Physics 123H - Midterm IIA - Solutions

Note: All quantities are measured in SI units unless otherwise stated.

1. An airplane moves with acceleration vector
\[ \vec{a} = a_x \hat{i} - (\frac{g}{7}) \hat{j}, \quad a_x > 0 \]
relative to the ground in uniform gravitational field. As usual \( \hat{j} \) is vertical
and points upward while \( \hat{i} \) is horizontal and points to the right. Inside the
airplane a pendulum of mass \( m \) makes an angle \( \theta \) with the vertical direction
such that
\[ \tan(\theta) = \frac{1}{3}. \]
The pendulum is at rest relative to the airplane. Then the ratio \( a_x / g \) is

(a) \( \frac{4}{7} \)
(b) 0
(c) \( \frac{1}{7} \)
(d) \( \frac{2}{7} \)
(e) \( \frac{3}{7} \)

Solution.
Since the pendulum is static relative to the airplane, it moves with ac-
celeration vector \( \vec{a} \) relative to the ground. Therefore Newton’s second law in
the ground reference frame reads
\[ \vec{F}_{\text{net}} = m \vec{a} \quad \vec{F}_{\text{net}} = \vec{F}_g + \vec{T} \]
The tension in the string:
\[ \vec{T} = T_x \hat{i} + T_y \hat{j} = T \sin \theta \hat{i} + T \cos \theta \hat{j} \]
The gravitational force: \( \vec{F}_g = -mg \hat{j} \).
The \( y \)-component of Newton’s 2nd law:
\[ T \cos \theta = mg + ma_y. \]
The $x$-component:

$$T \sin \theta = m a_x$$

Therefore

$$\tan \theta = \frac{a_x}{g + a_y}$$

$$a_x / g = (1 + a_y / g) \tan(\theta) = \frac{6}{7} \times \frac{1}{3} = \frac{2}{7}.$$

2. A puck of mass $m = 0.3$ kg placed on frictionless ice is subject to three forces acting in the horizontal plane. The first two forces, measured in Newtons, are

$$\vec{F}_1 = 3.5 \hat{i} - 4 \hat{j}, \quad \vec{F}_2 = -2 \hat{i} - 7.5 \hat{j}$$

What is the third force if the acceleration of the puck is

$$\ddot{a} = 4 \hat{i} + 6 \hat{j}$$

in m/s$^2$.

(a) $-0.3 \hat{i} + 13.3 \hat{j}$
(b) $0.4 \hat{i} - 11.4 \hat{j}$
(c) $-1.2 \hat{i} - 12.2 \hat{j}$
(d) $-0.8 \hat{i} + 11.6 \hat{j}$
(e) $0.6 \hat{i} - 9.2 \hat{j}$

Solution.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m \ddot{a}$$

$$\vec{F}_1 + \vec{F}_2 = 1.5 \hat{i} - 11.5 \hat{j}$$

$$\vec{F}_3 = 1.2 \hat{i} + 1.8 \hat{j} - 1.5 \hat{i} + 11.5 \hat{j} = -0.3 \hat{i} + 13.3 \hat{j}$$

3. A ball of mass $m$ is suspended from the ceiling in uniform gravitational field with two ideal cords as shown below.
The magnitude of the tension in cord 1 is $T_1 = 29.4$. Then the mass of the ball is

- (a) 4
- (b) 6
- (c) 5
- (d) 7
- (e) 3

For this problem note that

\[
\sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \cos(60^\circ) = \frac{1}{2}
\]

and

\[
\sin(30^\circ) = \frac{1}{2} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}.
\]

**Solution.**

\[
T_1 \cos(30^\circ) = T_2 \cos(60^\circ)
\]

\[
T_2 = T_1 \sqrt{3}
\]

\[
T_1 \sin(30^\circ) + T_2 \sin(60^\circ) = mg.
\]

\[
\left( \frac{1}{2} + \frac{3}{2} \right) T_1 = mg
\]

\[
mg = 2T_1.
\]
\[ m = 2 \times 29.4/9.8 = 6. \]

4. A pendulum of mass \( m \) attached to one end of an ideal cord of length \( \ell = 0.6 \) swings in a vertical plane in uniform gravitational field. The pivot point is fixed. The tension in the cord is \( T = mg/4 \) and the speed of the pendulum is \( v \) when the cord makes a 60° angle with the vertical as shown below.

Then \( v \) is:

(a) cannot be determined because the mass is not known.
(b) 2.6
(c) 1.8
(d) 3.2
(e) 2.1

Note that
\[ \sin(60°) = \frac{\sqrt{3}}{2}, \quad \cos(60°) = \frac{1}{2}. \]

**Solution.**

\[ \frac{mv^2}{\ell} = mg\cos(60°) + T = \frac{mg}{2} + \frac{mg}{4} = \frac{3mg}{4} \]
\[ v = \frac{\sqrt{3g\ell}}{2} = 2.1 \]

5. A particle moves on a circular trajectory centered at the origin in the \((x, y)\) plane with constant speed \(v = 4\) m/s. Suppose the particle passes through the point \((x, y) = (1, -2)\) at some point during the motion. What is the acceleration vector of the particle at this point?

