**Midterm II**

- Sunday November 18th 12:00pm in the Physics Lecture Hall

- Forces I and II i.e. Chapters 5 and 6 in textbook, Lectures 5,6,7 and first half of 8

- **No** energy and work
• Work done by a spring force

![Diagram of a spring system with direction arrows and notation for force components.]

• Hooke’s Law

\[ \vec{F}_s = -k\vec{d} \]

• always opposed to displacement (restoring force)

• \( k > 0 \) spring constant
\[ W_s = \int_{x_i}^{x_f} F_x \, dx \]
\[ = \int_{x_i}^{x_f} -kx \, dx \]
\[ = (-k) \int_{x_i}^{x_f} x \, dx \]
\[ = (-k/2) \left( x_f^2 - x_i^2 \right) \]

\[ W_s = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 \]
8. Potential energy. Conservation of energy

- **Potential energy**: energy associated with the configuration of a system of objects that exert forces on one another.

- can be converted into **kinetic energy** by allowing the system to evolve freely
Gravitational potential energy

\[ U_g = mgy + U_0 \]

where \( U_0 \) is an arbitrary constant depending on the choice of a reference configuration e.g.

\[ U_g = 0 \quad \text{at} \quad y = 0 \]

(ground level).
Elastic potential energy

\[ U_s = \frac{1}{2} kx^2 + U_0 \]

where \( U_0 \) is an arbitrary constant depending on the choice of a reference configuration e.g.

\[ U_0 = 0 \text{ at } x = 0 \]

(relaxed spring).
● Energy conservation:

In the absence of friction and air resistance

\[ K + U_g = \text{constant} \quad K + U_s = \text{constant} \]
• **Conservative Forces**

The work done by the force depends only on the initial and final position of the object, not on the path in between.

The net work done by a conservative force on a particle moving around any closed path is zero.
**Consequence:** when the configuration change is reversed the work changes **sign**:

\[ W_{a \to b} = -W_{b \to a} \]
Conservation of Mechanical Energy

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy $E_{mec}$ of the system, cannot change.

Conservative forces, isolated system $\Rightarrow U + K = \text{constant}$
Isolated: no external forces.

Isolated system

Moving object + Earth
Isolated system

Moving object + Earth + Wall + Floor + Spring
A boy is initially seated on the top of a hemispherical ice mound of radius $R = 13.8 \text{ m}$. He begins to slide down the ice, with a negligible initial speed (Fig. 8-45). Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?
1. Energy conservation:

\[ mgR = mgh + \frac{mv^2}{2} \]

\[ h = \text{height above the ground when losing contact} \]

\[ h = R \cos(\theta) \]

\[ v^2 = 2g(R - h) \]
2. Newton’s law:

\[ N + F_{gy} = -ma_c \]

\[ N = mg \cos(\theta) - \frac{mv^2}{R} \]

\[ N = \frac{mgh}{R} - \frac{2mg(R - h)}{R} \]

\[ N = mg \frac{3h - 2R}{R} \]

**Note:** the \( y \) direction is the radial direction.
\[ N = mg \frac{3h - 2R}{R} \]

Contact is lost when:

\[ N = 0 \]

\[ h = \frac{2R}{3} \]
Problem 22, Ch. 8: A skier starts from rest at height $H$ above the end of a ramp and leaves the ramp at angle $\theta$. No air resistance, no friction. What is the maximum height $h$ of his jump above the end of the ramp?
What is the **total** kinetic energy at the end of the ramp?

\[ A) \text{ mgh} \quad C) \text{ mgH} \]

\[ B) 0 \quad D) \text{ mg(H-h)} \]
Is **all** the kinetic energy at the end of the ramp converted back into gravitational potential energy?

