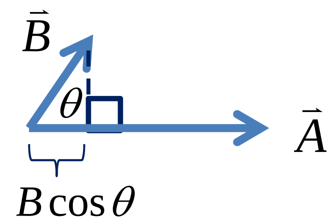
P123 F11 Lecture 9 28 October 2011

Power and Gravitational Potential Energy

REVIEW of Last Week's Lecture



- Scalar Product

$$\vec{A}\cdot\vec{B} = AB\cos\phi = A_xB_x + A_yB_y + A_zB_z$$

- Work

$W = Fs$ constant force parallel to displacement

$W = \vec{F}\cdot\vec{s}$ constant $\vec{F}$ not parallel to $\vec{s}$

$W = \int_{x_1}^{x_2} F_x dx$ variable force along x (area under F-x curve)

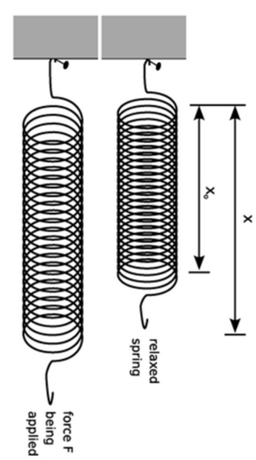
$W = \int_1^2 \vec{F}\cdot d\vec{s}$ in general

- Work-Energy Theorem

$$W_{tot} = \int_1^2 \vec{F}_{net}\cdot d\vec{s} = \Delta K = K_2 - K_1$$

Where $\vec{F}_{net} = \sum_r \vec{F}_i$; $K = \frac{1}{2}MV^2 \equiv$ Kinetic Energy

- Spring, Hooke's Law



$$F_x = kx$$

$$W = \tfrac{1}{2}kx^2$$

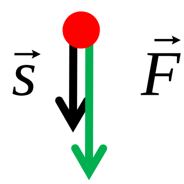
displacement from equilibrium

i-Clicker

cow

A ~~piece of fruit~~ falls straight down.
As it falls, the gravitational force:

cow



$$W = \vec{F} \cdot \vec{s}$$

A. Does positive work

B. Does negative work

C. Does no work

D. First does negative work, then positive work

E. First does positive work, then negative work

POWER

Rate at which work is done
(Rate at which energy changes)

$$P_{AV} = \frac{\Delta W}{\Delta t}$$

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Units: 1 Watt = 1 Joule/sec
1 Joule = 1 WS
(Light bulb ~ 100W)
(Humans ~ 100W)
(1 hp = 746W)

EXAMPLE: A 5 kg mass falls 3.2 meters.

- What is the average power generated by gravity?

$$\Delta W = F\Delta s = mg\Delta s = (5\,\text{kg})(10\,\text{m/s}^2)(3.2\,\text{m}) = 160\,\text{J}$$

$$\Delta s = \tfrac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2\Delta s}{g}} = \sqrt{\frac{2(3.2\,\text{m})}{(10\,\text{m/s}^2)}} = \sqrt{0.64}\,\text{s} = \boxed{0.8\,\text{s}}$$

$$\Rightarrow P_{AV} = \frac{(160\,\text{J})}{(0.8\,\text{s})} = 200\,\text{W}$$

The can stops the mass in 1/10 the time it takes to fall.

- How much power is supplied by the can?

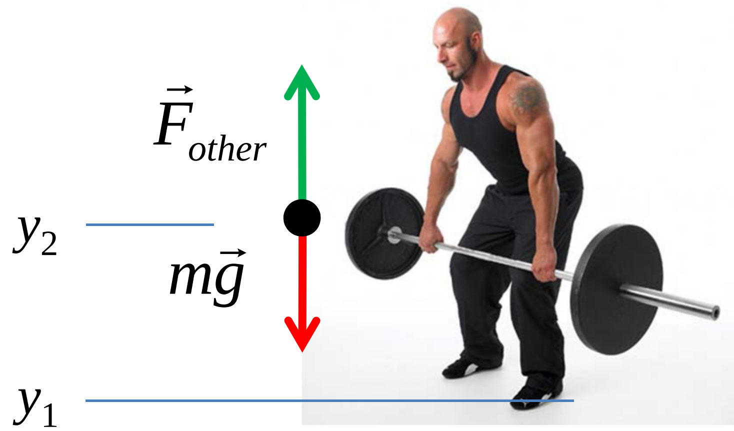
$$\Delta W = 160\,\text{J}$$

$$\Delta t = 0.08\,\text{s}$$

$$\Rightarrow P_{AV} = \frac{(160\,\text{J})}{(0.08\,\text{s})} = 2000\,\text{W} = 2\,\text{kW}$$

