravitational
potential energy stays the same:

\[ U = -\frac{GmM}{r}. \]

Therefore for the elliptical trajectory we have

\[ \frac{GmM}{4r} - \frac{GmM}{r} = -\frac{GmM}{2a}. \]

This yields

\[ a = \frac{2r}{3}. \]

10. A pointlike particle is placed in the gravitational field generated by two identical uniform spherical objects of mass \( M \) as shown below. The particle and the centers of the two spherical objects are a distance \( d \) away from the point \( O \). The two spherical objects
are assumed static and there is no friction or air drag. The particle has zero initial velocity.

Let $v$ be the speed of the particle when it reaches the point $O$. Then, rounded off to two significant digits, the ratio

$$\frac{v^2}{GM/d}$$

is:

a) 2.23
b) 1.17  
c) 0.89  
d) 1.57  
e) 2.31

\[
\frac{mv^2}{2} - 2\frac{GmM}{d} = -2\frac{GmM}{d\sqrt{2}}
\]

\[
v^2 = \frac{4GM}{d} \left(1 - \frac{1}{\sqrt{2}}\right) = 1.1715 \ldots
\]

11. A ball hangs at the end of an ideal cord of length \( l = 1 \text{m} \), and the other end of the cord is fixed. The ball is given a horizontal initial velocity of magnitude \( v \). What is the minimum value of \( v \) such that the ball moves on a circular trajectory of radius \( l \) in the vertical
plane.

(a) 7m/s  
(b) 4m/s  
(c) 5.6m/s  
(d) 6m/s  
(e) 3.2m/s

Solution. Since there is no friction, the mechanical energy is conserved. Initially the ball has kinetic energy

\[ K_0 = \frac{mv^2}{2} \]

Suppose it moves on a circular trajectory. At the top
it has both kinetic and potential gravitational energy

\[ K_{\text{top}} = 2mgl + \frac{mv^2_{\text{top}}}{2} \]

Newton’s 2nd law at the top yields

\[ ma_c = T + mg \]

where \( a_c = \frac{mv^2_{\text{top}}}{l} \) is the centripetal acceleration and \( T \) is the magnitude of the tension in the cord. Since \( T \geq 0 \), it follows that

\[ v^2_{\text{top}} \geq gl. \]

The lowest initial speed is obtained from energy conservation:

\[ \frac{mv^2}{2} = 2mgl + \frac{mgl}{2} = \frac{5mgl}{2} \]
Hence

\[ v = \sqrt{5gl} = 7\text{m/s}. \]

12. A ball of mass \( m = 0.1\text{kg} \) is dropped from a height \( h = 11\text{m} \) above the surface of a triangular block of mass \( M = 1\text{kg} \) as shown below. The triangular block is at rest on a frictionless table and the kinetic energy is conserved during the collision. Suppose the ball moves horizontally immediately after it hits the block. Find the speed of the block after collision.
(a) 0.5m/s
(b) 0.1m/s
(c) 1m/s
(d) 1.2m/s
(e) 1.4m/s

**Solution.** The velocity of the ball just before it hits the block is

\[ \vec{v}_0 = -\sqrt{2gh}\hat{j}. \]

Let \( \vec{v}_1 = v_1\hat{i} \) denote the velocity of the ball after collision, which is horizontal by assumption. Let \( \vec{u} = u_x\hat{i} \) denote the velocity of the block after collision. Since there is no external force along the \( x \)-axis, the
$x$-component of the total momentum is conserved. Therefore

$$Mu_x + mv_1 = 0$$

Moreover, since the kinetic energy is conserved during the collision, we have

$$Mu_x^2 + mv_1^2 = mv_2^2$$

The above equations imply

$$u = \sqrt{\frac{2m^2}{M(m + M)}} gh = 1.4\text{m/s}$$

13. A triangular plate of angle $\theta = 30^\circ$ is glued to the surface of a horizontal table. A ball moving with speed $v_0 = 10\text{m/s}$ hits the plate sideways as shown below.
The picture shows the collision seen from above such that the \((x, y)\) plane is horizontal. There is no friction between the ball and the plate and no friction between the ball and the table. Suppose the ball slides along the face of the plate after collision. Find the final speed of the ball.

