Solutions to Assignment 4

Due: 11:59pm on Thursday, September 29, 2011

4

3.12. **IDENTIFY:** The football moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. At the highest point in the trajectory, $v_y = 0$.

EXECUTE: **(a)** $v_y = v_{0y} + a_y t$. The time t is $\dfrac{v_{0y}}{g} = \dfrac{12.0\text{m/s}}{9.80\text{m/s}^2} = 1.224\text{ s}$, which we round to 1.22 s.

(b) Different constant acceleration equations give different expressions but the same numerical result:

$\frac{1}{2}gt^2 = \frac{1}{2}v_{y0}t = \dfrac{v_{0y}^2}{2g} = 7.35\text{ m}.$

(c) Regardless of how the algebra is done, the time will be twice that found in part (a), which is 2(1.224 s) = 2.45 s.

(d) $a_x = 0$, so $x - x_0 = v_{0x}t = (20.0\text{ m/s})(2.45\text{ s}) = 49.0\text{ m}.$

(e) The graphs are sketched in Figure 3.12.

EVALUATE: When the football returns to its original level, $v_x = 20.0\text{ m/s}$ and $v_y = -12.0\text{ m/s}$.

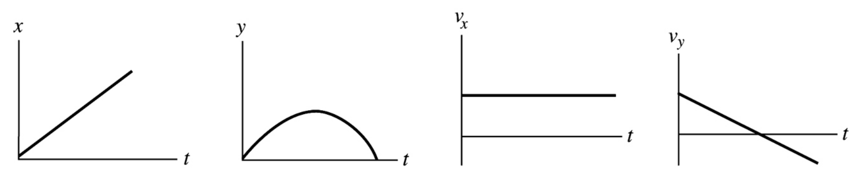


Figure 3.12

3.15. **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude v_0. The height h of the table and v_0 are the same; the acceleration due to gravity changes from $g_E = 9.80\text{ m/s}^2$ on earth to g_X on planet X.

SET UP: Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -g$, where g is either g_E or g_X.

EXECUTE: Use the vertical motion to find the time in the air: $y - y_0 = -h$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2h}{g}}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_{0x}t = v_0\sqrt{\dfrac{2h}{g}}$. $x - x_0 = D$ on earth and 2.76D on Planet X.

$(x - x_0)\sqrt{g} = v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_E} = 2.76D\sqrt{g_X}$. $g_X = \dfrac{g_E}{(2.76)^2} = 0.131g_E = 1.28\text{ m/s}^2$.

EVALUATE: On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor and it travels farther horizontally.

3.18. **IDENTIFY:** The shot moves in projectile motion.
SET UP: Let $+y$ be upward.
EXECUTE: **(a)** If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80\ \text{m/s}^2$ vertically downward.
(b) The x-component of velocity is constant at $v_x = (12.0\ \text{m/s})\cos 51.0° = 7.55\ \text{m/s}$. The y-component is $v_{0y} = (12.0\ \text{m/s})\sin 51.0° = 9.32\ \text{m/s}$ at release and $v_y = v_{0y} - gt = (9.32\ \text{m/s}) - (9.80\ \text{m/s})(2.08\ \text{s}) = -11.06\ \text{m/s}$ when the shot hits.
(c) $x - x_0 = v_{0x}t = (7.55\ \text{m/s})(2.08\ \text{s}) = 15.7\ \text{m}$.
(d) The initial and final heights are not the same.
(e) With $y = 0$ and v_{0y} as found above, Eq. (3.18) gives $y_0 = 1.81\text{m}$.
(f) The graphs are sketched in Figure 3.18.
EVALUATE: When the shot returns to its initial height, $v_y = -9.32\ \text{m/s}$. The shot continues to accelerate downward as it travels downward 1.81 m to the ground and the magnitude of v_y at the ground is larger than 9.32 m/s.

