P123 F11 Lecture 10 4 Nov 2011

ELASTIC POTENTIAL ENERGY AND ENERGY CONSERVATION

Review of last lecture

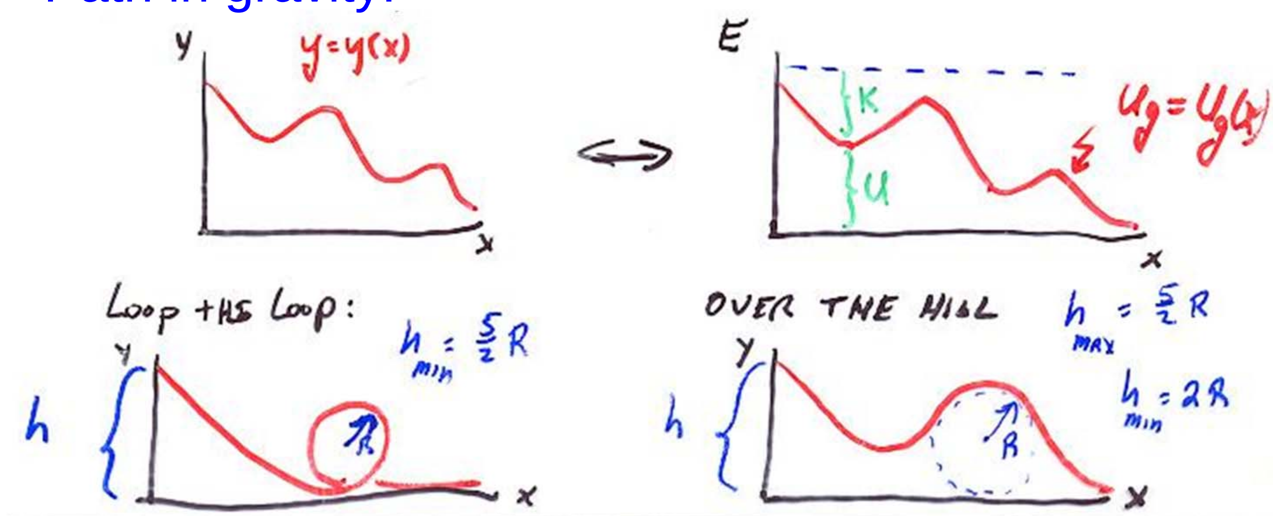
- Power: $P_{av} = \dfrac{\Delta W}{\Delta t}$ $\quad P = \dfrac{dW}{dt}$

  1 Watt = 1 Joule/Sec

- Gravitational Potential Energy: $U_g = mgy$
- Work – Energy Theorem: $W_{other} = K_2 - K_1 + U_{g\,2} - U_{g\,1}$
- Let E = K + U (Total Energy) : $W_{other} = E_2 - E_1$

  If: $W_{other} = 0$ then $E_2 - E_1 = 0$

- Curved path in gravity: $W_g = U_{g\,1} - U_{g\,2}$
- Path in gravity:



i-Clicker

Three identical balls are thrown with equals speeds from the top of a building at angles to the horizontal as shown. Neglect air resistance.

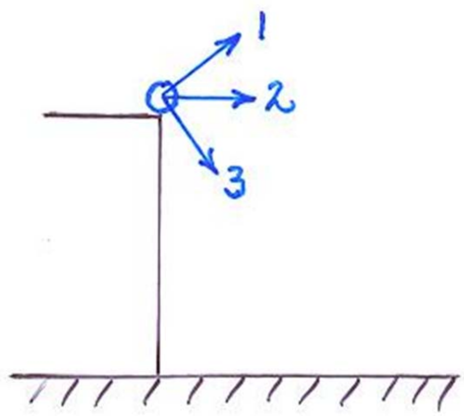
What is the relationship between their speeds when the balls hit the ground?

