# P123F11 - Lecture 2 - 9 Sept 2011

## 1 Dimensional Motion

Review:

- Displacement: $\Delta x = x_2 - x_1$

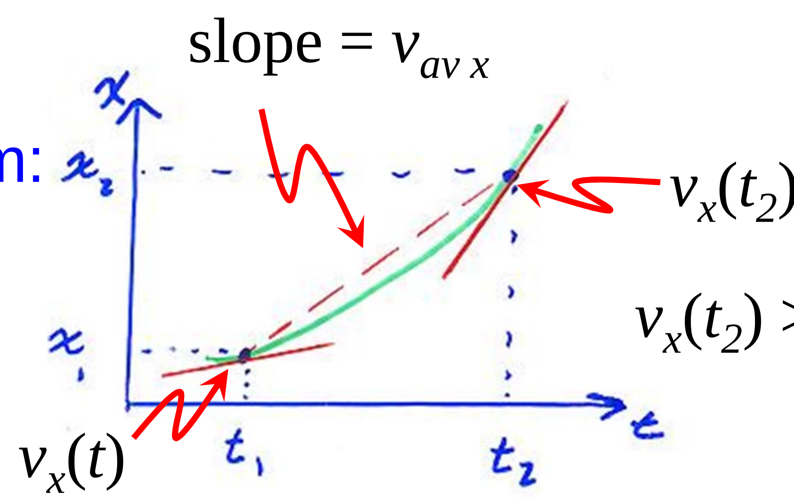
(Vector → If $\Delta x < 0$ then $x_1$ "to the right" of $x_2$)

- Average velocity: $v_{av_x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

(Not the same as average speed)


$x$-$t$ diagram:

$x = x(t)$




$v_x(t_2) > v_x(t_1)$

- Instantaneous velocity: $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

$v_x(t)$ is slope of tangent to $x$-$t$ plot at time $t$.

# *i-Clicker Question*

How many beans are in the 900 ml beaker?

A. Fewer than 1000

B. 1000-1500

C. 1500-2000

D. 2000-2500

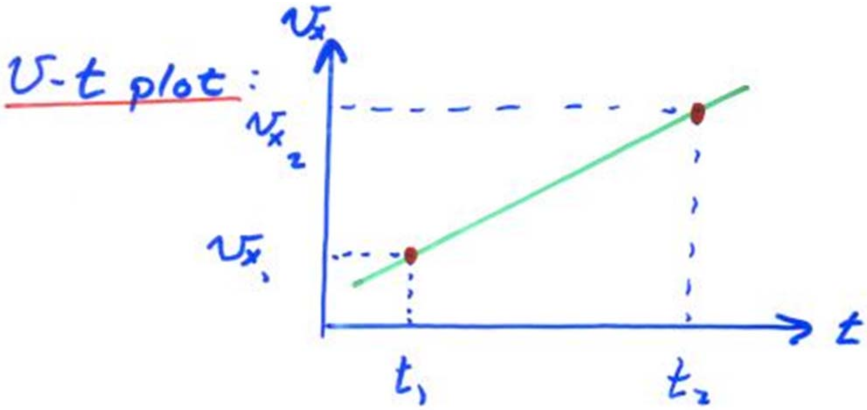
E. More than 2500

Consider $x$-$t$ plot on last slide: $v_x(t_1) < v_x(t_2)$

→ $v_x$ is not constant in time

→ $v_x = v_x(t)$ → acceleration

- acceleration: time rate of change of velocity




- average acceleration: $a_{av_x} = \frac{v_{x2} - v_{x1}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$

- instantaneous acceleration: $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$

(slope of line tangent to $v_x(t)$ at time $t$ )

