

P123 F11 Lecture 5 30 September 2010

Newton's Laws of Motion

TOPICS FROM PREVIOUS LECTURE (motion in 2-D)

- Components of Equation of Motion

$$v_x = v_{ox} + a_x t$$

$$v_y = v_{oy} + a_y t$$

$$x = x_o + v_{ox} t + 1/2 a_x t^2$$

$$y = y_o + v_{oy} t + 1/2 a_y t^2$$

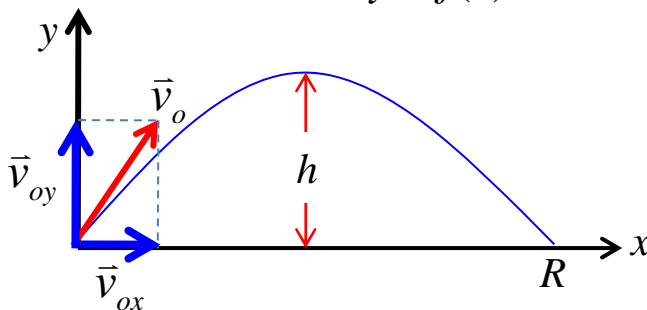
etc., ...

- Projectile Motion

Superposition of two motions:

Constant v_x and free-fall v_y

Then solve for : $y = f(x)$ or trajectory:



$$h = \frac{v_o^2 \sin^2 \alpha_o}{2g}$$

$$R = \frac{v_o^2 \sin 2\alpha_o}{g}$$

- Relative Velocity:

(e.g., cars on highway, airplane in wind, boat on river, ...)

Fixed Frame: A

Moving Frame: B

Moving Object: P

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

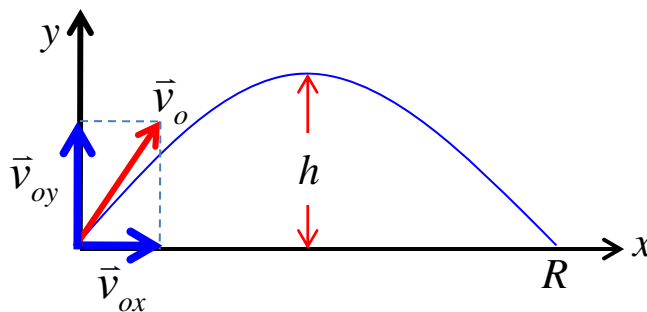
i-Clicker

A projectile is launched at a 30° angle above the horizontal. Ignore air resistance. The projectile's acceleration is greatest:

- A. at a point between the launch point and the high point of the trajectory.
- B. at the high point of the trajectory.
- C. at a point between the high point of the trajectory and where it hits the ground.

D. misleading question — the acceleration is the same (but nonzero) at all points along the trajectory

E. misleading question — the acceleration is zero at all points along the trajectory



$$a_x = 0 ; a_y = -g$$

NEWTON'S LAWS OF MOTION

So Far: Follow motion in 1-D and 2-D based solely on definitions of v and a in terms of position as a function of time. (KINEMATICS)

NOW: Determine motion based on the underlying physical cause of that motion. (DYNAMICS)

“Laws of Motion” formulated by Newton over 300 years ago

Forces and interactions

Common experience: Force is a “push” or a “pull”



Different types of forces:

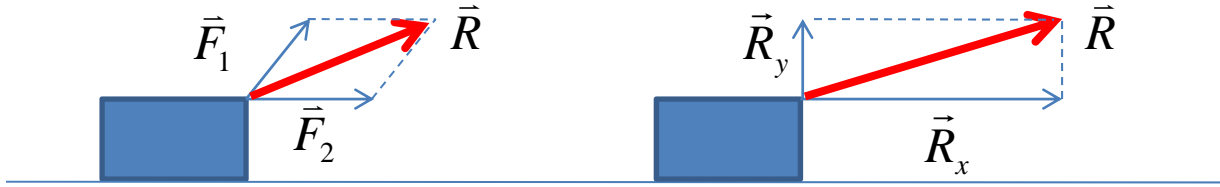
- Contact Force (kick soccer ball, push book, tow car, ...)
- Long range forces, act through “empty” space (electric force, magnetic force, gravitational force, ...)

Unit of force in SI units: Newton [N]

We will focus on contact forces and gravity (near the earth’s surface)

Superposition of forces

Forces are vectors → add to give resultant.