A) $v_1 = v_2 = v_3$

B) $v_1 = v_2 < v_3$

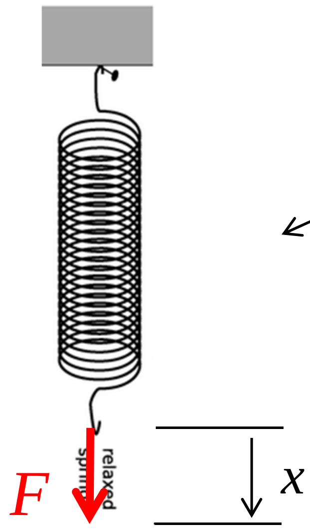
C) $v_1 < v_2 < v_3$

D) $v_1 > v_2 > v_3$



$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = \left(mgy_f - mgy_i\right)$$

ELASTIC POTENTIAL ENERGY



Spring at equilibrium

x = Displacement from equilibrium

Work done on spring by force F to create displacement x :

$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx^2 - 0 \qquad (x_1 = 0)$$

BUT

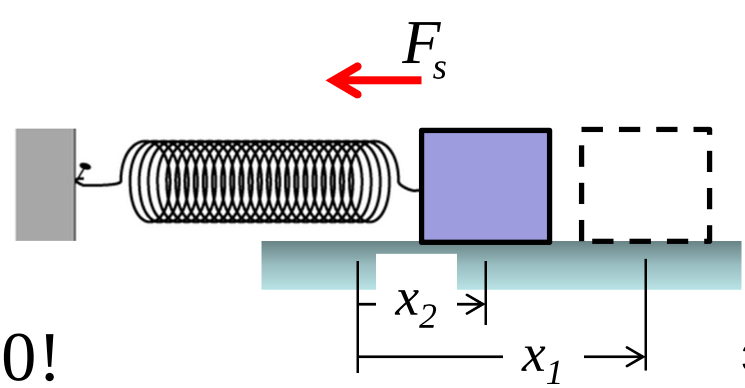
As spring is stretched, it applies an equal and opposite force on whatever is causing the stretching. → $W_{el} = -W$

Where W_{el} is work done by the spring.

$$\boxed{W_{el} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2}$$

→ If $x_1 < x_2 \Rightarrow W_{el} < 0$

When spring retracts:



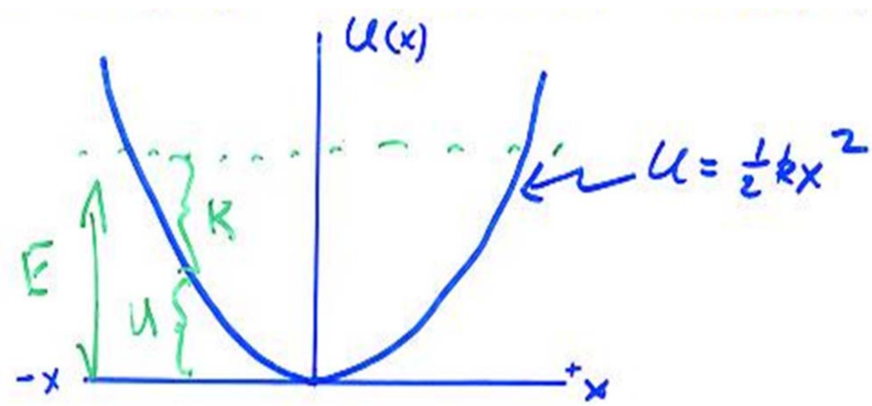
$$W_{el} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 > 0!$$

As with gravity, work done by spring can be characterized as change in potential energy:

$$U_{el} = \tfrac{1}{2}kx^2$$

$$W_{el} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 = U_{el_1} - U_{el_2} = -\left(U_{el_2} - U_{el_1}\right) = -\Delta U_{el}$$

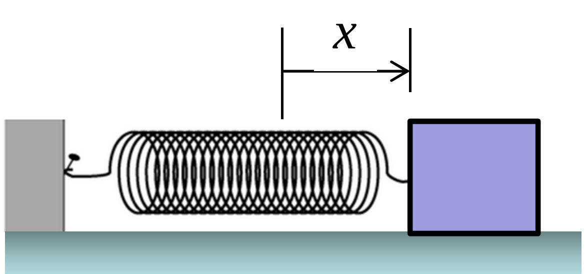
Recall Energy – Displacement diagram:



Note: $U(x)$ must be referenced to $\underline{x = 0}$!
(different from gravity)

(This is because the elastic force depends on position while the force of gravity does not)

So, if a spring acts on a body:



$$W_{TOT} = W_{el} = U_{el_1} - U_{el_2}$$

$$= \Delta K = K_2 - K_1$$

[Elastic Force Only]

If ONLY elastic force acts on body:

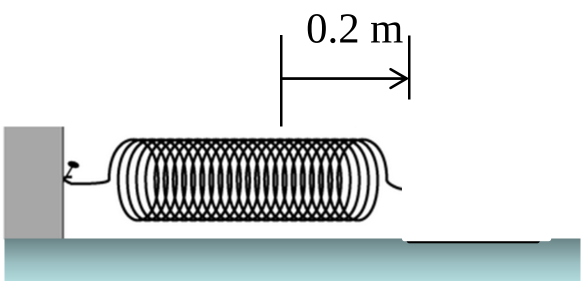
$$W_{TOT} = \Delta K = K_2 - K_1 = W_{el} = U_{el_1} - U_{el_2}$$

$$\Rightarrow K_2 - K_1 = U_{el_1} - U_{el_2} \Rightarrow K_2 + U_{el_2} = K_1 + U_{el_1}$$

$$\tfrac{1}{2} m v_2^2 + \tfrac{1}{2} k x_2^2 = \tfrac{1}{2} m v_1^2 + \tfrac{1}{2} k x_1^2$$

EXAMPLE 1:

It takes a force of 800 N to extend a spring 0.20 m.



(a) What is the PE stored in the spring?

$F = kx \rightarrow 800\ \text{N} = k\,(0.20\ \text{m}) \rightarrow k = 4000\ \text{N/m}$

$U_{el} = (1/2)kx^2 = (1/2)(4000\ \text{N/m})(0.04\ \text{m}^2) = 80\ \text{J}$

(b) A 5 kg mass is attached to the spring and then released. What is the speed of the mass at the instant the spring returns to its equilibrium position? (no friction)

$E_2 - E_1 = 0$: $E_1 = K_1 + U_{el1} = 0 + 80\ \text{J}$

$E_2 = K_2 + 0 = (1/2)\,mv^2$

$E_2 = E_1 \rightarrow (1/2)(5\ \text{kg})\,v^2 = 80\ \text{J}$

$\rightarrow v^2 = 2(80\ \text{J})/(5\ \text{kg}) = 32\ \text{m}^2/\text{s}^2 \rightarrow v = 5.6\ \text{m/s}$

Suppose gravity and elastic forces are both present?

$$W_{TOT} = W_g + W_{el} + W_{other} = \Delta K = K_2 - K_1$$

$$mgy_1 - mgy_2 + \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 + W_{other} = K_2 - K_1$$

$$W_{other} = (K_2 + mgy_2 + \tfrac{1}{2}kx_2^2) - (K_1 + mgy_1 + \tfrac{1}{2}kx_1^2)$$