GRAVITATIONAL POTENTIAL ENERGY

Suppose you lift an object of mass m in the presence of gravity:



$$W_{tot} = \int_{y_1}^{y_2} F_{net}\,dy = \int_{y_1}^{y_2} (F_{other} - mg)\,dy = \underbrace{\int_{y_1}^{y_2} F_{other}\,dy}_{W_{other}} - \underbrace{\int_{y_1}^{y_2} mg\,dy}_{W_{grav}}$$

$$W_{tot} = W_{other} - mg(y_2 - y_1) = \Delta K = \tfrac{1}{2} m v_2{}^2 - \tfrac{1}{2} m v_1{}^2$$

Clearly mgy has units of energy

mgy = $U_g(y)$ = gravitational potential energy of mass m at height y.

$$W_{other} = K_2 - K_1 + U_{g_2} - U_{g_1}$$

Let $K + U_g = E$ = Total Energy

Then: $W_{other} = E_2 - E_1$ = Change in Total Energy

Suppose: $F_{other} = 0 \Rightarrow W_{other} = 0$

Then: $(K_2 + U_{g2}) - (K_1 + U_{g1}) = 0$

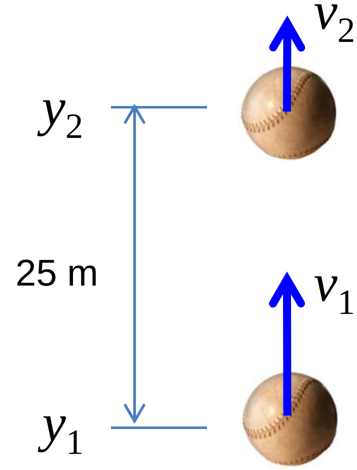
$$E_2 \quad - \quad E_1 \quad = 0$$

TOTAL ENERGY IS CONSERVED

(if $W_{other} = 0$)

EXAMPLE: A 0.15 kg baseball is thrown upward. After traveling 25m, it has a speed of 20 m/s.

What was the initial speed of the baseball?



$$K_1 + U_1 = (1/2)mv_1^2 + mgy_1 = E_1$$

$$K_2 + U_2 = (1/2)m(20 \text{ m/s})^2 + mgy_2 = E_2$$

$$E_2 - E_1 = 0 = (1/2)m(20 \text{ m/s})^2 + mgy_2 - (1/2)mv_1^2 - mgy_1$$

$$(1/2)mv_1^2 = (1/2)m(20 \text{ m/s})^2 + mg(y_2 - y_1)$$

$$v_1^2 = 400 \text{ m}^2/\text{s}^2 + 2(10 \text{ m/s}^2)(25 \text{ m}) = 900 \text{ m}^2/\text{s}^2$$

$$v_1 = 30 \text{ m/s}$$

Example (continued)

What is the maximum height the ball will reach?