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

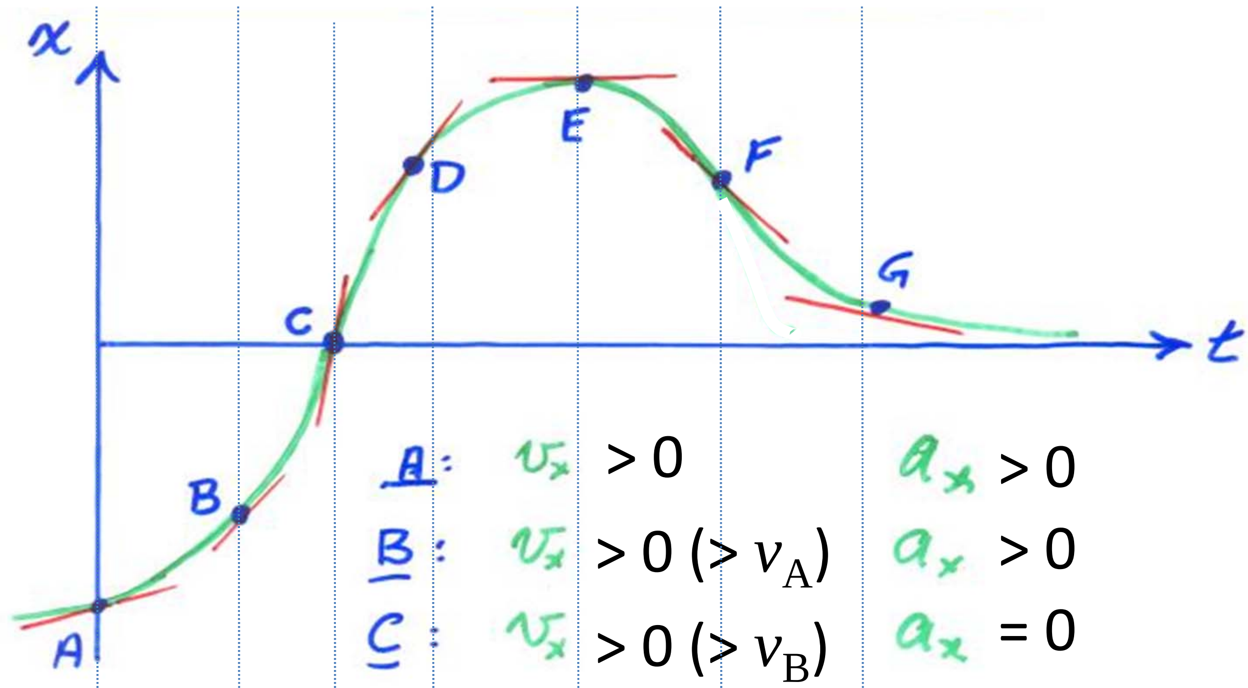
In $v$-$t$ plot above, $v_x(t)$ is a line → constant acceleration