For this problem note that

\[
\sin(30^\circ) = \frac{1}{2}, \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}.
\]
(a) 10m/s
(b) 7m/s
(c) 12m/s
(d) 0m/s
(e) 5m/s

Solution. Choose a coordinate system as follows.
The only force acting on the ball during the collision is the normal force $\vec{N}$ which is orthogonal to the surface. Therefore the parallel component of the velocity must be preserved during the collision. This yields

$$mv_{\parallel} = mv_0 \sin \theta$$

after the collision. Hence $v_{\parallel} = v_0 \sin \theta = 5\text{m/s}$.

14. In problem 13 suppose the mass of the ball is $m = (\sqrt{3}/10)\text{kg}$. Compute the magnitude of the impulse of the normal force acting on the ball during the collision.

(a) $1\text{kg} \cdot \text{m/s}$

(b) $1.5\text{kg} \cdot \text{m/s}$
(c) 0.5 kg · m/s
(d) 0.7 kg · m/s
(e) 1.7 kg · m/s

Solution. The impulse of the normal force acting on the ball is given by

\[ \vec{J}_N = \Delta \vec{p}_\perp. \]

Since the ball slides along the surface after collision,

\[ \Delta p_\perp = mv_0 \cos \theta = 1.5 \text{ kg} \cdot \text{ m/s}. \]

Therefore

\[ J_N = 1.5 \text{ kg} \cdot \text{ m/s}. \]

15. A spring of constant \( k \) is compressed a distance
\( \Delta l \) m between two blocks of masses \( m \) and \( 2m \) respectively, lying on a smooth horizontal surface. The spring is assumed to be ideal and massless.

The blocks are free to move off the spring once it reaches its underformed configuration. Let \( v \) be the final speed of the block of mass \( m \) after the spring is released. Then the ratio \( v/\Delta l \) is

(a) \( \sqrt{2k/3m} \)

(b) \( \sqrt{k/m} \)

(c) \( \sqrt{3k/4m} \)
(d) $\sqrt{\frac{k}{2m}}$

(e) $\sqrt{\frac{4k}{5m}}$

Energy conservation:

$$k(\Delta l)^2/2 = mv^2/2 + (2m)v_2^2/2$$

Momentum conservation:

$$mv = 2mv_2 \Rightarrow v_2 = v/2$$

Hence

$$k(\Delta l)^2 = m(1 + 1/2)v^2 = 3mv^2/2.$$  

Therefore

$$\frac{v}{\Delta l} = \sqrt{2k/3m}.$$  

16. A ball moves on a smooth horizontal surface
with constant initial velocity vector

$$\vec{v}_i = 3\hat{i} - 2\hat{j}$$

The ball hits a wall parallel with the $x$ axis and bounces back with velocity vector $\vec{v}_f$ such that

$$v_{f,x} = \frac{v_{i,x}}{2}.$$ 

During the collision the $x$ and $y$ components of the force acting on the ball are related by

$$F_x = -\frac{F_y}{2}.$$ 

What is the $y$ component of the velocity of the ball $\vec{v}_f$ after collision?

(a) 2 m/s
(b) 0.5 m/s
(c) 0 m/s
(d) 1 m/s
(e) 3 m/s

Using Newton’s 2nd law we have

\[ m(v_{fx} - v_{ix}) = J_x \quad m(v_{fy} - v_{iy}) = J_y \]

where \( \vec{J} = \int \vec{F} dt \) is the impulse of the force acting on the ball during the collision. Since \( F_x = -F_x/2 \) one has \( J_x = -J_y/2 \) as well. Hence

\[ v_{fy} - v_{iy} = -2(v_{fx} - v_{ix}) = v_{ix} \]

\[ v_{fy} = v_{iy} + v_{ix} = 1 \text{ m/s}. \]
17. A particle of mass $m$ moving with constant velocity $\vec{v} = 3\hat{i}$ in the $(x, y)$ plane collides with a second particle of mass $2m$ initially at rest. The velocity of the first particle after collision is

$$\vec{v}_1 = \hat{i} - 2\hat{j}$$

Then the velocity vector of the second particle after collision is

(a) $-\hat{i} + 2\hat{j}$
(b) $\hat{i} - \hat{j}$
(c) $\hat{i} + \hat{j}$
(d) $2\hat{i} - \hat{j}$
(e) 0
Momentum conservation

\[ m_1 \vec{v} = m_1 \vec{v}_1 + m_2 \vec{v}_2 \]

where \( m_1 = m \), \( m_2 = 2m \). Hence

\[ \vec{v}_2 = \frac{1}{2} (\vec{v} - \vec{v}_1) = \hat{i} + \hat{j}. \]