$$K_3 + U_3 = 0 + mgy_{max}$$

$$K_1 + U_1 = (1/2)m(30 \text{ m/s})^2 + mgy_1$$

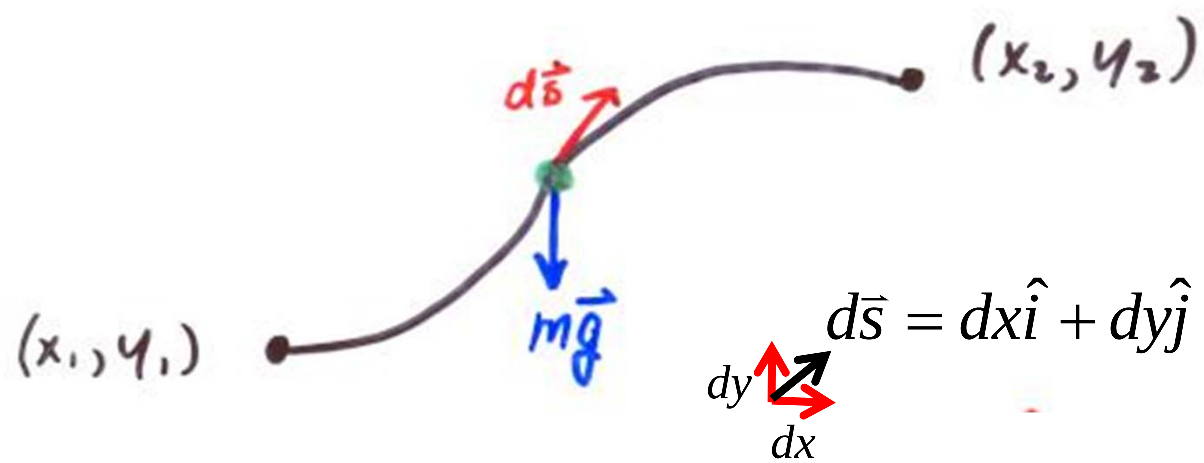
$$mgy_{max} - (1/2)m(900 \text{ m}^2/\text{s}^2) - mgy_1 = 0$$

$$mg(y_{max} - y_1) = (1/2)m(900 \text{ m}^2/\text{s}^2)$$

$$(y_{max} - y_1) = (450 \text{ m}^2/\text{s}^2)/(10 \text{ m/s}^2)$$

$$(y_{max} - y_1) = h = 45 \text{ m}$$

WHAT ABOUT MOTION ALONG A CURVED PATH?



$$d\vec{s} = dx\hat{i} + dy\hat{j}$$

$$W_{grav} = \int_1^2 \vec{F}_{grav} \cdot d\vec{s} = \int_1^2 (-mg\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= \int_1^2 (-mg)dy = -mg(y_2 - y_1) = U_{g_1} - U_{g_2}$$

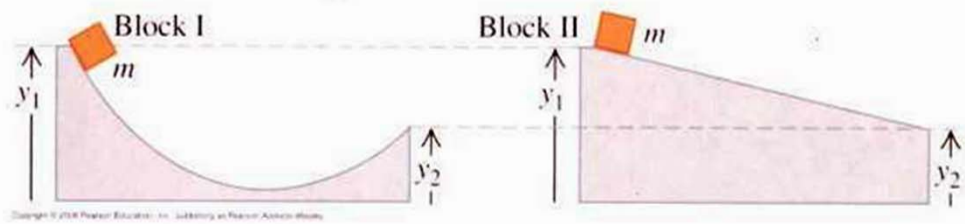
CHANGE of gravitational potential energy only depends on change of elevation

(NOTE: If other forces are present

THEN: $W_{other} = E_2 - E_1$)

i-Clicker

The two ramps shown are both frictionless. The heights y_1 and y_2 are the same for each ramp. A block of mass m is released from rest at the left-hand end of each ramp. Which block arrives at the right-hand end with the greater speed?



A. the block on the curved track

B. the block on the straight track

C. Both blocks arrive at the right-hand end with the same speed.

D. The answer depends on the shape of the curved track.

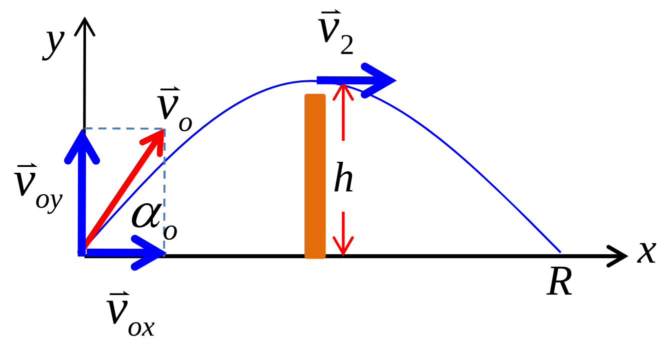
$$(K_2 + U_{g2}) - (K_1 + U_{g1}) = 0$$

$$(K_2 - K_1) = -(U_{g2} - U_{g1})$$

$$\left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) = (mgy_2 - mgy_1)$$

EXAMPLE: PROJECTILE MOTION

My snow blower ejects snow with a velocity $\vec{v}$. That makes an angle α_o with the horizontal.