A) Yes

B) No
At the end of ramp:

\[ v = \sqrt{2gH} \quad v_x = v\cos(\theta) \quad v_y = v\sin(\theta) \]

\[ K = \frac{mv^2}{2} = mgH\cos^2(\theta) + mgH\sin^2(\theta) \]
At the top of jump:

\[ v_x = v \cos(\theta) \quad v_y = 0 \]

\[ K = \frac{mv_x^2}{2} = mgH \cos^2(\theta) \]
Is **all** the kinetic energy at the end of the ramp converted back into gravitational potential energy?

A) Yes

B) No  \[ K_{\text{top}} = mgH\cos^2(\theta) \]
Energy conservation during jump:

\[ mgH = mgh + mgH \cos^2(\theta) \]

\[ h = H(1 - \cos^2(\theta)) = H\sin^2(\theta) \]
Problems 29, 35, Ch. 8 The distance between the spring and the upper end of the relaxed spring, measured along the ramp, is \( d \). No friction. What is the maximum deformation \( x \) of the spring relative to its relaxed configuration?
What is the gravitational potential energy relative to the turning point?

A) \( mgd \)

B) \( mgdsin(\theta) \)

C) \( mg(d + x)sin(\theta) \)

D) 0
What is the gravitational potential energy relative to the turning point?

A) $mgd$

B) $mgd \sin(\theta)$

C) $mg(d + x)\sin(\theta)$

D) 0
Energy conservation:

\[ mg(d + x)\sin(\theta) = \frac{kx^2}{2} \]
Second degree equation for $x$

\[ \frac{kx^2}{2} - mgx\sin(\theta) - mgd = 0 \]
What is the kinetic energy of the block when it hits the spring?

A) $mgd$
B) $mgdsin(\theta)$
C) $mg(d + x)$
D) $mg(d + x)sin(\theta)$
What is the kinetic energy of the block when it hits the spring?

A) \( mgd \)

B) \( mgdsin(\theta) \)

C) \( mg(d + x) \)

D) \( mg(d + x)sin(\theta) \)
• Reading a potential energy curve

In 1D motion along the $x$-axis, conservative force:

$$\Delta U = -W = -\int_{x_i}^{x_f} F_x dx \quad \Rightarrow \quad dU(x) = -F_x(x) dx$$

$$F_x(x) = -\frac{dU}{dx}(x)$$
A plot of $U(x)$, the potential energy of a particle confined to move along an $x$ axis. There is no friction, so mechanical energy is conserved.

A plot of the force $F(x)$ acting on the particle, derived from the potential energy plot by taking its slope at various points.
The total mechanical energy $E_{\text{mec}}$ is conserved $\Rightarrow$ flat line.

$K(x) = E_{\text{mec}} - U(x)$
Can the particle ever be at $x < x_1$?

A) Yes.

B) No.

C) Cannot be determined.
**Answer**

Can the particle ever be at $x < x_1$?

- **A**) Yes.
- **B**) No.
- **C**) Cannot be determined.

\[
K = E_{\text{mec}} - U < 0 \quad \text{impossible!} \quad K = \frac{mv^2}{2} \geq 0
\]
- Turning point:
  \[ K(x_1) = 0 \]

- Classically forbidden region:
  \[ x < x_1 \]
For either of these three choices for $E_{\text{mec}}$, the particle is trapped (cannot escape left or right).

- $E_{\text{mec}} \leq 4J \Rightarrow$ Trapped particle
• **Equilibrium Points**: extrema of $U(x)$
Example:

- particle of mass $m = 2\text{kg}$ moving along the $x$-axis
- conservative force derived from attached $U(x)$ graph
- $x = 6.5\text{m}, \ v_{0x} = -4\text{m/s}$
- $x_1 = 4.5\text{m}, \ v_1 = ?$
\[ x = 6.5 \text{m} \Rightarrow U = 0 \]

\[ E_{\text{mec}} = K_0 = \frac{mv_0^2}{2} = 16\text{J} \]

\[ K_1 = E_{\text{mec}} - U_1 = 16\text{J} - 7\text{J} = 9\text{J} \]

\[ v_1 = \sqrt{\frac{2E_1}{m}} = 3\text{m/s} \]
A particle is initially at \( x = d \) and moves in the negative \( x \)-direction. Where does the particle have the greatest speed?