$$= (K_2 + U_{g_2} + U_{el_2}) - (K_1 + U_{g_1} + U_{el_1})$$

$$= (K_2 + U_2) - (K_1 + U_1)$$

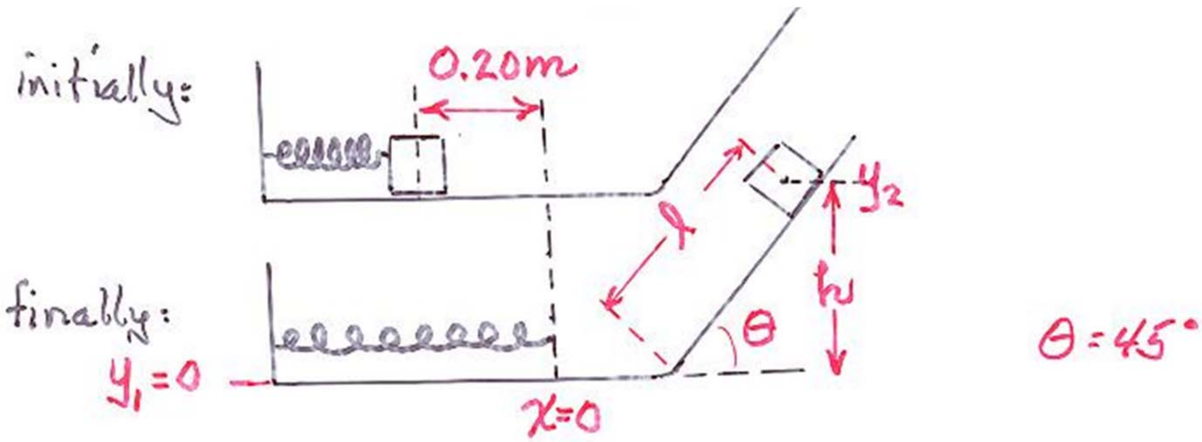
where $U_i = U_{g_i} + U_{h_i} = mgy_i + \tfrac{1}{2}kx_i^2$ $[i = 1,2]$

$W_{other} =$ Change in total mechanical energy
(For example, work done by friction)

STRATEGY

- Define system. Make a sketch.
- Define initial state (1) and final state (2)
- Define coordinate system (especially $y = 0$ and $+y$ = up)
- Identify forces that cannot be described by potential (such as friction, normal). If they do work, they contribute to W_{other} (may be < 0!).

Example 2: A 2.0 kg block is pushed against a spring ($k = 500$N/m), compressing it 0.20 m. Block is released and slides along a frictionless surface up a 45° incline. How far up does it go?



Identify: Use conservation of mechanical energy, potential energies for gravity and spring, no "other" forces

Set up: See drawings

Execute: $$K_1 + U_{g_1} + U_{el_1} = K_2 + U_{g_2} + U_{el_2}$$

Initial state: $K_1 = 0$, $U_{g_1} = 0$, $U_{el_1} = \frac{1}{2}kx_1^2$

Final state: $K_2 = 0$, $U_{g_2} = mgh = mgl\sin\theta$, $U_{el_2} = 0$

$$E_1 = E_2 \Rightarrow \tfrac{1}{2}kx_1^2 = mgl\sin\theta$$

$$l = \frac{\frac{1}{2}kx_1^2}{mg\sin\theta} = \frac{\frac{1}{2}(500\ \text{N/m})(0.20\ \text{m})^2}{(2.0)(9.8\ \text{m/s}^2)\sin 45^0} = 0.72\ \text{m}$$

Evaluate: l proportion to k, inversely proportion to m.

i-Clicker

A block is released from rest on a frictionless incline as shown. When the moving block is in contact with the spring and compressing it, what is happening to the gravitational potential energy U_{grav} and the elastic potential energy U_{el}?

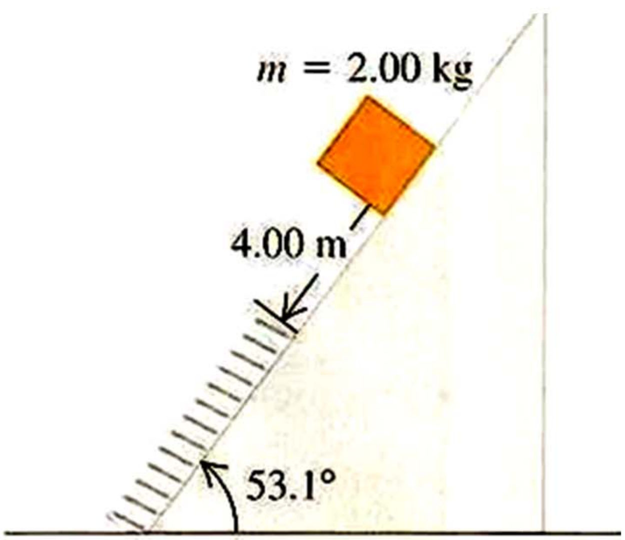
A. U_{grav} and U_{el} are both increasing.