i.e.: $v_x(t) = [\text{const}]t$ → $dv_x/dt = [\text{const}]$

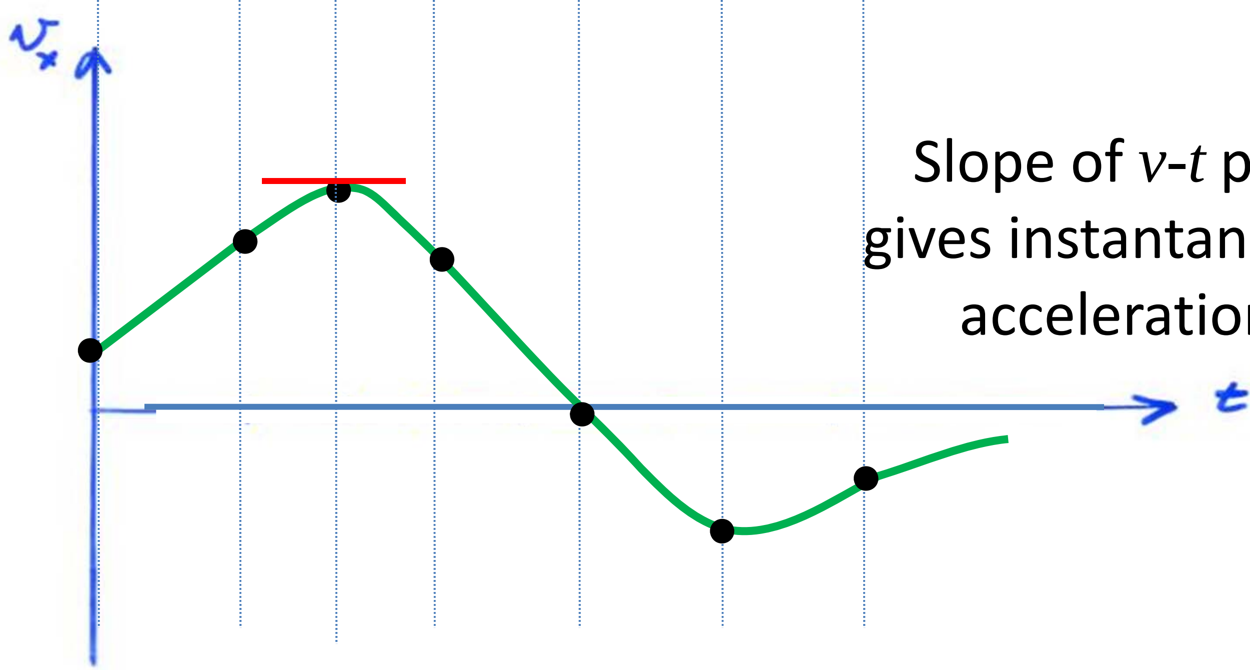
[Not always true! Suppose: $v_x(t) = Ct^3$

Then: $a_x = a_x(t) = dv_x/dt = 3Ct^2 \neq$ constant in time!]

# Acceleration from $x$-$t$ plot:




| | $v_x$ | $a_x$ |
|---|---|---|
| A: | $> 0$ | $> 0$ |
| B: | $> 0$ ($> v_A$) | $> 0$ |
| C: | $> 0$ ($> v_B$) | $= 0$ |
| D: | $> 0$ ($< v_C$) | $< 0$ |
| E: | $= 0$ | $< 0$ |
| F: | $< 0$ | $= 0$ |
| G: | $< 0$ | $> 0$ |

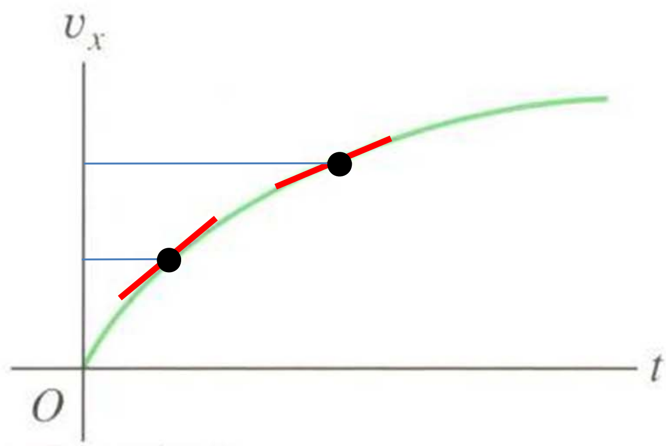



Slope of $v$-$t$ plot gives instantaneous acceleration

# *i-Clicker Question*

This is the $v_x$–$t$ graph for an object moving along the $x$-axis.

Which of the following descriptions of the motion is most accurate?