- \((a)\) \(2.2\hat{i} - 1.8\hat{j}\)
- \((b)\) \(-3.2\hat{i} + 6.4\hat{j}\)
- \((c)\) \(4.6\hat{i} - 5.2\hat{j}\)
- \((d)\) \(-1.4\hat{i} - 2.8\hat{j}\)
- \((e)\) \(-3.2\hat{i} + 4.8\hat{j}\)

**Solution.**

The position vector at the given moment is 
\[ \vec{r} = \hat{i} - 2\hat{j}. \]

Hence the radius is \(R = 5\). The centripetal acceleration is given by 
\[ \vec{a}_c = -\frac{v^2}{R}\vec{r} = -\frac{v^2}{5R^2}(\hat{i} - 2\hat{j}) = -\frac{16}{5}(\hat{i} - 2\hat{j}) = -3.2\hat{i} + 6.4\hat{j} \]

6. A pendulum hanging from the ceiling is in uniform circular motion along a circular trajectory of radius \(r = 1.2\) in the *horizontal* plane with constant speed \(v = 3.5\). The pivot point is fixed and located at height \(h\) above the horizontal plane of motion. The magnitude of the free fall acceleration is \(g = 9.8\) and air resistance is negligible.
Then $h$ is

a) 0.844  
b) 2.012  
c) 1.324  
d) 0.984  
e) 1.152

**Solution.**
The force diagram is shown below, where $\tan(\theta) = h/r$. 
Note that

\[ T \cos(\theta) = F_c = ma_c = \frac{mv^2}{r}. \]

Moreover,

\[ T \sin(\theta) = mg \]

\[ T \cos(\theta) = ma_c = \frac{mv^2}{r} \]

\[ \tan(\theta) = \frac{rg}{v^2} = \frac{h}{r} \]

\[ h = \frac{gr^2}{v^2} = 1.152 \]

7. In the figure below the pulleys and the ropes are ideal and the pulleys are fixed. The mass of the suspended square block is 1 kg and the mass of the rectangular block placed on the floor is 3 kg. The distance \( h \) is \( h = 1 \). The static friction coefficient between the larger block and the floor is \( \mu_s = 0.75 \). The suspended block swings like a pendulum such that its velocity at the lowest point is horizontal and has magnitude \( v \). Find the maximum value of \( v \) such that the larger block does not slip along the floor during the motion.

\[ a) \ 1.5 \]
\[ b) \ 5.5 \]
\[ c) \ 3.5 \]
\[ d) \ 2.5 \]
\[ e) \ 4.5 \]
Solution.

Newton’s law for the suspended block

\[
\frac{m_1v^2}{h} = T - m_1g \Rightarrow T = m_1g + \frac{m_1v^2}{h}
\]

Newton’s law for the block on the floor

\[
T = f_s \leq \mu_s m_2 g
\]

Therefore

\[
m_1g + \frac{m_1v_{\text{max}}^2}{h} = \mu_s m_2 g
\]

\[
v_{\text{max}}^2 = (3\mu_s - 1)gh \Rightarrow v_{\text{max}} = 3.5.
\]

8. Two blocks of masses \(m_1 = 1\), \(m_2 = 3\) connected by massless rigid rod slide down a slope of angle \(\theta\) as shown below.
The kinetic friction coefficient between the first block and the ramp is $\mu_k = 0.25$ while the kinetic friction coefficient between the second block and the ramp is zero. Find the magnitude of the common acceleration of the two blocks provided that

$$\sin(\theta) = \frac{3}{5}, \quad \cos(\theta) = \frac{4}{5}.$$ 

a) 5.39  

b) 4.71  

c) 2.89  

d) 6.23  

e) 4.37  

**Solution.**

\[
(m_1 + m_2)a = (m_1 + m_2)gsin(\theta) - \mu_k m_1 g\cos(\theta) 
\]

\[
4ma = 4mg \times \frac{3}{5} - \frac{1}{4} \times (mg) \times \frac{4}{5} 
\]

Hence

\[
a = \frac{3g}{5} - \frac{g}{20} = \frac{11g}{20} = 5.39. 
\]
9. In the figure below the pulley and the ramp are fixed, the pulley and the cord are ideal, and the ramp angle is $\theta = 45^\circ$. The static friction coefficient between the first block and the ramp is $\mu_s = 0.35$ and the mass of the first blocks is $m_1 = 1$. What is the smallest value of $m_2$ that prevents the first block from sliding down the ramp. Note that

$$\sin(45^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2}.$$ 

The result should be computed up to significant digits.

\[\begin{array}{c}
\text{a)} \ 0.84 \\
\text{b)} \ 0.46 \\
\text{c)} \ 0.64 \\
\text{d)} \ 1.12 \\
\text{e)} \ 0.24 \\
\end{array}\]

**Solution.**

\[\begin{align*}
m_2g + \mu sm_1g\cos(\theta) &= m_1gsin(\theta) \\
m_2 &= (1 - \mu_s) \frac{\sqrt{2}}{2} \\
m_2 &= 0.4596 \ldots
\end{align*}\]
10. A ball of mass $m$ attached to one end of a rigid rod of length $\ell$ swings in a vertical plane in uniform gravitational field. The square of its speed when the rod makes a $30^\circ$ angle with the vertical is 

$$v^2 = \frac{g\ell\sqrt{3}}{3}$$

The gravitational acceleration is vertical and points downward in the figure. Air resistance and the mass of the rod are negligible.