B. U_{grav} and U_{el} are both decreasing.

C. U_{grav} is increasing, U_{el} is decreasing.

D. U_{grav} is decreasing, U_{el} is increasing.

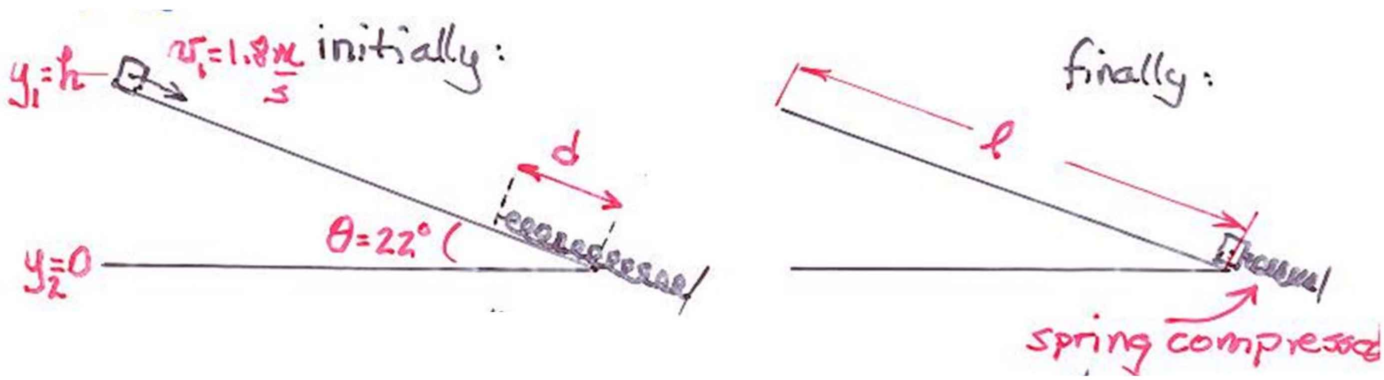
E. The answer depends on how the block's speed is changed



$$U_g = mgy$$

$$U_{el} = \tfrac{1}{2}kx^2$$

Example 3: [Gravity, spring, other forces (friction)]
You are designing a delivery ramp for heavy crates (1470 N) moving at top with v_1 = 1.80m/s.
The ramp is 8.00m long, inclined at 22° and exerts frictional force f_k = 550N. At the bottom, the crate compresses spring and stops; static friction f_s = 550N also. What is spring constant k?



Identify: Use potential energies for gravity, spring (U_g, U_{el}) and include W_{other} (friction). For equilibrium at bottom, spring force balanced by friction + component of weight.

Execute: $$K_1 + U_{g1} + U_{el1} + W_{other} = K_2 + U_{g2} + U_{el2}$$

Initial: $K = \frac{1}{2} m v_1^2$
$U_{g1} = mgh = wl\sin\theta$
$U_{el1} = 0$

Final: $K_2 = 0$
$U_{g2} = 0$
$U_{el2} = \frac{1}{2} k d^2$

Also: $W_{other} = -f_k l$ (minus because force opposite displacement)

- Substitute into: $K_1 + U_{g1} + U_{el1} + W_{other} = K_2 + U_{g2} + U_{el2}$

$$\frac{1}{2} m v_1^2 + wl\sin\theta - f_k l = \frac{1}{2} k d^2$$

- Solve for kd^2:

$$\frac{1}{2}\left(\frac{1470\ \mathrm{N}}{9.8\ \mathrm{m/s^2}}\right)(1.80\ \mathrm{m/s})^2 + (1470\ \mathrm{N})(8.00\ \mathrm{m})\sin(22^o) - (550\ \mathrm{N})(8.00\ \mathrm{m}) = \frac{1}{2} k d^2$$

$$\frac{1}{2} k d^2 = 248\ \mathrm{J}$$ (eqn. 1)