What is the maximum height the snow flies?

$$\vec{v}_o = v_o \cos\alpha_o \hat{i} + v_o \sin\alpha_o \hat{j} = v_{ox}\hat{i} + v_{oy}\hat{j}$$

$$\vec{v}_2 = v_2\hat{i} + 0\hat{j}$$

$$a_x = 0 \Rightarrow v_2 = v_o \cos\alpha_o = v_{ox}$$

$$K_o = (1/2)mv_o^2 \quad U_o = 0$$

$$K_2 = (1/2)mv_2^2 \quad U_2 = mgy_{max}$$

$$K_2 = (1/2)m(v_o \cos\alpha_o)^2$$

$$K_o + U_o = K_2 + U_2$$

$$(1/2)mv_o^2 = (1/2)m(v_o \cos\alpha_o)^2 + mgy_{max}$$

$$(1/2)\cancel{m}[\cancel{(v_o \cos\alpha_o)^2} + (v_o \sin\alpha_o)^2] = (1/2)\cancel{m}\cancel{(v_o \cos\alpha_o)^2} + \cancel{m}gy_{max}$$

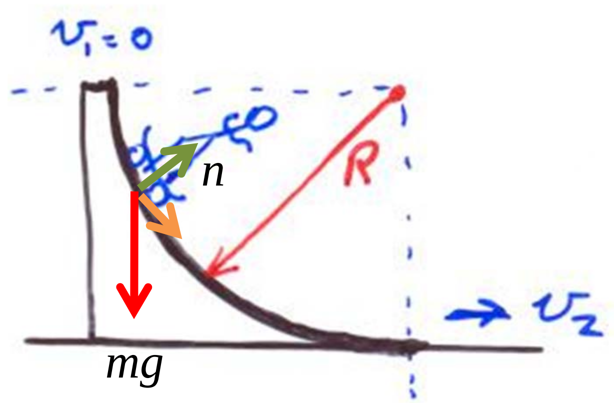
$$(1/2)(v_o \sin\alpha_o)^2 = gy_{max}$$

$$\rightarrow v_{oy}^2 = 2gy_{max}$$

MOTION ALONG A CIRCULAR ARC

Skate boarder drops into a (frictionless) quarter-pipe:

What is her speed when she exits?



$$\cancel{W_{other}} = E_2 - E_1 \quad = 0$$

What is W_{normal}? $= 0$

$$E_2 = K_2 + U_2 = (1/2)mv_2^2 + 0$$

$$E_1 = K_1 + U_1 = 0 + mgR$$

$$E_2 = E_1 \rightarrow (1/2)mv_2^2 = mgR$$

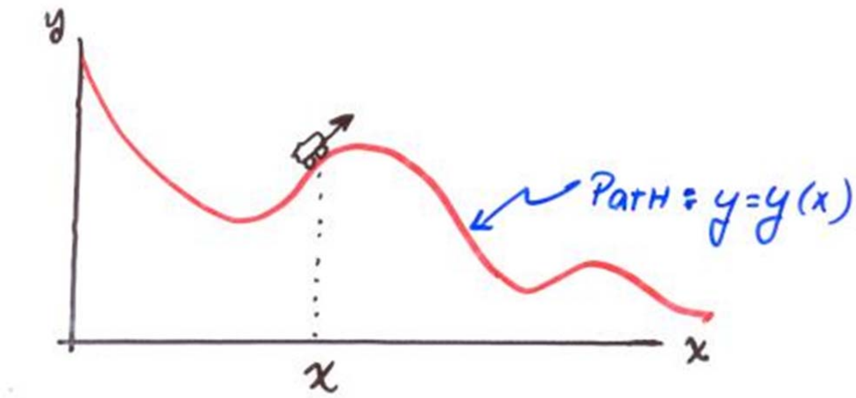
$$v_2^2 = 2gR$$

$$v_2 = \sqrt{2gR}$$

If friction were present?