18. An open cart is rolling in the rain on a horizontal surface without friction. The cart moves in a straight line along the \( x \)-axis. The rain droplets fall vertically into the cart such that the mass of the cart increases at the rate

\[ \frac{1}{m} \frac{dm}{dt} = 0.2 \text{ s}^{-1}. \]

Suppose at some moment the \( x \)-component of the velocity of the cart is \( v_x = 1.2 \text{ m/s} \). What is the
magnitude of the acceleration of the cart at the same time, measured in $m/s^2$?

(a) 0.24
(b) 0.42
(c) 0.18
(d) 1.2
(e) 0.86

Let $dm$ be the increase in the mass of the cart and the water in it for an infinitesimal time interval, $t$ to $t + dt$. Let $v_x$ be the horizontal component of the velocity of the cart at time $t$. Momentum conservation yields

$$mv_x = (m + dm)(v_x + dv_x)$$
where \( u_x \) is the horizontal component of the velocity vector of the water droplets relative to the ground and \( dv_x \) is the infinitesimal variation of the horizontal velocity of the cart in the same time interval. This yields

\[
(dm)v_x + mdv_x = 0,
\]
hence

\[
a_x = \frac{dv_x}{dt} = -v_x \frac{1}{m} \frac{dm}{dt} = 0.24 \, m/s^2.
\]

19. A compressed spring launches a block of mass \( m = 0.1\, \text{kg} \) up a circular ramp of radius \( R = 0.5\, \text{m} \) such that the top point \( P \) is a height \( R = 0.5\, \text{m} \) above the ground. The horizontal surface is frictionless, but there is nonzero kinetic friction between the block and
the ramp. The spring constant is $k = 100\text{N/m}$ and the spring is compressed a distance $\Delta l = 0.2\text{m}$. Suppose the block reaches the point $P$ with speed $v = 4\text{m/s}$. What is the work done by kinetic friction during the motion?

(a) 0.93J  
(b) $-0.71\text{J}$  
(c) $-0.49\text{J}$
(d) 1.22J
(e) −1.18J

The work done by friction equals the variation of the mechanical energy:

\[ W_{fk} = mgR + \frac{mv^2}{2} - \frac{k(\Delta l)^2}{2} = 0.49 + 0.8 - 2 = -0.71J \]

20. A block of mass \( m = 1\text{kg} \) is placed at height \( h = 11\text{m} \) on the frictionless surface of a triangular block of mass \( M = 10\text{kg} \) such that the system is initially at rest. The triangular block can slide without friction on a horizontal table. Find the final speed of the triangular block.
(a) 2.6m/s
(b) 3.4m/s
(c) 4.1m/s
(d) 1.8m/s
(e) 1.4m/s

Since there is no friction in the system, the mechanical energy is conserved. Since there is no external horizontal force, the $x$-component of the total mo-
mentum is also conserved. Therefore we have

\[ mgh = \frac{1}{2}mv^2 + \frac{1}{2}Mu^2 \]

and

\[ mv_x + Mu_x = 0 \Rightarrow mv = Mu. \]

The second equation yields \( v = Mu/m \). By substitution in the first we obtain

\[ 2mgh = \frac{m + M}{m}Mu^2. \]

This implies

\[ u = \sqrt{\frac{2m^2gh}{M(m + M)}} = \sqrt{1.96} = 1.4 \text{m/s}. \]

21. A ball tied at one end of a massless cord of
length \( l = 1\text{m} \) is free to move on a frictionless horizontal surface. The other end of the cord is fixed at the origin \( O \) and the ball is initially at \((x_0, y_0) = (0, 0.5\text{m})\) such that the cord is slack. Suppose the ball is given an initial velocity \( \vec{v}_0 = (2\text{m/s})\hat{i} \) and it starts moving on a circular trajectory of radius \( l \) as soon as the cord is stretched taut. Find the speed of the ball on the circular trajectory.

