Practice Midterm I Solutions

1. A particle moves 30 cm west and then changes direction, moving $50\sqrt{2}$ cm north-east. The unit vector $\hat{i}$ points east, while the unit vector $\hat{j}$ points north. What is the displacement vector of the particle measured in meters?

   (a) $0.5\hat{j}$
   (b) $0.3\hat{i}$
   (c) $-0.3\hat{i} + 0.5\hat{j}$
   (d) $0.2\hat{i} + 0.5\hat{j}$
   (e) $-0.2\hat{i} + 0.5\hat{j}$

   **Solution**
   \[
   \vec{\Delta r} = -0.3\hat{i} + 0.5\sqrt{2}\frac{\hat{i} + \hat{j}}{\sqrt{2}} = 0.2\hat{i} + 0.5\hat{j}
   \]

2. A car moves along the $x$-axis with constant acceleration $2 \text{m/s}^2$ for 10 minutes and then slows down with constant acceleration $-1 \text{m/s}^2$ for 6 minutes. What is the average acceleration of the car during this time interval measured in $\text{m/s}^2$?

   (a) 0.875
   (b) 1
   (c) 1.125
   (d) 0.5
   (e) cannot be determined; need to know the initial velocity of the car.

   **Solution**
   
   $$v_1 = v_0 + a_{x,1}t_1,$$
   $$v_2 = v_1 + a_{x,2}t_2$$

   $$a_{x,\text{avg}} = \frac{v_2 - v_0}{t_1 + t_2} = \frac{a_{x,1}t_1 + a_{x,2}t_2}{t_1 + t_2} = \frac{20 - 6}{16} = 0.875 \text{ m/s}^2$$
3. A particle \( A \) moves with constant velocity \( v_{Ax} = 20 \text{ m/s} \) along the \( x \)-axis. At time \( t = 0 \) a second particle \( B \) starts moving from rest in the \((x,y)\)-plane with constant acceleration

\[
\vec{a}_B = 12\hat{i} - 7\hat{j}
\]

measured in \( \text{m/s}^2 \). What is the relative velocity of particle \( B \) with respect to particle \( A \) at time \( t = 2 \text{ s} \)?

(a) \(-7\hat{j}\)
(b) \(4\hat{i} - 14\hat{j}\)
(c) \(5\hat{i} - 12\hat{j}\)
(d) \(12\hat{i}\)
(e) \(-4\hat{i} + 14\hat{j}\)

Solution

\[
\vec{v}_B = t\vec{a}_B = 24\hat{i} - 14\hat{j}
\]

\[
\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 4\hat{i} - 14\hat{j}
\]

4. A car moving along the \( x \) axis with velocity \( v_{0x} = 20 \text{ m/s} \) starts slowing down when it crosses the origin with constant acceleration of magnitude 4 \( \text{m/s}^2 \). What is the velocity of the car at \( x = 32 \text{ m} \).

(a) 6m/s
(b) 8m/s
(c) 10m/s
(d) 14m/s
(e) 12m/s

\[
v_x^2 - v_{0x}^2 = 2a_x\Delta x = -2|a_x|\Delta x \quad \Rightarrow \quad v_x^2 = v_{0x}^2 - 2|a_x|\Delta x = 400 - 8 \times 32 = 144
\]

Hence \( v_x = 12 \text{ m/s} \).
5. A particle moving with constant acceleration $a_x = -2 \text{m/s}^2$ along the $x$-axis is at the origin at time $t = 0$. The initial velocity of the particle is $v_{0x} = 4 \text{ m/s}$. How long will it take for the particle to reach $x = 4\text{m}$?

(a) 2s  
(b) 5 s  
(c) 8 s  
(d) 3 s  
(e) it will never reach $x = 4 \text{ m}$.