A. The object is slowing down at a decreasing rate.

B. The object is slowing down at an increasing rate.

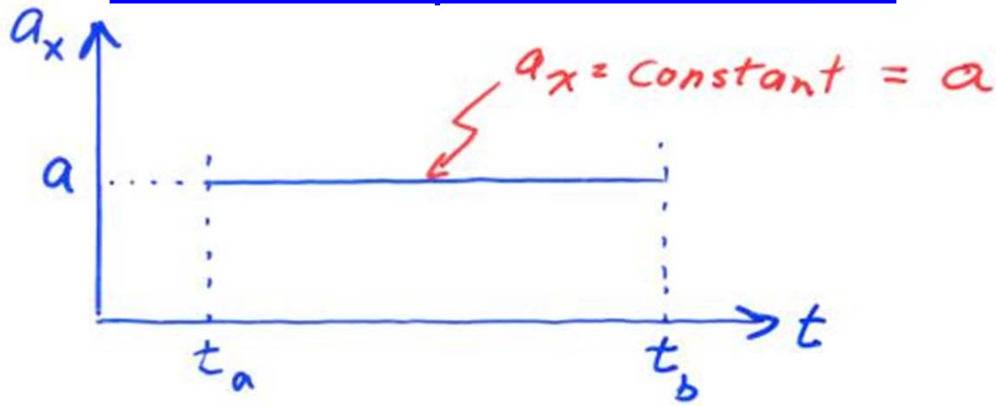
C. The object is speeding up at a decreasing rate.

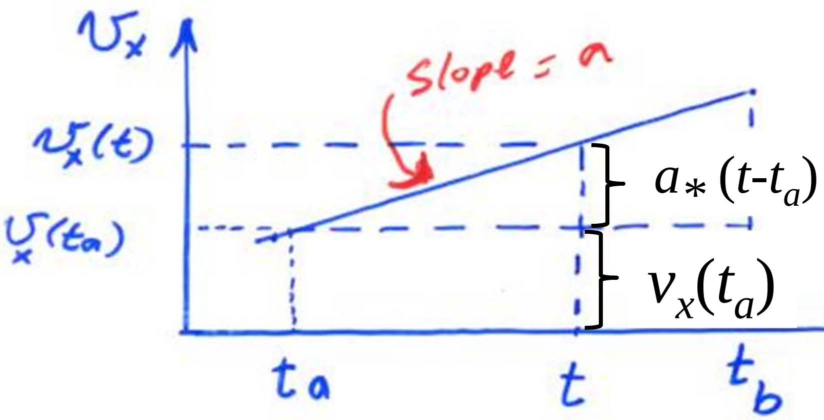
D. The object is speeding up at an increasing rate.

E. The object's speed is changing at a steady rate.

Constant acceleration is an important special case!