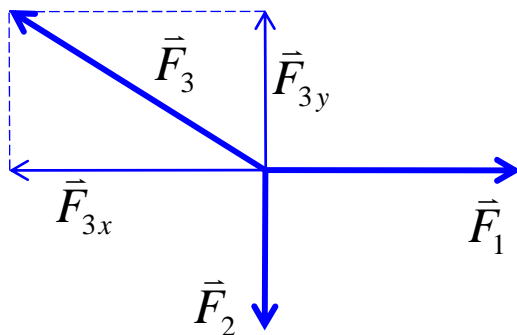


NET FORCE: $\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$

$$R_x = \sum F_x = F_{1x} + F_{2x} + F_{3x} + \dots \quad R_y = \sum F_y = F_{1y} + F_{2y} + F_{3y} + \dots$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

EQUILIBRIUM: Net force = 0



$$\sum \vec{F} = 0 ?$$

$$\sum F_x = 0; \quad \sum F_y = 0; \quad \sum F_z = 0$$

Newton's first law

~~Ancient Idea: The natural state of an object is at rest.
If an object moves, it's because a force acts on it.~~

NEWTON: The nature of a body is to resist change in motion.

Newton's First Law

When no net force acts on an object, its acceleration is zero

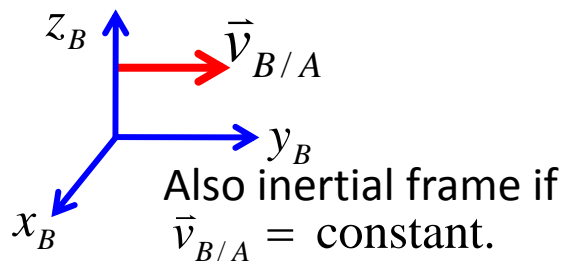
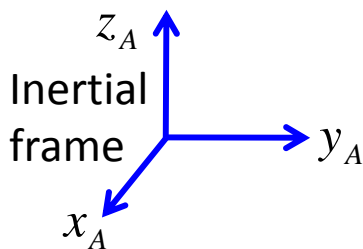
$$\text{If } \sum \vec{F} = 0 \text{ then } \vec{a} = 0$$

→ An isolated object with **no net force** acting on it is either **at rest**, or is moving with a **constant velocity**.

INERTIA: The tendency of a body to resist change in \vec{v} .

INERTIAL FRAMES OF REFERENCE

An inertial frame of reference is one in which Newton's 1st Law is valid (The frame is not accelerating!!)



- A reference frame that moves at constant velocity relative to a distant star is the best approx. of an inertial frame.
- Earth is approximately an inertial frame
- Accelerating elevator is **NOT** an inertial frame.

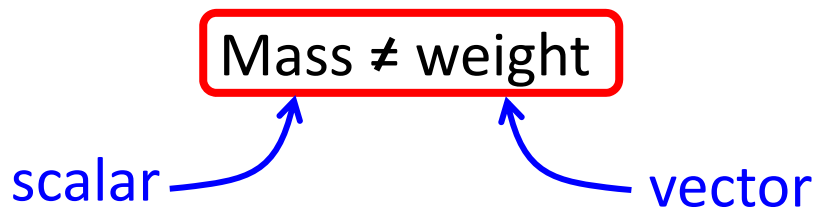
Newton's Second Law

Background: From Newton's 1st Law, $\vec{a} = 0$ when $\sum \vec{F} = 0$
But when $\sum \vec{F} \neq 0$ the net force causes the object to accelerate in the same direction as the net force. The magnitude of \vec{a} is directly proportional to $\sum \vec{F}$

Mass and Force:

- Force changes motion of an object
- Mass (inertia) resists change in motion

$$m = \frac{|\sum \vec{F}|}{a} \quad \text{or} \quad |\sum \vec{F}| = ma$$



BUT.... Weight is proportional to mass.

- The mass of a body is the same everywhere in the universe.
- The weight of a body is not!!