But, crate is in equilibrium at the bottom, $\sum \vec{F} = 0$

$$\Rightarrow f_s + w\sin\theta - F_{el} = 0$$

But $F_{el} = kd$

$$\Rightarrow kd = f_s + w\sin\theta = 550\ \mathrm{N} + (1470\ \mathrm{N})(\sin 22^\circ)$$

$$kd = 1100\ \mathrm{N}$$ (eqn. 2)

Solving (1) and (2), $$k = \frac{60{,}500\ \mathrm{N}^2}{248\ \mathrm{J}} = 2440\ \mathrm{N/m}$$

$$d = \frac{1{,}100\ \mathrm{N}}{2{,}440\ \mathrm{N/m}} = 0.45\ \mathrm{m}$$

CONSERVATIVE AND NON-CONSERVATIVE FORCES

A conservative force allows two-way conversion between kinetic and potential energy.

Properties of work done by conservative force:

- Work done can be expressed as difference between initial and final states of a potential energy function
- Work done is reversible
- Work done on a body is independent of path; it depends only on starting and ending points
- Work done through a closed path (same beginning and end point) is zero.

$$W_{cons.\,force} = -\Delta U$$

(gravity, spring; also electrical forces)

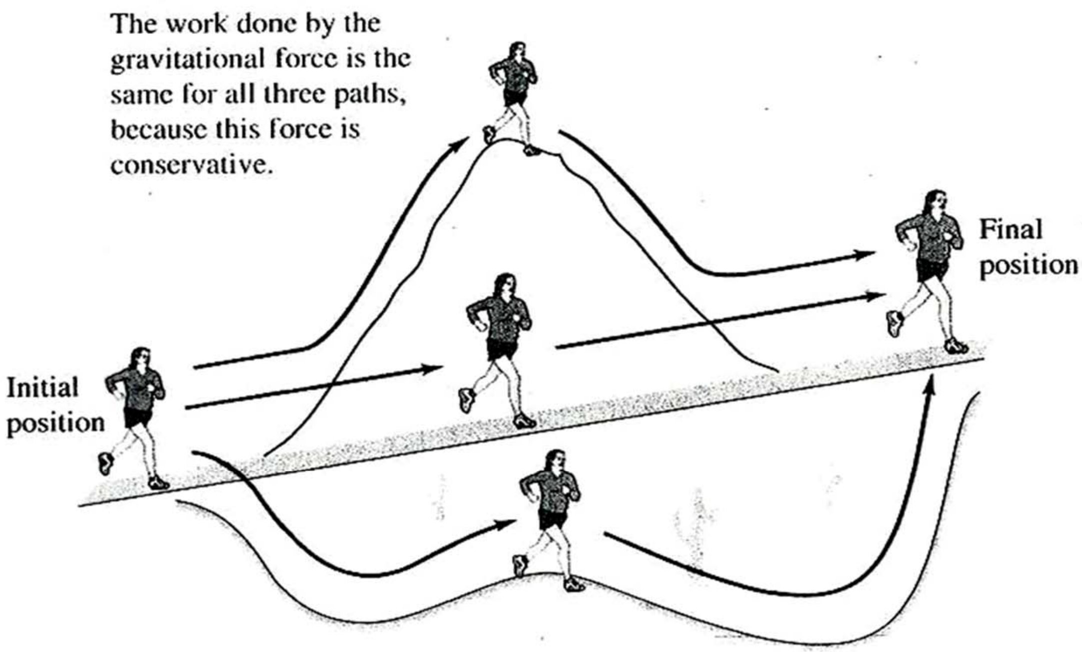
Total mechanical energy is conserved:

$$E = K + U = \text{ constant}$$

Non-conservative (dissipative) forces include friction, air resistance, chemical reactions,…

Conservative forces

- The work by a conservative force like gravity does not depend on the path your hiking team chooses, only how high you climb.



LAW OF CONSERVATION OF ENERGY

Non-conservative forces cause change of internal energy of solid or gas

(e.g., friction causes heat, and temp. change is related to int. energy).