$$W_f = E_2 - E_1$$

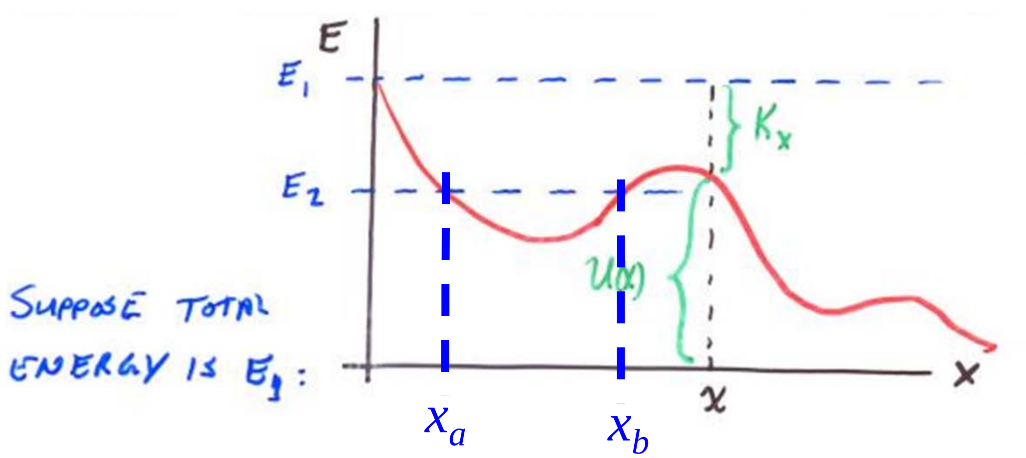
EXAMPLE: ROLLER COASTER



Potential energy is function of x because y is $y(x)$:

$$U(x) = mgy(x)$$

→ Plot $U(x)$ on energy vs. x diagram

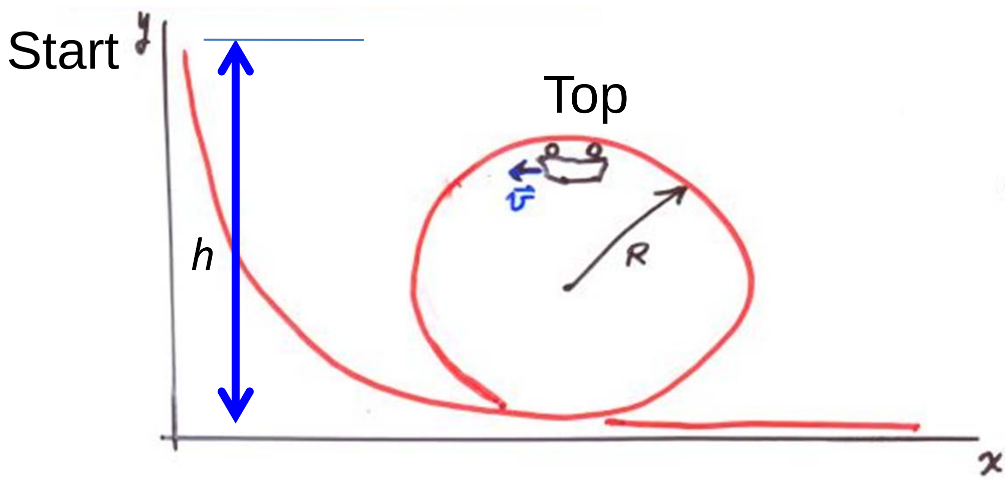


At any point x, $K_x + U(x) = E_1$

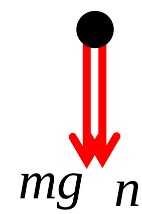
$$\Rightarrow K_x = \tfrac{1}{2} m v^2(x) = E_1 - U(x) = E_1 - mgy(x)$$