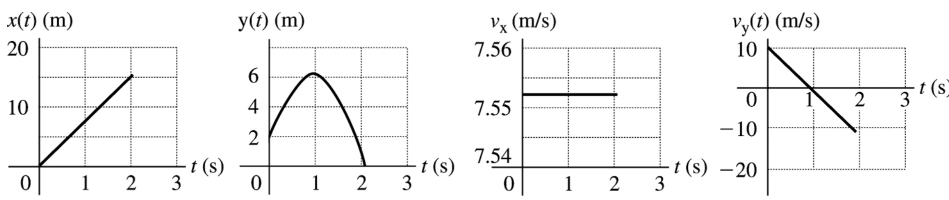
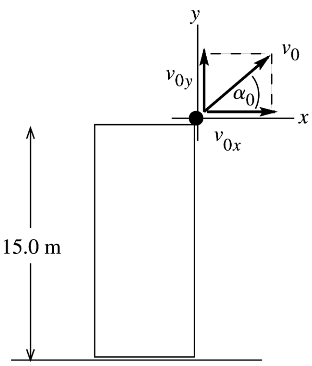


Figure 3.18

3.21. **IDENTIFY:** Take the origin of coordinates at the roof and let the $+y$-direction be upward. The rock moves in projectile motion, with $a_x = 0$ and $a_y = -g$. Apply constant acceleration equations for the x and y components of the motion.
SET UP:



$v_{0x} = v_0 \cos\alpha_0 = 25.2\ \text{m/s}$

$v_{0y} = v_0 \sin\alpha_0 = 16.3\ \text{m/s}$

Figure 3.21a

(a) At the maximum height $v_y = 0$.

$a_y = -9.80\ \text{m/s}^2,\ \ v_y = 0,\ \ v_{0y} = +16.3\ \text{m/s},\ \ y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y\,(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (16.3\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = +13.6\ \text{m}$

(b) SET UP: Find the velocity by solving for its *x* and *y* components.

$v_x = v_{0x} = 25.2\ \text{m/s}$ (since $a_x = 0$)

$v_y = ?,\ \ a_y = -9.80\ \text{m/s}^2,\ \ y - y_0 = -15.0\ \text{m}$ (negative because at the ground the rock is below its initial position), $v_{0y} = 16.3\ \text{m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ (v_y is negative because at the ground the rock is traveling downward.)

EXECUTE: $v_y = -\sqrt{(16.3\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-15.0\ \text{m})} = -23.7\ \text{m/s}$

Then $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2\ \text{m/s})^2 + (-23.7\ \text{m/s})^2} = 34.6\ \text{m/s}$.

(c) SET UP: Use the vertical motion (*y*-component) to find the time the rock is in the air:

$t = ?,\ \ v_y = -23.7\ \text{m/s}$ (from part (b)), $a_y = -9.80\ \text{m/s}^2,\ \ v_{0y} = +16.3\ \text{m/s}$

EXECUTE: $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-23.7\ \text{m/s} - 16.3\ \text{m/s}}{-9.80\ \text{m/s}^2} = +4.08\ \text{s}$

SET UP: Can use this *t* to calculate the horizontal range:

$t = 4.08\ \text{s},\ \ v_{0x} = 25.2\ \text{m/s},\ \ a_x = 0,\ \ x - x_0 = ?$

EXECUTE: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2\ \text{m/s})(4.08\ \text{s}) + 0 = 103\ \text{m}$

(d) Graphs of *x* versus *t*, *y* versus *t*, v_x versus *t* and v_y versus *t:*

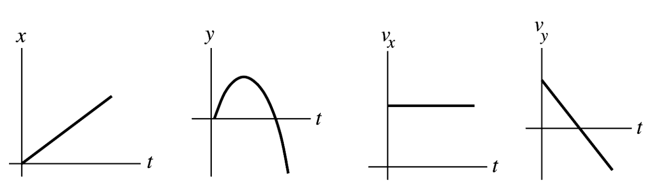


Figure 3.21b

EVALUATE: The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y} = +16.3$ m/s the time it takes the rock to return to the level of the roof $(y = 0)$ is $t = 2v_{0y}/g = 3.33$ s. The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

3.32. **IDENTIFY:** The relative velocities are $\vec{v}_{S/F}$, the velocity of the scooter relative to the flatcar, $\vec{v}_{S/G}$, the scooter relative to the ground and $\vec{v}_{F/G}$, the flatcar relative to the ground. $\vec{v}_{S/G} = \vec{v}_{S/F} + \vec{v}_{F/G}$. Carry out the vector addition by drawing a vector addition diagram.

SET UP: $\vec{v}_{S/F} = \vec{v}_{S/G} - \vec{v}_{F/G}$. $\vec{v}_{F/G}$ is to the right, so $-\vec{v}_{F/G}$ is to the left.

EXECUTE: In each case the vector addition diagram gives

(a) 5.0 m/s to the right

(b) 16.0 m/s to the left

(c) 13.0 m/s to the left.

EVALUATE: The scooter has the largest speed relative to the ground when it is moving to the right relative to the flatcar, since in that case the two velocities $\vec{v}_{S/F}$ and $\vec{v}_{F/G}$ are in the same direction and their magnitudes add.