Earlier we saw: $K_1 + U_1 + W_{other} = K_2 + U_2$

e.g., for friction, $W_{other} = -\Delta U_{\text{internal}}$

$$K_1 + U_1 - \Delta U_{\text{internal}} = K_2 + U_2$$

or $\Delta K + \Delta U + \Delta U_{\text{internal}} = 0$

So, if you account for all forms of energy the total energy of the universe is conserved.

Energy is neither created or destroyed, it only changes form.

Force and Potential Energy

Conservative force is intimately related to potential energy function.

For conservation force in 1-D,

$$W_c = -\Delta U$$

For small displacement Δx, $W_c = F_x \Delta x = -\Delta U$

$$F_x = \frac{-\Delta U}{\Delta x} \Rightarrow F_x = \frac{-dU(x)}{dx}$$

Check:

- For spring, $U = \frac{1}{2}kx^2$ $F = \frac{-dU}{dx} = -kx$

  "Restoring Force"

- For gravity: $U = mgy$ $F = \frac{-dU}{dy} = -mg$

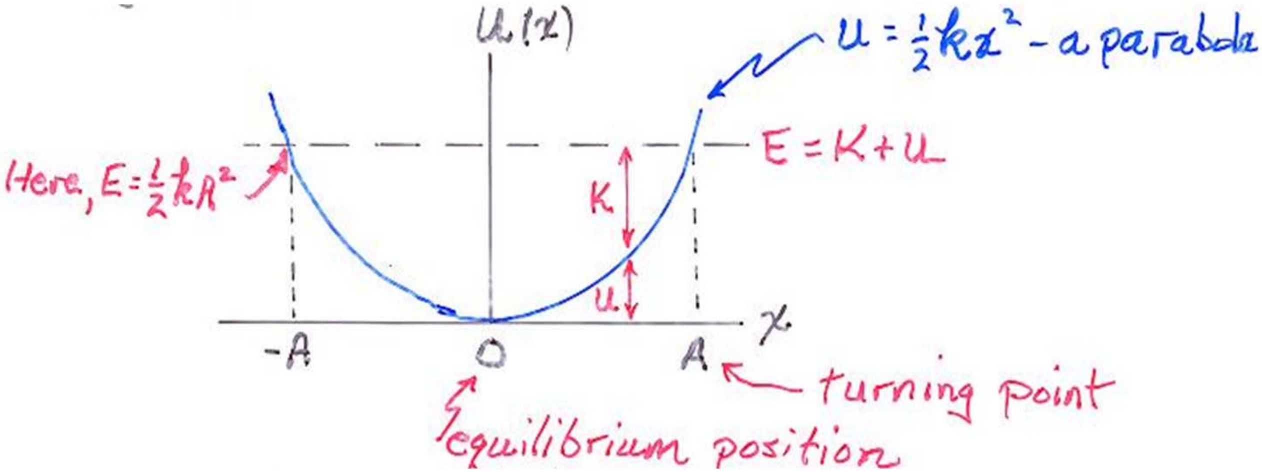
  (weight is in - y direction)

- In 3-dimensions: $\vec{F} = -(\frac{dU}{dx}\vec{i} + \frac{dU}{dy}\vec{j} + \frac{dU}{dz}\vec{k})$

ENERGY DIAGRAMS

For a spring, potential energy function: $U(x) = \frac{1}{2}kx^2$

Energy diagram:

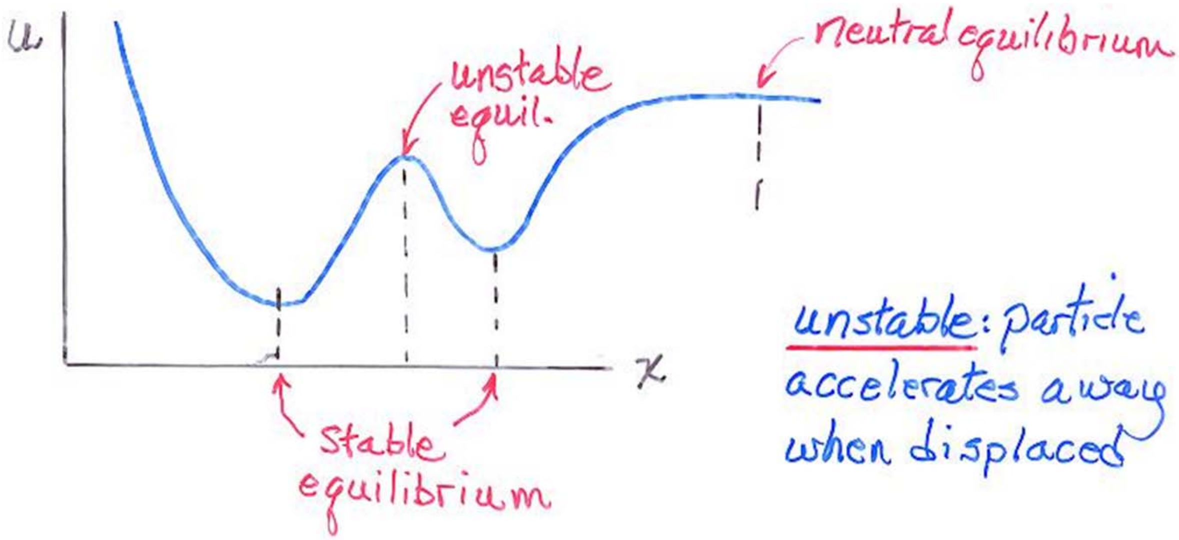


At equilibrium:

$$F = \frac{-dU}{dx} = -kx = 0$$

For spring, equilibrium is <u>stable</u>, because F attracts displaced object back to $x = 0$

General case:



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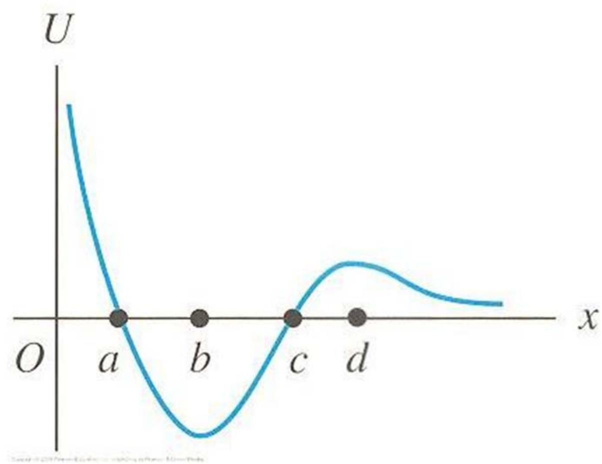
The graph shows the potential energy U for a particle that moves along the x-axis.

The particle is initially at $x = d$ and moves in the negative x-direction. At which of the labeled x-coordinates does the particle have the greatest *speed*?



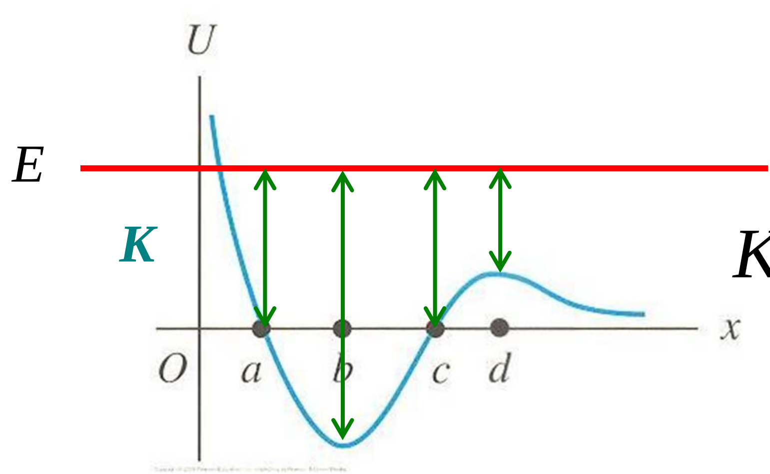
A. at $x = a$

B. at $x = b$

C. at $x = c$

D. at $x = d$

E. more than one of the above



$$K = \frac{1}{2}mv^2$$