Statement Of Newton's Second Law

Background: The acceleration of an object is directly proportional to the net force acting on that object, and inversely proportional to its mass.

$$a = \frac{|\sum \vec{F}|}{m} \quad \text{or} \quad |\sum \vec{F}| = ma$$

$$\sum \vec{F} = m\vec{a} \Rightarrow \sum F_x = ma_x; \quad \sum F_y = ma_y; \quad \sum F_z = ma_z$$

Unit of force: Newton [N]

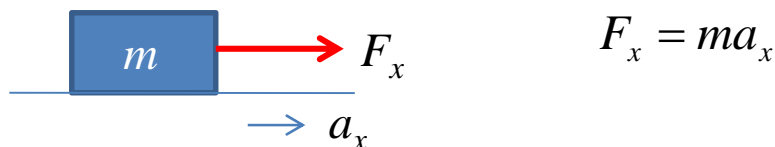
$$1 \text{ N} = (1 \text{ kg}) \times (1 \text{ m/s}^2) = 1 \text{ kg}\cdot\text{m/s}^2$$

(in cgs units: 1 dyne = 1 g.cm/s²)

(in British units: 1 lb = 1 slug X 1 ft/s²)

$$(1 \text{ lb} = 4.448 \text{ N})$$

EXAMPLE: A block ($m = 5.0 \text{ kg}$) is at rest at $t = 0$. Then, a constant force F_x acts on it. At $t = 3.0 \text{ s}$, the block has moved 2.3 m. Find F_x .



Have: m , need a_x . But, also have t and $(x - x_o)$.

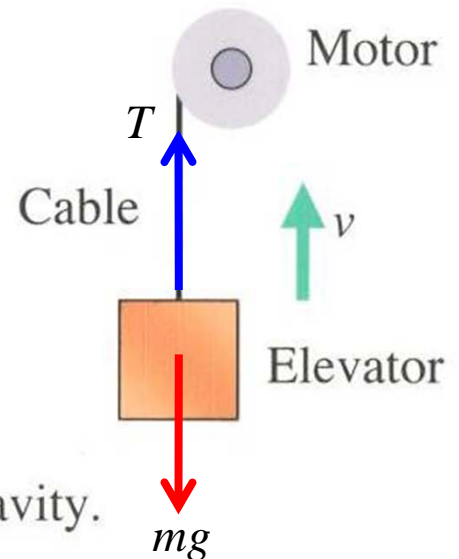
$$(x - x_o) = v_o t + \frac{1}{2} a_x t^2 \rightarrow \frac{2(x - x_o)}{t^2} = a_x \rightarrow \frac{2(2.3 \text{ m})}{(3.0 \text{ s})^2} = a_x = \frac{4.6}{9} \text{ m/s}^2 = 0.51 \text{ m/s}^2$$

$$\text{So: } F_x = (5.0 \text{ kg})(0.51 \text{ m/s}^2) = 2.6 \text{ kg}\cdot\text{m/s}^2 = 2.6 \text{ N}$$

i-Clicker

An elevator is being lifted at a constant speed by a steel cable attached to an electric motor. There is no air resistance, nor is there any friction between the elevator and the walls of the elevator shaft.

The upward force exerted on the elevator by the cable is



- A. greater than the downward force of gravity.
- B. equal to the force of gravity.**
- C. less than the force of gravity.
- D. any of the above, depending on the speed of the elevator.

Elevator moves, but at constant velocity $\rightarrow a = 0$

$$F = ma, \text{ therefore } \sum \vec{F} = 0 \text{ in particular } \sum F_y = 0$$

Therefore, $T - mg = 0$; $T = mg$.

Mass And Weight

Weight is the attractive force, \vec{F}_g , exerted by the earth's gravity on an object.

$$\vec{F}_g = \text{weight of object} = \vec{w}$$

Because $\sum \vec{F} = m\vec{a}$ and $\vec{a} = \vec{g}$ then in free fall (i.e., no force other than gravity acting on object) we have

$$\vec{w} = m\vec{g}$$

So we can compare masses by measuring weight at earth's surface.

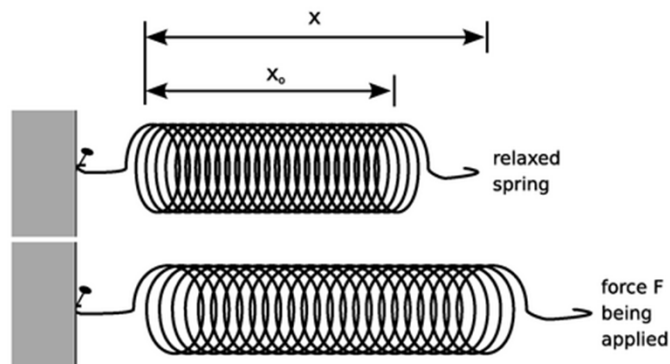
Note:

- Weight is a force that acts on a body at all times
- \vec{g} varies with location, we will use $g = 9.8 \text{ m/s}^2$.
- Avoid incorrect usage (i.e. an object's weight is 2 kg)

Measuring forces:

Convenient to use elongation of a spring.