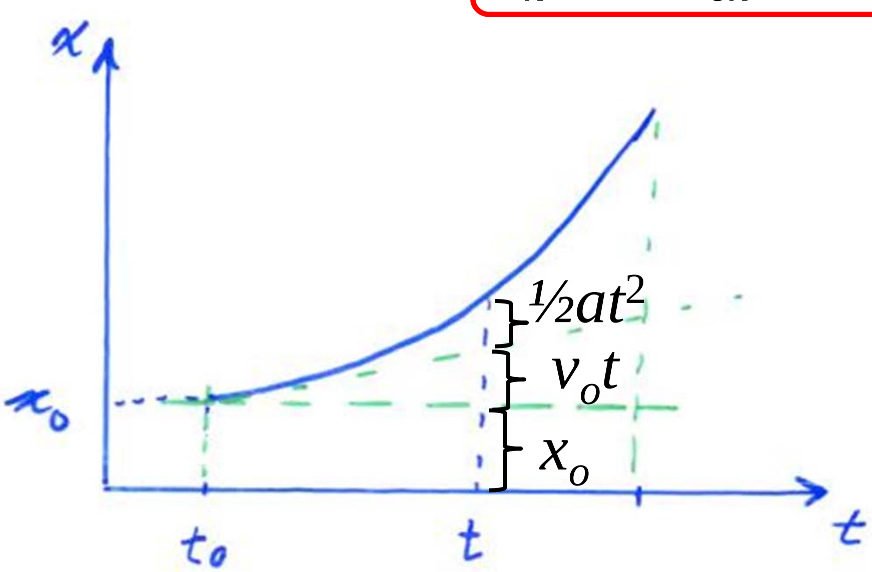
<u>Deserves special attention!!</u>




$v(t) = v_x(t_a) + a_*(t-t_a)$

Let $t_a = 0$ → $v_x(t) = v_{ox} + at$




$$x(t) = x_o + v_o t + ½at^2$$

# IMPORTANT EQUATIONS

(1) $v_x(t) = v_{ox} + at$

(2) $x(t) = x_o + v_{ox}(t) + ½\, at^2$

} constant acceleration

## Other helpful relationships:

(3) $v_{av_x} = \frac{x - x_o}{t}$ ; (4) $v_{av_x} = \frac{v_{ox} + v_x}{2}$

} const. acc. only

## ALGEBRA:

(3) → $x - x_o = v_{av}\, t$

sub $v_{av_x}$ from (4)

$$(x - x_o) = \left[\frac{v_x + v_o}{2}\right] t$$ (5)

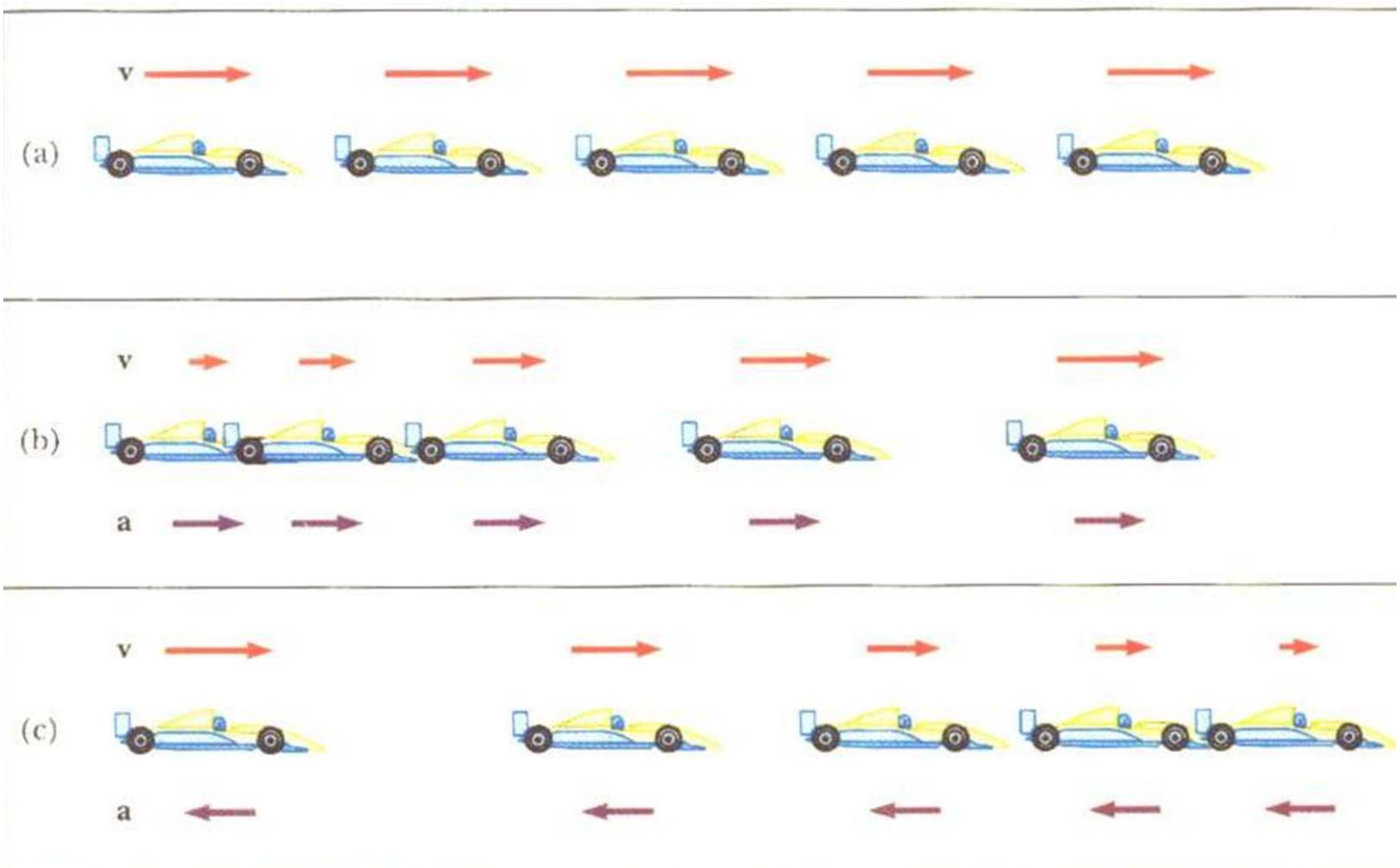
rewrite (1) → $t = (v_x - v_o) / a$ (6)