A) \( x = a \)

B) \( x = b \)

C) \( x = c \)

D) \( x = d \)
A particle is initially at $x = d$ and moves in the negative $x$-direction. Where does the particle have the greatest speed?

- $A) x = a$
- $B) x = b$
- $C) x = c$
- $D) x = d$

$E_{\text{mec}} = U + K = \text{constant}$

$K_{\text{max}} \Leftrightarrow U_{\text{min}}$
A particle is initially at \( x = d \) and moves in the negative \( x \)-direction. Where is the particle slowing down?

A) \( x = a \)

B) \( x = b \)

C) \( x = c \)

D) nowhere
A particle is initially at $x = d$ and moves in the negative $x$-direction. Where is the particle slowing down?

A) $x = a$

B) $x = b$

C) $x = c$

D) nowhere

$$a_x > 0 \iff F_x > 0 \iff \frac{dU}{dx} < 0$$
What if:

- **External forces**

- **Dissipative forces:**
  - $W$ depends on the path
  - There is **no** potential energy $U$ associated to a configuration such that
    $$\Delta U = -W$$

- Examples: kinetic friction, drag
### Work done on a system by an external force

Work is energy transferred to or from a system by means of an external force acting on that system.
Example:

\[ W = \Delta K + \Delta U = \Delta E_{\text{mec}} \]

Applied external force

\[ \vec{F} = F \hat{j} \]

Newton’s 2nd law:

\[ ma_y = F - mg \]

Constant acceleration model:

\[ v^2 = v_0^2 + 2a_y \Delta y \]

\[ F \Delta y = \frac{mv^2}{2} - \frac{mv_0^2}{2} + mg \Delta y \]

\[ = \Delta K + \Delta U \]
• **Conservative forces inside the system**

\[ W_{\text{ext}} = \Delta K + \Delta U = \Delta E_{\text{mec}} \]

\( W_{\text{ext}} \): total work done on the system by external forces
Example:

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

\[
F - f_k = ma_x \quad v^2 = v_0^2 + 2a_x \Delta x
\]

\[
F \Delta x = \frac{mv^2}{2} - \frac{mv_0^2}{2} + f_k \Delta x
\]

\[
F \Delta x = \Delta K + f_k \Delta x
\]
Example:

\[ m a_x = F - f_k - m g \sin \theta \]

\[ v^2 = v_0^2 + 2a_x \Delta x \]

\[ F \Delta x = \frac{mv^2}{2} - \frac{mv_0^2}{2} + m g \Delta x \sin \theta + f_k \Delta x \]

\[ = \Delta K + \Delta U + f_k \Delta x \]

\[ W_F = \Delta K + \Delta U + |W_{\text{friction}}| \]
Conservative and dissipative forces inside the system:

\[ W_{\text{ext}} = \Delta U + \Delta K + |W_{\text{dissipative}}| \]

\[ W_{\text{ext}} = \Delta E_{\text{mec}} + |W_{\text{dissipative}}| \]

**Note:** work of friction or drag forces

\[ \Downarrow \]

heat

\[ \Downarrow \]

Increase in **thermal** energy

\[ |W_{\text{dissipative}}| = \Delta E_{\text{th}} \]
Law of Conservation of Energy

The total energy $E$ of a system can change only by amounts of energy that are transferred to or from the system.

\[ W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} \]

- $\Delta E_{\text{mec}}$ is any change in the mechanical energy of the system,
- $\Delta E_{\text{th}}$ is any change in the thermal energy of the system,
- $\Delta E_{\text{int}}$ is any change in any other type of internal energy of the system.
The total energy $E$ of an isolated system cannot change.

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad (\text{isolated system})$$