Let $N$ be the magnitude of the normal force from the rod acting on the pendulum at this point? Then the ratio \( \frac{N}{mg} \) is:

(a) $\sqrt{3}/3$

(b) $\sqrt{3}/6$

(c) 1

(d) $1/6$

(e) $\sqrt{3}/2$

Note that

$$\sin(30^\circ) = \frac{1}{2}, \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}.$$
Solution.

\[ \frac{mv^2}{\ell} = mg\cos(30^\circ) - N \]

\[ N = \frac{mg\sqrt{3}}{2} - \frac{mg\sqrt{3}}{3} = \frac{mg\sqrt{3}}{6} \]

11. A car moves with constant velocity \( \vec{v} = \hat{v}\hat{i} \) along a horizontal surface. At time \( t = 0 \) the driver presses the brakes and the car starts sliding along the surface. The total distance it takes to stop is \( d = 28 \text{ m} \). The kinetic friction coefficient between the tires and the pavement is 0.3. Find the speed \( v \) measured in m/s. The result should be rounded off to two significant digits.

(a) 9.87
(b) 11.18
(c) 10.59
(d) 12.83
(e) 13.42

Solution.

\[ v^2 = 2ad, \quad a = \mu_k g \]

\[ v = \sqrt{2\mu_kgd} = 12.8312 \ldots \text{ m/s} \]

12. A car moves around a circular horizontal track of radius \( R = 60 \text{ m} \) with constant speed \( v \text{ m/s} \). The static friction coefficient between the tires and the pavement is \( \mu_s = 0.4 \). What is the maximum value of the speed in m/s such that the car can round the curve without slipping? The result should be rounded off to two significant digits.

(a)10.86
(b) 14.23
(c) 15.34
(d) 11.67
(e) 9.58

Solution.

\[ \mu_s mg = \frac{mv^2}{R} \]

\[ v = \sqrt{\mu_s gR} = 15.3362 \ldots \text{ m/s} \]

13. In the figure below the pulleys and the cord are ideal. The first pulley is fixed while the second is free to roll without friction along the cord. The cord loops around the second pulley making symmetric 45° degree angles with the vertical direction. A vertical pull force is applied to the loose end such that the tension in the string has magnitude \( T = 11.2 \). Find the magnitude of the acceleration of the block attached to the second pulley if its mass is \( m = 1 \). The result should be rounded off to two significant digits.

\[ \begin{align*}
\text{a)} & \ 6.96 \\
\text{b)} & \ 5.24 \\
\text{c)} & \ 3.88 \\
\text{d)} & \ 6.04
\end{align*} \]
4.72

Solution.

\[ ma = 2 \times F \cos(45^\circ) - mg = F \sqrt{2} - 9.8 = 6.03919 \ldots \]

14. A kite of weight \( mg = 2.5 \) tied to an ideal cord is held static by the wind. The other end of the cord is held fixed at ground level and the cord makes an angle \( \theta \) with the vertical direction. The drag force acting on the kite is

\[ \vec{D} = 3\hat{i} + 4\hat{j} \]

where \( \hat{i} \) is horizontal and \( \hat{j} \) is vertical as usual. Then \( \tan(\theta) \) is:

a) 2
b) 1
c) 0.5
d) 2.5
e) 1.5

Solution.

Let \( \vec{T} = T_x\hat{i} + T_y\hat{j} \) be the tension in the cord. Then

\[ D_x + T_x = 0, \quad D_y - mg + T_y = 0. \]

Hence

\[ T_x = -3, \quad T_y = -1.5 \]

Then

\[ \tan(\theta) = \frac{|T_x|}{|T_y|} = 2. \]

15. In the figure below the pulleys and the cord are ideal and the pulleys are fixed. The cord is looped around both pulleys and both ends are attached to the block. The block does not move along the ramp. The mass of the block is \( m \) and there is no friction.
The magnitude of the tension force is

\[ T = \frac{3}{5}mg \]

Then the magnitude of the normal force acting on the block from the ramp is:  

- a) 6/5  
- b) 7/5  
- c) 4/5  
- d) 3/5  
- e) 1

The following trigonometric identity may be useful.

\[ \sin^2(\theta) + \cos^2(\theta) = 1. \]

**Solution.**
\[ T = mg \sin(\theta) \]

\[ \sin(\theta) = \frac{3}{5}, \quad \cos(\theta) = \frac{4}{5} \]

\[ N = T + mg \cos(\theta) = \left( \frac{3}{5} + \frac{4}{5} \right) mg = \frac{7}{5} mg. \]