plug (6) into (5)

$$(x - x_o) = \left[\frac{v_x + v_o}{2}\right]\left[\frac{v_x - v_o}{a}\right]$$

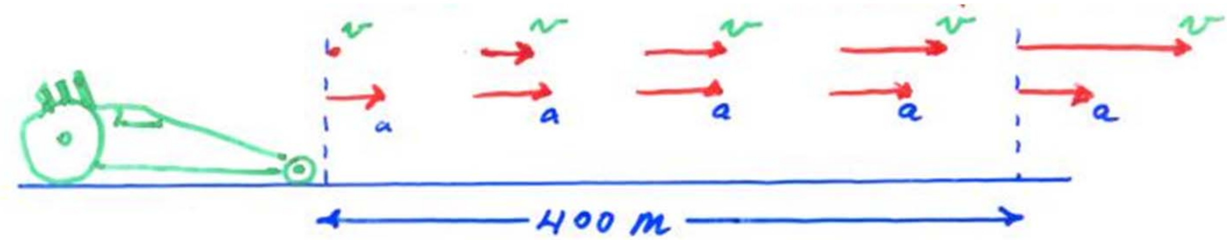
$$(x - x_o) = \frac{v_x^2 - v_o^2}{2a}$$

# Motion diagrams showing position, velocity and acceleration



# Lets put these equations to work!

Drag race: Constant acceleration along 400 m track. $v_x$ = 150 m/s at end.




- What is the acceleration?

Known: $(x - x_o) = 400$ m; $v_x = 150$ m/s; $v_o = 0$

Need: $a = ?$

$$(x - x_o) = \frac{v_x^2 - v_o^2}{2a} \qquad a = \frac{v_x^2 - \cancelto{0}{v_o^2}}{2(x - x_o)}$$

$$a = \frac{(150\ \text{m/s})^2}{2(400\ \text{m})} = 28\ \text{m/s}^2$$

- How long does the race take?

Known: $(x - x_o)$, $v_x$, $v_o$, and $a$

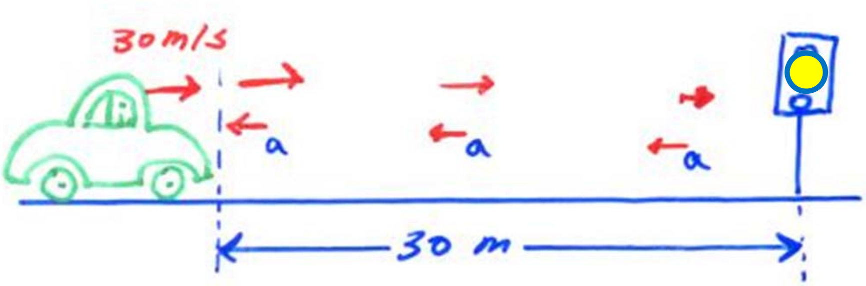
Need: $t = ?$

$$x(t) = x_o + v_o t + \tfrac{1}{2}at^2 \qquad x - x_o = \cancelto{0}{v_o t} + \tfrac{1}{2}at^2$$