**Solution**

$$x = x_0 + v_{0x}t + a_x t^2 / 2 = x_0 + v_{0x}t - |a_x| t^2 / 2$$

This is a quadratic equation for $t$

$$|a_x| t^2 / 2 - v_{0x} t + (x - x_0) = 0.$$ 

Hence $t^2 - 4t + 4 = 0$, which yields $t = 2 \text{ s}$.

6. Particle A moves with constant velocity $\vec{v}_{A,x} = 160 \text{ m/s}$ along the $x$ axis and it crosses the origin at time $t = 0$. At the same time particle B starts moving from $x_{0B} = 640$ with constant acceleration $a_{B,x} = 20 \text{ m/s}^2$ and zero initial velocity. How long will it take for the two particles to collide?

(a) 12s  
(b) 5 s  
(c) 16 s  
(d) 10 s  
(e) 8 s

**Solution**

$$x_A = tv_{0A}, \quad x_B = x_{0B} + a_{B,x} t^2 / 2$$

$$x_A = x_B \Rightarrow 10t^2 - 160t + 640 = 0 \Rightarrow (t - 8)^2 = 0$$
Hence \( t = 8 \) s.

7. A skydiver jumps out of an airplane moving horizontally with constant acceleration \( a = 5 \text{ m/s}^2 \) along the \( x \)-axis. What is the relative acceleration of the skydiver with respect to the plane (measured in \( \text{m/s}^2 \)) during free fall? The \( y \) axis is vertical and points upward.

(a) \( 5\hat{i} \)
(b) \( -9.8\hat{j} \)
(c) \( 5\hat{i} + 9.8\hat{j} \)
(d) \( -5\hat{i} - 9.8\hat{j} \)
(e) cannot be determined from the data

\[ \vec{a}_{rel} = -9.8\hat{j} - 5\hat{i} \]

8. A particle moves on a curved path on in the \((x, y)\) plane. The velocity vector of the particle at time \( t_1 = 2s \) is

\[ \vec{v}_1 = -13\hat{i} + 8\hat{j} \]

and the velocity vector at time \( t_2 = 6 \) s is

\[ \vec{v}_2 = 11\hat{i} - 4\hat{j} \]

All components are measured in \( \text{m/s} \). What is the magnitude of the average acceleration of the particle in this time interval, measured in \( \text{m/s}^2 \)?

(a) \( 2\sqrt{5} \)
(b) \( \sqrt{7} \)
(c) \( 3\sqrt{5} \)
(d) \( 6\sqrt{3} \)
(e) \( 4\sqrt{7} \)
At $t = 0$ two particles A and B start moving simultaneously in the $(x, y)$ plane. The initial velocity vectors of the two particles at $t = 0$ are
\[ \vec{v}_{0A} = \hat{i} + \hat{j}, \quad \vec{v}_{0B} = -\hat{i}. \]
The particles move with constant acceleration vectors
\[ \vec{a}_A = 2\hat{i}, \quad \vec{a}_B = -\hat{j}. \]
What is the relative velocity $\vec{v}_{BA}$ of particle B with respect to A at time $t = 1$ s?

(a) $\vec{v}_{BA} = -\hat{i} + 2\hat{j}$

(b) $\vec{v}_{BA} = 2\hat{i} - \hat{j}$

(c) $\vec{v}_{BA} = -4\hat{i} - 2\hat{j}$

(d) $\vec{v}_{BA} = 0$

(e) $\vec{v}_{BA} = -\hat{j}$.

At time $t = 1$ s we have
\[ \vec{v}_A = \vec{v}_{0A} + t\vec{a}_A = 3\hat{i} + \hat{j} \]
\[ \vec{v}_B = \vec{v}_{0B} + t\vec{a}_B = -\hat{i} - \hat{j} \]
Therefore
\[ \vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -4\hat{i} - 2\hat{j}. \]
where \( v_0 > 0 \). At the same time a second ball \( B \) is dropped with zero initial velocity from the point \( \vec{r}_{0B} = 20(\hat{i} + \hat{j}) \).