(later, we will discuss Hooke's law which states that the elongation of a spring is directly proportional to the force exerted on it.)

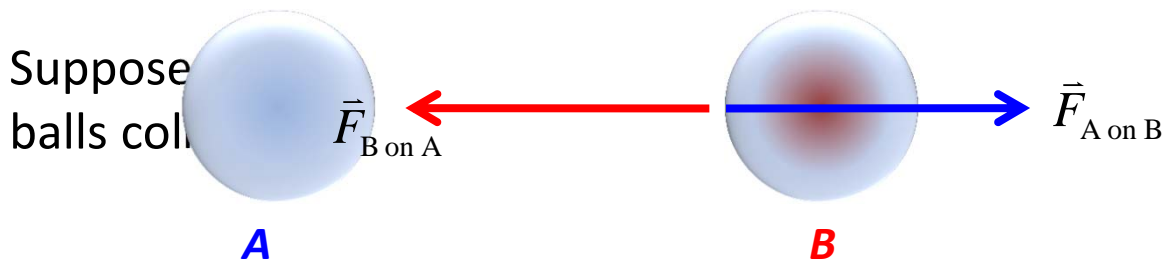


Newton's Third Law

Whenever two **isolated** objects interact, the forces that the objects exert on each other are always equal in magnitude and opposite in direction.

Restated: If object A exerts a force on B (an “action”) then B exerts a force on A (a “reaction”). These **two** forces:

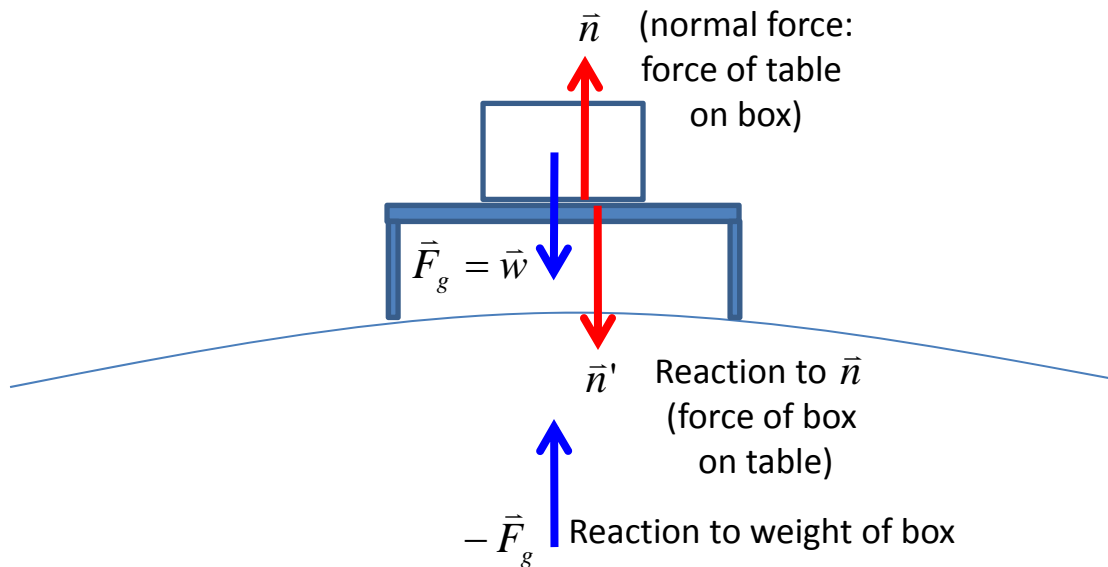
- Have the same magnitude
- Are opposite in direction
- Act on different bodies!!!!



“action-reaction pair” $\vec{F}_{B \text{ on } A} = -\vec{F}_{A \text{ on } B}$

Forces exist in pairs, acting on different objects

Consider a box on a table on the earth

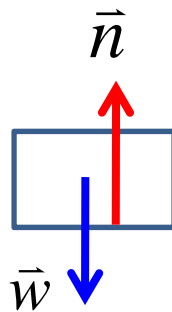


Simpler to understand if consider forces on box alone:

Box is at rest, so:

$$\sum \vec{F} = 0$$

$$\sum \vec{F}_x = 0 \quad \sum \vec{F}_y = 0$$



\vec{n} and \vec{w} are **NOT** action/reaction pair. They act on same object.