$$400\ \text{m} = \tfrac{1}{2}\,(28\ \text{m/s}^2)\,t^2 \rightarrow$$

$$t = 5.3\ \text{s}$$

# Yellow Light




- Driving at 30 m/s
- Light turns yellow when you are 30 m from int.
- Decelerate at 10 m/s$^2$.
- ***Will you stop before intersection?*** ***No!***

Known: $v_o = 30$ m/s; $a = -10$ m/s$^2$ ; $v_f = 0$ m/s;

Need: $(x\text{-}x_o) = ?$

$$(x - x_o) = \frac{v_x^2 - v_o^2}{2a} = \frac{0 - (30\ \text{m/s})^2}{2(-10\ \text{m/s}^2)} = 45\ \text{m}$$

- ***What should a be?***

Known: $(x\text{-}x_o) = 30$ m; $v_o = 30$ m/s; $v_f = 0$ m/s

Need: $a = ?$

$$a = \frac{v_x^2 - v_o^2}{2(x - x_o)} = \frac{0 - (30\ \text{m/s})^2}{2(30\ \text{m})} = -15\ \text{m/s}^2$$

- ***If a = -30 m/s$^2$, where will I stop?***

$(x\text{-}x_o) \sim 1/a$ so $(x\text{-}x_o) = 15$ m

# FREE FALL

## Motion in 1-D under the influence of gravity.

- acceleration due to gravity is constant (at Earth's surface)

$$a = -g \quad \text{where} \quad g = 9.80 \text{ m/s}^2$$

- gravity acts vertically downward (choose $y$-axis as vertical)

→ Same equations of motion..

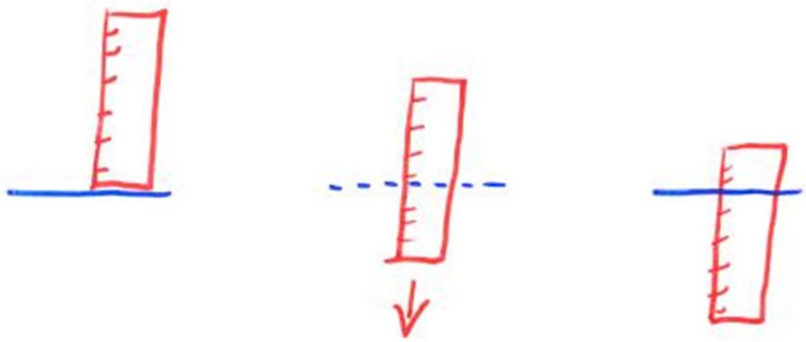
$$(y - y_o) = \tfrac{1}{2}(v_{yo} + v_y)\, t$$