3.34. **IDENTIFY:** Calculate the rower's speed relative to the shore for each segment of the round trip.

SET UP: The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.

EXECUTE: The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).

The total time the rower takes is $\dfrac{1.5\text{ km}}{6.8\text{ km/h}} + \dfrac{1.5\text{ km}}{1.2\text{ km/h}} = 1.47\text{ h} = 88.2\text{ min}$.

EVALUATE: It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

3.37. **IDENTIFY:** Relative velocity problem in two dimensions.

(a) SET UP: $\vec{v}_{P/A}$ is the velocity of the plane relative to the air. The problem states that $\vec{v}_{P/A}$ has magnitude 35 m/s and direction south.

$\vec{v}_{A/E}$ is the velocity of the air relative to the earth. The problem states that $\vec{v}_{A/E}$ is to the southwest ($45°$ S of W) and has magnitude 10 m/s.

The relative velocity equation is $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

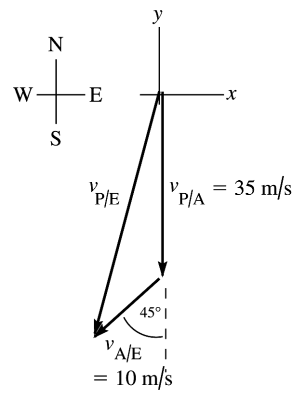


Figure 3.37a

EXECUTE: **(b)** $(v_{P/A})_x = 0$, $(v_{P/A})_y = -35$ m/s

$(v_{A/E})_x = -(10 \text{ m/s})\cos 45° = -7.07$ m/s,

$(v_{A/E})_y = -(10 \text{ m/s})\sin 45° = -7.07$ m/s

$(v_{P/E})_x = (v_{P/A})_x + (v_{A/E})_x = 0 - 7.07 \text{ m/s} = -7.1$ m/s

$(v_{P/E})_y = (v_{P/A})_y + (v_{A/E})_y = -35 \text{ m/s} - 7.07 \text{ m/s} = -42$ m/s

(c)

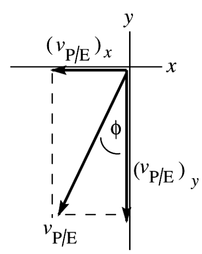


Figure 3.37b

$$v_{P/E} = \sqrt{(v_{P/E})_x^2 + (v_{P/E})_y^2}$$

$$v_{P/E} = \sqrt{(-7.1 \text{ m/s})^2 + (-42 \text{ m/s})^2} = 43 \text{ m/s}$$

$$\tan\phi = \frac{(v_{P/E})_x}{(v_{P/E})_y} = \frac{-7.1}{-42} = 0.169$$

$\phi = 9.6°$; (9.6° west of south)

EVALUATE: The relative velocity addition diagram does not form a right triangle so the vector addition must be done using components. The wind adds both southward and westward components to the velocity of the plane relative to the ground.

3.39. **IDENTIFY:** The resultant velocity, relative to the ground, is directly southward. This velocity is the sum of the velocity of the bird relative to the air and the velocity of the air relative to the ground.

SET UP: $v_{B/A} = 100$ km/h. $\vec{v}_{A/G} = 40$ km/h, east. $\vec{v}_{B/G} = \vec{v}_{B/A} + \vec{v}_{A/G}$.

EXECUTE: We want $\vec{v}_{B/G}$ to be due south. The relative velocity addition diagram is shown in Figure 3.39.

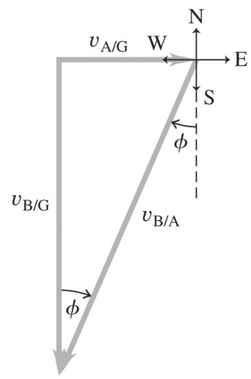


Figure 3.39

(a) $\sin\phi = \dfrac{v_{A/G}}{v_{B/A}} = \dfrac{40\text{ km/h}}{100\text{ km/h}}$, $\phi = 24°$, west of south.

(b) $v_{B/G} = \sqrt{v_{B/A}{}^2 - v_{A/G}{}^2} = 91.7\text{ km/h}$. $t = \dfrac{d}{v_{B/G}} = \dfrac{500\text{ km}}{91.7\text{ km/h}} = 5.5\text{ h}$.

EVALUATE: The speed of the bird relative to the ground is less than its speed relative to the air. Part of its velocity relative to the air is directed to oppose the effect of the wind.