\[
\begin{array}{c}
\begin{align*}
\vec{v}_0 & \quad \text{slack cord} \\
\end{align*}
\end{array}
\]

(a) 1\text{m/s}
(b) 2m/s
(c) 0.5m/s
(d) 1.5m/s
(e) 2.5m/s

The picture below shows the configuration of the system when the cord is stretched taut. Let $\vec{v}_0\parallel$ and $\vec{v}_0\perp$ denote the components of $\vec{v}_0$ with respect to the direction of the cord. Note that the tension in the cord acts only along the parallel direction, and there is no force acting perpendicular to the cord. Therefore the perpendicular component of the momentum
is conserved. This yields

\[ mv = mv_0 \sin \theta. \]

From the right triangle we have \( \sin \theta = \frac{d}{l} = \frac{1}{2} \). Therefore

\[ v = \frac{v_0}{2} = 1 \text{m/s}. \]

22. Suppose the mass of the ball is \( m = 0.1 \text{kg} \) in problem 21. Compute the magnitude of the impulse of the tension force in the cord during the transition
from linear to circular motion.

(a) $\sqrt{3}/5$ kg · m/s
(b) $\sqrt{3}/2$ kg · m/s
(c) $1/3$ kg · m/s
(d) $1/5$ kg · m/s
(e) $\sqrt{3}/10$ kg · m/s

The impulse of the tension force is given by

$$\vec{J} = \Delta \vec{p}_\parallel$$

since tension only acts along the cord. Therefore

$$J = mv_0\cos\theta = \frac{\sqrt{3}}{10} \text{ kg} \cdot \text{m/s}.$$
23. A completely inelastic collision occurs among two particles with initial velocity vectors

\[ \vec{v}_1 = 5\hat{i} - 7\hat{j}, \quad \vec{v}_2 = -3\hat{i} + 4\hat{j}. \]

measured in m/s. The two particles have the same mass \( m = 0.1 \) kg. What is the total kinetic energy of the system of two particles after collision, measured in Joules?

a) 1.0  
b) 1.8  
c) 1.6  
d) 1.2  
e) 0.325
\[ \vec{V} = \frac{1}{2}(\vec{v}_1 + \vec{v}_2) = \frac{1}{2}(2\hat{i} - 3\hat{j}) \]

\[ K_f = m\vec{V}^2 = \frac{0.1}{4}(4 + 9) = 0.325 \text{ J.} \]

24. In a two dimensional collision, a particle of mass \( m \) moving with initial velocity \( \vec{v} \) collides a second particle of mass \( 2m \) initially at rest. The velocity vectors \( \vec{v}_{1f}, \vec{v}_{2f} \) of the two particles after collision are perpendicular and the final speed of the first one is \( v_{1f} = v/2 \). Find the ratio \( v_{2f}/v \) where \( v_{2f} \) is the final speed of the second particle.

a) \( \sqrt{3}/4 \)
b) \( \sqrt{3}/2 \)
c) $\sqrt{2}/4$

d) $\sqrt{2}/3$

e) $2/3$

$$m_1 \vec{v} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$
$$\vec{v} = \vec{v}_{1f} + 2\vec{v}_{2f}$$

Since $\vec{v}_1 \cdot \vec{v}_2 = 0$,

$$v^2 = v^2_{1f} + 4v^2_{2f} = \frac{v^2}{4} + 4v^2_{2f}$$

$$\frac{v_{2f}}{v} = \frac{\sqrt{3}}{4}$$

25. A pendulum consists of two small blocks tied together with an ideal cord while the first block is
tied to the pivot point \( O \) with a second ideal cord. The pendulum swings in a vertical plane in uniform gravitational field about the pivot. Suppose at some point during the motion both blocks are static and the cords are aligned and form a \( 30^\circ \) angle with the vertical. What is the acceleration of the center of mass at this moment?
The sum of the external forces acting along the tan-

a) $g$

b) 0

c) $g/2$

d) $g\sqrt{3}/2$

e) $g\sqrt{3}$
gential direction is

\[ m_1 g \sin(30^\circ) + m_2 g \sin(30^\circ) = (m_1 + m_2) g / 2 \]

The tangential acceleration of the COM is

\[ a_t = g / 2 \]

The centripetal acceleration is 0 since \( v_{COM} = 0 \) at this point.

