Solutions to Assignment 8

Due: 11:59pm on Thursday, October 27, 2011

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1.45. **IDENTIFY:** $\vec{A}\cdot\vec{B} = AB\cos\phi$

SET UP: For $\vec{A}$ and $\vec{B}$, $\phi = 150.0°$. For $\vec{B}$ and $\vec{C}$, $\phi = 145.0°$. For $\vec{A}$ and $\vec{C}$, $\phi = 65.0°$.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = (8.00\text{ m})(15.0\text{ m})\cos150.0° = 2\ 104\text{ m}^2$

(b) $\vec{B}\cdot\vec{C} = (15.0\text{ m})(12.0\text{ m})\cos145.0° = -148\text{ m}^2$

(c) $\vec{A}\cdot\vec{C} = (8.00\text{ m})(12.0\text{ m})\cos65.0° = 40.6\text{ m}^2$

EVALUATE: When $\phi < 90°$ the scalar product is positive and when $\phi > 90°$ the scalar product is negative.

1.87. **IDENTIFY:** We know the scalar product and the magnitude of the vector product of two vectors and want to know the angle between them.

SET UP: The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\theta$ and the vector product is $|\vec{A}\times\vec{B}| = AB\sin\theta$.

EXECUTE: $\vec{A}\cdot\vec{B} = AB\cos\theta = -6.00$ and $|\vec{A}\times\vec{B}| = AB\sin\theta = +9.00$. Taking the ratio gives $\tan\theta = \dfrac{9.00}{-6.00}$, so $\theta = 124°$.

EVALUATE: Since the scalar product is negative, the angle must be between 90° and 180°.

1.51. **IDENTIFY:** Apply Eqs. (1.18) and (1.22).

SET UP: The angle between the vectors is $20° + 90° + 30° = 140°$.

EXECUTE: **(a)** Eq. (1.18) gives $\vec{A}\cdot\vec{B} = (3.60\text{ m})(2.40\text{ m})\cos140° = -6.62\text{ m}^2$.

(b) From Eq. (1.22), the magnitude of the cross product is $(3.60\text{ m})(2.40\text{ m})\sin140° = 5.55\text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$-direction).

EVALUATE: We could also use Eqs. (1.21) and (1.27), with the components of $\vec{A}$ and $\vec{B}$.

6.2. **IDENTIFY:** In each case the forces are constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: In part **(a)**, when the cable pulls horizontally $\phi = 0°$ and when it pulls at 35.0° above the horizontal $\phi = 35.0°$. In part **(b)**, if the cable pulls horizontally $\phi = 180°$. If the cable pulls on the car at 35.0° above the horizontal it pulls on the truck at 35.0° below the horizontal and ϕ 145.0°. For the gravity force $\phi = 90°$, since the force is vertical and the displacement is horizontal.

EXECUTE: **(a)** When the cable is horizontal, $W = (850\text{ N})(5.00\times10^3\text{ m})\cos0° = 4.26\times10^6\text{ J}$. When the cable is 35.0° above the horizontal, $W = (850\text{ N})(5.00\times10^3\text{ m})\cos35.0° = 3.48\times10^6\text{ J}$.

(b) $\cos180° = -\cos0°$ and $\cos145.0° = -\cos35.0°$, so the answers are -4.25×10^6 J and -3.48×10^6 J.

(c) Since $\cos\phi = \cos90° = 0$, $W = 0$ in both cases.

EVALUATE: If the car and truck are taken together as the system, the tension in the cable does no net work.

6.7. **IDENTIFY:** All forces are constant and each block moves in a straight line, so $W = Fs\cos\phi$. The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

SET UP: Since the 12.0 N block moves at constant speed, $a = 0$ for it and the tension T in the string is $T = 12.0$ N. Since the 20.0 N block moves to the right at constant speed the friction force f_k on it is to the left and $f_k = T = 12.0$ N.

EXECUTE: **(a)** (i) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00$ J. (ii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00$ J.

(b) (i) $\phi = 90°$ and $W = 0$. (ii) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00$ J. (iii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00$ J. (iv) $\phi = 90°$ and $W = 0$.

(c) $W_{tot} = 0$ for each block.

EVALUATE: For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

6.10. **IDENTIFY** and **SET UP:** Use $W = F_p s = (F\cos\phi)s$ to calculate the work done in each of parts (a) through (c). In part (d), the net work consists of the contributions due to all three forces, or $w_{net} = w_{grav} + w_n + w_f$.