Suppose ball \( A \) collides with \( B \) exactly at the same moment the latter hits the ground. What is \( v_0 \)? Air resistance is negligible.

\( a \) \( v_0 = 15 \text{ m/s} \)

\( b \) \( v_0 = 7 \text{ m/s} \)

\( c \) \( v_0 = 16 \text{ m/s} \)

\( d \) \( v_0 = 14 \text{ m/s} \)

\( e \) \( v_0 = 10 \text{ m/s} \)

**Solution** The falling time of \( B \) is

\[
\Delta t = \sqrt{\frac{2h}{g}}
\]

where \( h = d = 20 \text{ m} \). The horizontal component of the velocity vector of \( A \) is constant, \( v_{Ax} = \frac{v_0}{\sqrt{2}} \). In order for the balls to collide as said in the problem

\[
\frac{d}{\Delta t} = v_0 \frac{v_0}{\sqrt{2}}
\]

This yields

\[
v_0 = \sqrt{gd} = 14 \text{ m/s}.
\]

11. An elevator descends with constant acceleration \( \vec{a}_{\text{elevator}} = -(6.8 \text{ m/s}^2) \hat{j} \) relative to the Earth’s reference system, which is assumed inertial. A physicist inside the elevator drops a ball with zero initial velocity from height \( h = 1.5 \text{ m} \) above the floor. How long will it take for the ball to hit the floor? The magnitude of the free fall acceleration is \( 9.8 \text{ m/s}^2 \).

\( a \) \( 2 \text{ s} \)

\( b \) \( 3 \text{ s} \)
Note that the acceleration of the ball relative to the Earth’s reference frame is

\[ \vec{a} = -g \hat{j} \]

since the only force acting on the ball is gravity. Then the relative acceleration of the ball with respect to the elevator is

\[ \vec{a}_{rel} = \vec{a} - \vec{a}_{elevator} = 3 \hat{j}. \]

Therefore the falling time measured by an observer inside the elevator is

\[ \Delta t = \sqrt{\frac{2h}{3}} = 1 \text{ s}. \]

12 A river is 50 meters wide flows with constant speed 0.6 m/s. A swimmer crossing the river swims with constant relative velocity of magnitude 2 m/s directed straight across the river with respect to the water. What is the total displacement of the swimmer along the \( x \) axis?

(a) 10 m
(b) 5 m
(c) 12 m
(d) 0 m
(e) 15 m

Solution

\[ \Delta t = 50/2 = 25 \text{ s} \Rightarrow \Delta x = 0.6 \Delta t = 15 \text{ m}. \]

13. A ball falls from height \( h = 2 \) in uniform gravitational field such that the total falling time is \( T \). How high above the floor is the ball at time \( t = T/2 \).
14. A bus moves horizontally with constant acceleration of magnitude $g/5$. Inside the bus a ball falls straight down from a height $h$ above the floor with zero initial velocity relative to the bus. Let $d$ be the total horizontal displacement of the ball during the fall. Then the ratio $d/h$ is

(a) 1
(b) 1/5
(c) 1/4
(d) 5
(e) 2/5

Solution

\[ y = h - gt^2/2 \]
\[ y = 0 \Rightarrow T = \sqrt{2h/g} \]
\[ y(T/2) = h - (g/2) \times (1/4) \times (2h/g) = h - h/4 = 3h/4 \]

15. A projectile is launched from the origin in uniform gravitational field with velocity vector $\mathbf{v} = 4\hat{i} + 5\hat{j}$ (measured in m/s). What is the ratio $x/y$ at time $t = 1$ after the launch?

(a) 0.1
(b) 30
(c) 20
(d) 0.5
(e) 40

**Solution**

\[ x = 4t = 4, y = 5t - \frac{g}{2}t^2 = 5 - 4.9 = 0.1 \]

Hence

\[ x/y = 40 \]