$$v_y = v_{oy} - gt$$

$$y = y_o + v_{oy}t - \tfrac{1}{2}gt^2$$

$$(y - y_o) = \frac{v_y^2 - v_o^2}{-2g}$$

# EXAMPLE: REACTION TIME (red rulers)



Known: $y_o = 0$ m; $v_{yo} = 0$ m/s ; $a = -g$ ; $y_f = -0.010$ m

Need: $t = ??$

$$y = y_o^{\,0} + v_{oy}^{\,0} t - \tfrac{1}{2} g t^2$$

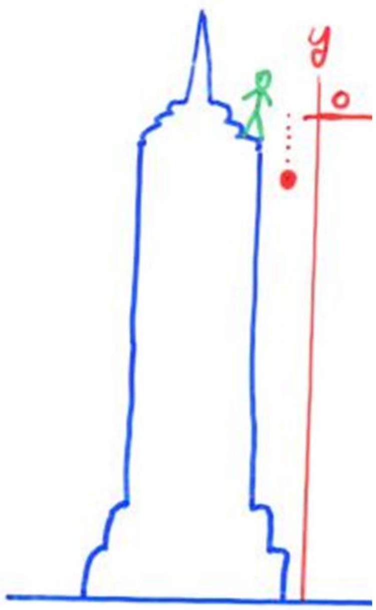
$$y_f = -\tfrac{1}{2} g t^2$$

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-0.10\text{ m})}{9.8\text{ m/s}^2}}$$

$$t = \sqrt{0.02\text{ s}^2} = 0.14\text{ s}$$

EXAMPLE: Drop a penny from top of the Empire State Building !

(DO NOT TRY THIS!)




Observe: The penny takes 8.1 s to hit ground

- How tall is building?

Known: $v_{yo}$ = 0 m/s; $a = -g$; $t$ = 8.1 s; $y_o = 0$

Need: $y - y_o$

$$y = \cancel{y_o}^{\,0} + \cancel{v_{oy}}^{\,0}t - \tfrac{1}{2}gt^2$$

$$y = -\tfrac{1}{2}gt^2 = -(\tfrac{1}{2})(9.8\text{ m/s}^2)(8.1\text{ s})^2$$

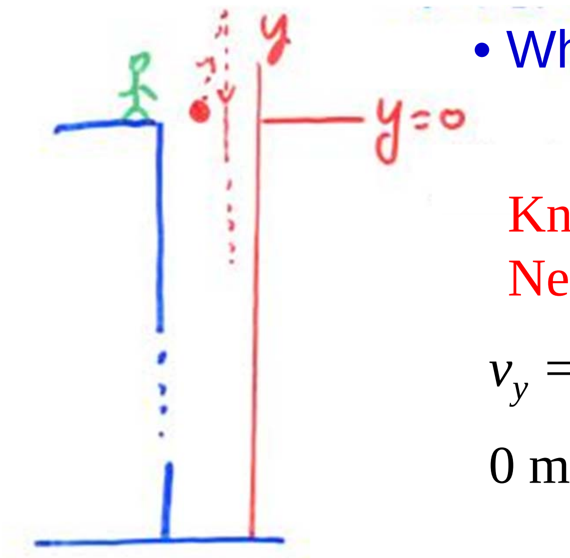
$$y = -320\text{ m}$$

- What's the velocity of the penny just before it hits the ground?

Known: $v_{yo}$ = 0 m/s; $a = -g$; $t$ = 8.1 s; and $(y - y_o)$= -320 m

$$v = -gt = -(9.8\text{ m/s}^2)(8.1\text{ s}) = -79\text{ m/s}$$

What if I first throw coin upward with speed of 67 mi/hr (=30 m/s)?




- When will coin reach max height? (above starting point)

Known: $v_{yo}$ = +30 m/s; $a$ = -g
Need: $t$ when $v_y = 0$

$$v_y = v_{oy} - gt$$

$$0 \text{ m} = 30 \text{ m/s} - (9.8 \text{ m/s}^2)t$$

$$t = \frac{v_{yo}}{g} = \frac{30\text{ m/s}}{9.8\text{ m/s}^2} = 3\text{ s}$$

- When will it pass me on the way down?

$$y = y_o + v_{oy}t - \tfrac{1}{2}gt^2 \quad \text{but } y = y_o = 0$$

$$0 = v_{oy}t - \tfrac{1}{2}gt^2 = t\,(v_{oy} - \tfrac{1}{2}gt)$$

$$t = 0 \quad \text{or} \quad t = 6\text{ s}$$

- What is speed just before hitting ground?

$$(y - y_o) = \frac{v_y^2 - v_o^2}{-2g} \qquad v_y = -85\text{ m/s}$$