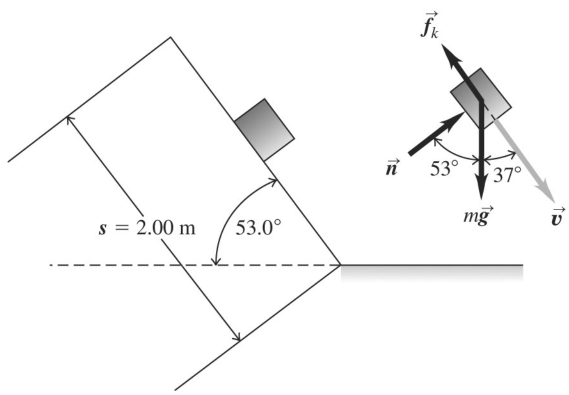

Figure 6.10

EXECUTE: **(a)** As the package slides, work is done by the frictional force which acts at $\phi = 180°$ to the displacement. The normal force is $mg\cos 53.0°$. Thus for $\mu_k = 0.40$,

$W_f = F_p s = (f_k\cos\phi)s = (\mu_k n\cos\phi)s = [\mu_k(mg\cos 53.0°)](\cos 180°)s.$

$W_f = (0.40)[(8.00\text{ kg})(9.80\text{ m/s}^2)(\cos 53.0°)](\cos 180°)(2.00\text{ m}) = -38$ J.

(b) Work is done by the component of the gravitational force parallel to the displacement. $\phi = 90° - 53° = 37°$ and the work of gravity is $W_{grav} = (mg\cos\phi)s = [(8.00\text{ kg})(9.80\text{ m/s}^2)(\cos 37.0°)](2.00\text{ m}) = +125$ J.

(c) $W_n = 0$ since the normal force is perpendicular to the displacement.

(d) The net work done on the package is $W_{net} = W_{grav} + W_n + W_f = 125\text{ J} + 0.0\text{ J} - 38\text{ J} = 87$ J.

EVALUATE: The net work is positive because gravity does more positive work than the magnitude of the negative work done by friction.

6.13. **IDENTIFY:** Find the kinetic energy of the cheetah knowing its mass and speed.

SET UP: Use $K = \frac{1}{2}mv^2$ to relate v and K.

EXECUTE: **(a)** $K = \frac{1}{2}mv^2 = \frac{1}{2}(70\text{ kg})(32\text{ m/s})^2 = 3.6\times 10^4\text{ J}.$

(b) K is proportional to v^2, so K increases by a factor of 4 when v doubles.

EVALUATE: A running person, even with a mass of 70 kg, would have only 1/100 of the cheetah's kinetic energy since a person's top speed is only about 1/10 that of the cheetah.

6.20. **IDENTIFY:** From the work-energy relation, $W = W_{\text{grav}} = \Delta K_{\text{rock}}$.

SET UP: As the rock rises, the gravitational force, $F = mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, s, the work is negative. Let h be the vertical distance the rock travels.

EXECUTE: **(a)** Applying $W_{\text{grav}} = K_2 - K_1$ we obtain $-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. Dividing by m and solving for v_1, $v_1 = \sqrt{v_2^2 + 2gh}$. Substituting $h = 15.0$ m and $v_2 = 25.0$ m/s,

$$v_1 = \sqrt{(25.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(15.0\text{ m})} = 30.3\text{ m/s}$$

(b) Solve the same work-energy relation for h. At the maximum height $v_2 = 0$.

$$-mgh = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \text{ and } h = \frac{v_1^2 - v_2^2}{2g} = \frac{(30.3\text{ m/s})^2 - (0.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 46.8\text{ m}.$$

EVALUATE: Note that the weight of 20 N was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, m. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

6.30. **IDENTIFY:** We know (or can calculate) the change in the kinetic energy of the crate and want to find the work needed to cause this change, so the work-energy theorem applies.

SET UP: $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$.

EXECUTE: $W_{\text{tot}} = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}(30.0\text{ kg})(5.62\text{ m/s})^2 - \frac{1}{2}(30.0\text{ kg})(3.90\text{ m/s})^2$.

$W_{\text{tot}} = 473.8\text{ J} - 228.2\text{ J} = 246\text{ J}$.

EVALUATE: Kinetic energy is a scalar and does not depend on direction, so only the initial and final speeds are relevant.