What if energy = E_2? Then: $x_a < x < x_b$

INSIDE THE LOOP



Free body diagram



At top of loop: $\sum_i F_{y_i} = ma_r = \frac{mv^2}{R} = mg + n$

$$n = \frac{mv^2}{R} - mg$$

$$n = 0 \Rightarrow v_{\min} \Rightarrow \frac{mv^2}{R} = mg$$

$$\Rightarrow v_{\min} = \sqrt{gR}$$

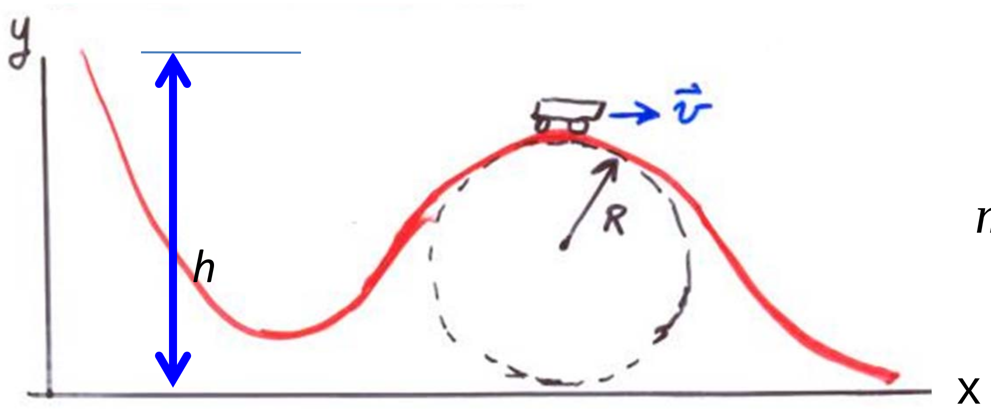
$$E_{\text{top}} = K_{\text{top}} + U_{\text{top}} = (1/2)mv_{\min}{}^2 + mg2R$$

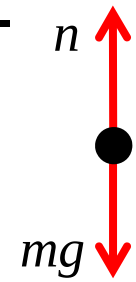
$$= (1/2)mgR + mg2R = (5/2)mgR$$

$$E_{\text{start}} = K_{\text{start}} + U_{\text{start}} = 0 + mgh = E_{\text{top}}$$

$$\rightarrow mgh = (5/2)mgR; \quad h = (5/2)R$$

OUTSIDE THE LOOP





Free body diagram

$$\text{At top of hill: } \sum_i F_{y_i} = ma_r \Rightarrow \frac{mv^2}{R} = mg - n$$

$$n = mg - \frac{mv^2}{R} \qquad v_{max} \text{ when } n = 0 \Rightarrow mg = \frac{mv^2}{R}$$

$$\Rightarrow v_{max} = \sqrt{gR}$$

So: $E_{TOPmax} = (1/2)mv_{max}^2 + mg2R = (5/2)mgR$

$$h_{max} = (5/2)R$$

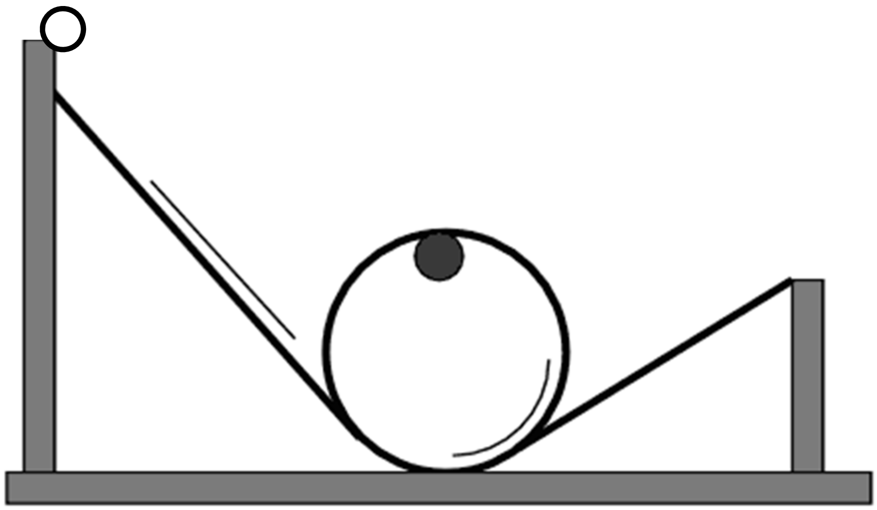
BUT: Need enough energy to get over the hill:

→ $E_{TOPmin} = U_{TOP} = mg2R = mgh_{min}$

$$h_{min} = 2R$$

$$2R < h < (5/2)R$$

i-Clicker



A ball starts at rest at the top of the ramp and is allowed to roll down the frictionless track through the loop-the-loop as shown. At the instant it is at the top of the loop, the ball's total mechanical energy:

A. Is greater than when is started
B. Is less than when is started.
C. Is equal to when it started.
D. Is greater than it is at the bottom of the loop.
E. Cannot tell without knowing the mass of the ball.

No "other" forces, so :

$$\left(K_2 + U_{g2}\right) - \left(K_1 + U_{g1}\right) = 0$$

$$E_2 \quad - \quad E_1 \quad = 0$$