\vec{n} and \vec{n}'

ARE action/reaction pair

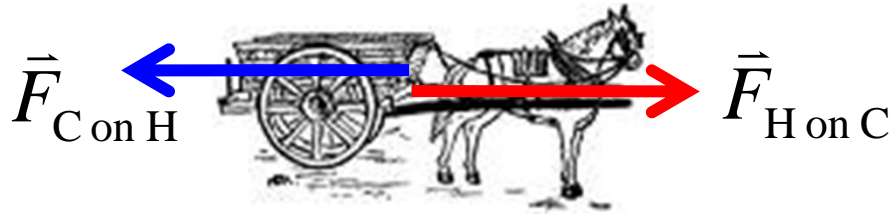
x : no forces

$$y: n - mg = 0 \rightarrow n = mg$$

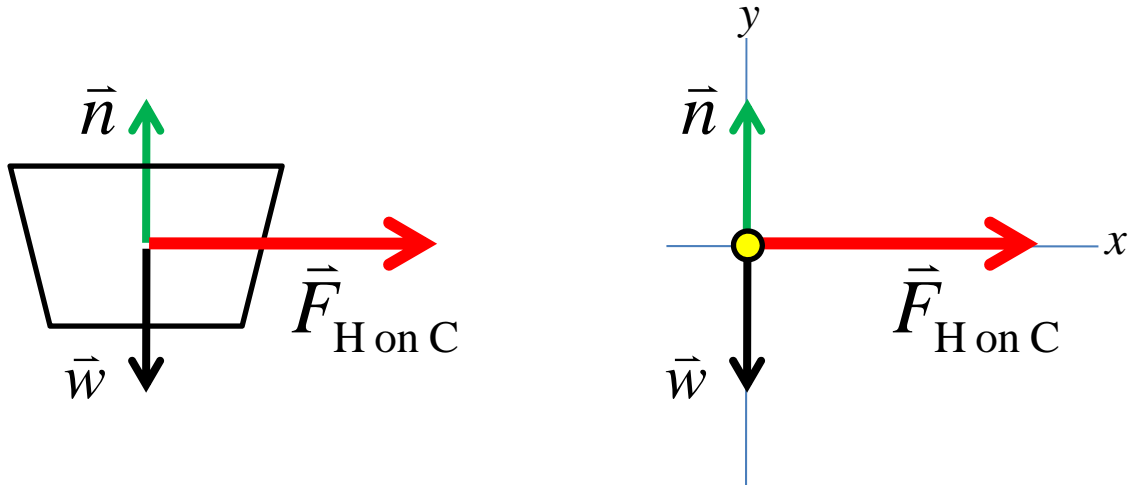
Free Body Diagrams: an aid to solving problems

- $\sum \vec{F} = m\vec{a}$ applied to one object with mass m (or a group moving as one).
- Use only forces applied to this body
- Draw diagram for each body separately
- Choose convenient coordinate system
- Apply Newton's Laws in component form.

EXAMPLE: Apparent contradiction – a horse pulling a cart.
 If the force of horse on cart equals the force of cart on horse
 how does the cart move???



Isolate cart and draw free body diagram



At the instant the cart moves, it accelerates:

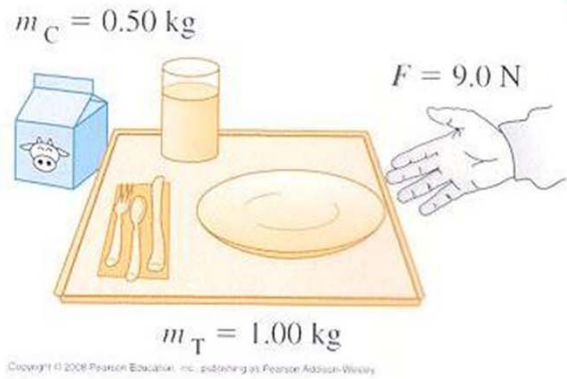
1st law: $\sum \vec{F} \neq 0$ so \vec{v} is NOT constant (acceleration)

2nd law: $\sum \vec{F} = m\vec{a}$
 $\sum F_y = n - w = 0$
 $\sum F_x = ma_x = F_{HC} \Rightarrow a_x = \frac{F_{HC}}{m}$