26. In the picture below the spring is ideal and the blocks are tied together with an ideal cord while the upper block is connected to the loose end of the spring. The other end of the spring is glued to the ceiling. The spring constant is \( k = 10 \) N/m and the two blocks have equal masses \( m_1 = m_2 = 0.1 \) kg. What is the magnitude of the acceleration of the center of
mass of the two blocks when the elongation of the spring is $\Delta l = 15$ cm relative to its underformed configuration. The result should be expressed in $m/s^2$.

a) 3.2  
b) 2.7  
c) 2.3  
d) 4.1  
e) 1.8
\[ a_{\text{com}} = \frac{(m_1 + m_2)g - k\Delta l}{m_1 + m_2} = g - \frac{k\Delta l}{m_1 + m_2} = 9.8 - \frac{1.5}{0.2} = 9.8 - 7.5 = 2.3 \]

27. In a two particle elastic collision, the masses of the two particles are \( m_1 = 0.3 \) g and \( m_2 = 0.2 \) g. The initial velocity vectors are

\[ \vec{v}_1 = 13\hat{i}, \quad \vec{v} = -5\hat{i} \]
in m/s. What is the final velocity vector of particle 1 measured in m/s?

- a) $2.6\hat{i}$
- b) $-1.4\hat{i}$
- c) $-1.8\hat{i}$
- d) $1.4\hat{i}$
- e) $0.6\hat{i}$

28. In a two particle completely inelastic collision the masses of the two particles are $m_1 = 13$ g and
\( m_2 = 7 \text{ g} \). The initial velocity vectors are
\[
\vec{v}_1 = -6\hat{i}, \quad \vec{v} = 4\hat{i}
\]
in \( \text{m/s} \). How much kinetic energy, measured in Joules, is converted into heat?

a) 0.53  
b) 0.43  
c) 0.63  
d) 0.33  
e) 0.23

\[
v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{-78 + 28}{20} = -2.5 \text{ m/s}
\]

\[
K_i - K_f = \frac{1}{2} \times 0.13 \times 36 + \frac{1}{2} \times 0.07 \times 16 - \frac{1}{2} \times 0.2 \times 25 = 0.33 \text{ J}.
\]
29. An empty spherical cavity is cut inside a uniform hollowed sphere of mass density $\rho$ and radius $R$ as shown below. The spherical cavity has radius $R$ and is centered at distance $R$ from the center. A pointlike particle of mass $m$ is placed right at the center of the hollowed sphere.

Let $F_g$ denote the magnitude of the gravitational
force acting on the particle. Then the ratio 
\[ \frac{F_g}{\pi G m \rho R} \]
is:

a) 3/4  
b) 3/2  
c) 1/2  
d) 1/3  
e) 2/3

In the absence of the cavity the force acting on a particle at the center is 0. Therefore, by superposition,
\[ \vec{F}_g + \vec{F}_{\text{small sphere}} = 0 \]
where $\vec{F}_{\text{small sphere}}$ is the gravitational force acting on a particle at the surface of a uniform spherical distribution of mass of radius $R/2$. This yields

$$F_g = \frac{4\pi m \rho G R}{3} \frac{1}{2} = \frac{2\pi m \rho G R}{3}$$

30. A spring pulls a block of mass $m$ along a frictionless horizontal table as shown below. The block is subject to air drag of magnitude

$$D = cv^2$$

for some coefficient $c$. 
Suppose at some moment during the motion the spring is elongated, and the velocity and the acceleration of the block are parallel to the spring and point to the left. At the same time the magnitude of the acceleration is

\[ a = \frac{2cv^2}{5m}. \]

Then the instantaneous power output of the spring at this moment is

a) \( \frac{2cv^3}{5} \).
b) $7cv^3/5$

 c) $cv^3/5$

d) $6cv^3/5$

e) $cv^3$

Newton’s second law:

$$F_{spring} - cv^2 = ma$$

$$F_{spring} = cv^2 + ma = \frac{7}{5}cv^2$$

Power

$$P = \vec{F} \cdot \vec{v} = \frac{7}{5}cv^3$$