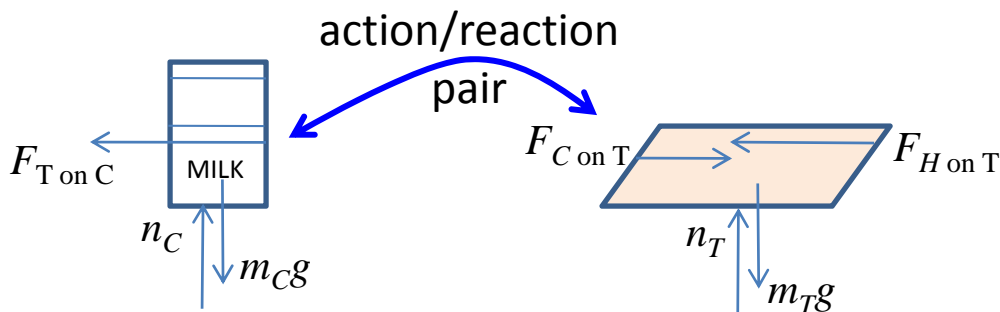
3rd law: Note that \vec{n}, \vec{w} are NOT reaction pair!!!

i-Clicker

You are pushing a 1.00-kg food tray through the cafeteria line with a constant 9.0-N force. As the tray moves, it pushes on a 0.50-kg milk carton. If the food tray and milk carton are *accelerating to the left*,

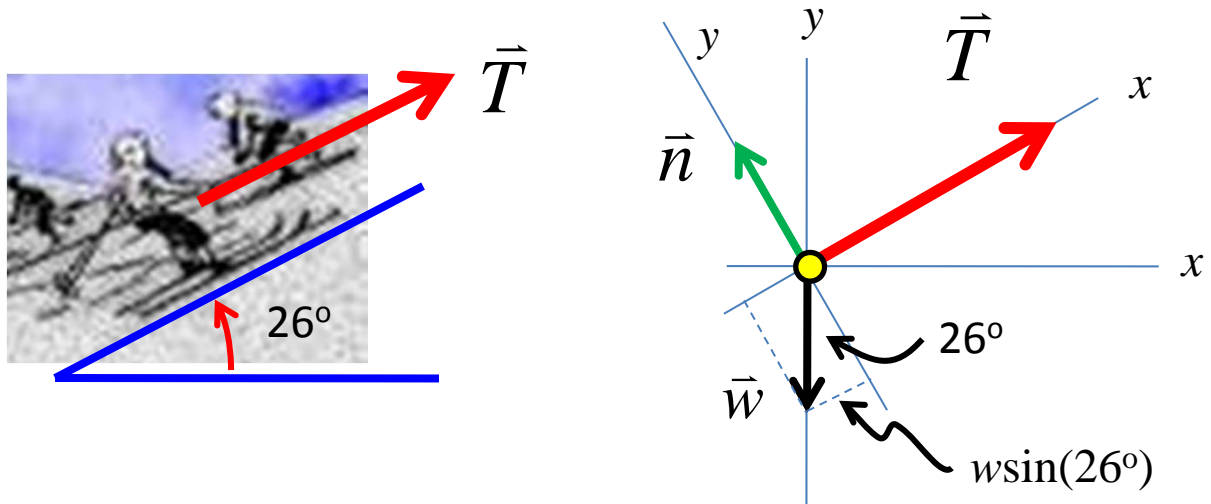


- A. the tray exerts more force on the milk carton than the milk carton exerts on the tray.
- B. the tray exerts less force on the milk carton than the milk carton exerts on the tray.
- C. the tray exerts as much force on the milk carton as the milk carton exerts on the tray.



$$\vec{F}_{T \text{ on } C} = -\vec{F}_{C \text{ on } T}$$

EXAMPLE: A skier of mass 65.0 kg is pulled by a tow rope up a snow-covered ski slope at a constant speed. The rope and the slope are inclined by 26° . Draw a free body diagram and calculate the tension in the tow rope.



moving, but no acceleration $\rightarrow \sum \vec{F} = 0$

consider components:

$$\sum F_y = 0 = n - w \cos(26^\circ)$$

$$\sum F_x = 0 = T - w \sin(26^\circ)$$

$$T = w \sin(26^\circ) = mg \sin(26^\circ)$$

$$T = (65.0 \text{ kg}) * (9.80 \text{ m/s}^2) * \sin(26^\circ)$$

$$T = 279 \text{ N}$$