Lecture 3

Motion in 2- and 3-Dimensions

REVIEW OF 1-D MOTION

$$\Delta x = x_2 - x_1$$

$$v_{av} = \frac{\Delta x}{\Delta t}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

speed = $|v|$ (magnitude of $v$)

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

• For CONSTANT acceleration in 1-D

$$x = x_o + v_o \ t + \frac{1}{2} at^2$$

$$v = v_o + at$$

$$\left( x - x_o \right) = \frac{v^2 - v_o^2}{2a}$$

• Free Fall: replace $a$ with $-g$
**i-Clicker**

This is a motion diagram of an object moving along the $x$-direction with constant acceleration. The dots 1, 2, 3, ... show the position of the object at equal time intervals $\Delta t$.

$x = 0$

Which of the following $a_x-t$ graphs best matches the motion shown in the motion diagram?

A. ![Graph A](image)

B. ![Graph B](image)

C. ![Graph C](image)

D. ![Graph D](image)

E. ![Graph E](image)
A student obtains a graph of an object’s velocity versus time and then draws the graph of the acceleration versus time for the same time interval.

What, if anything, is wrong with the graph of the acceleration versus time?

A) The last acceleration segment should not be zero when the object is moving.

B) The positive acceleration segment should be further from the time axis than the negative acceleration segment is.

C) The sign of the first acceleration segment is incorrect.

D) The first acceleration segment should steadily decrease in magnitude.

E) The graph is correct.
Ariel and Byron are moving along a straight hallway. If the vertical axis is position, which of the following statements is TRUE?

A) The two children are never at the same position at the same time.

B) The two children are at the same place at \( t = 0 \).

C) The two children never have the same velocity simultaneously.

D) The two children never have the same acceleration.

E) None of the above are TRUE.
Ariel and Byron are moving along a straight hallway. If the vertical axis is velocity, which of the following statements is TRUE?

A) The two children never change direction.

B) The two children never have the same velocity.

C) The two children are moving at the same speed at \( t = 0 \).

D) The two children never have the same acceleration.

E) None of the above.
The graphs show the velocity versus time for six boats traveling along a narrow channel. In each graph, a point which is being discussed is marked with a dot. Which of the following is INCORRECT (at the time indicated by the dots)?

A) The boats in A and C have the same velocity.

B) The boats in B and F are moving in the same direction.

C) The boats in D and E are moving with negative velocity.

D) The dot on graph E corresponds to a larger speed than the dot on graph A.

E) The dot on graph E corresponds to a larger speed than the point on graph D.
MOTION IN 2 AND 3 DIMENSIONS

Recall: Displacement is a vector!
- Has magnitude and direction
  So are velocity and acceleration

- Represent vector by arrow:

\[ \vec{A} \quad \vec{B} \quad \vec{C} \]

Here: \( \vec{B} = \vec{A} \) (Same magnitude and direction)

Arrows do not need to lie on top of each other.

Furthermore: \( \vec{C} = -\vec{A} \) \[= -\vec{B} \]

(Same Magnitude, Opposite Direction)

Notation: \( |\vec{A}| = \text{Magnitude of vector} \quad \vec{A} \)

Write: \( |\vec{A}| = A \)

NOTE: For vectors above:
  \( |\vec{B}| = B = A \)

and \( |\vec{C}| = C = A ! \) ( - sign indicates direction)
Adding two vectors graphically

- Two vectors may be added graphically using either the *parallelogram* method or the *head-to-tail* method.

(a) We can add two vectors by placing them head to tail.

\[ \vec{C} = \vec{A} + \vec{B} \]

(b) Adding them in reverse order gives the same result.

\[ \vec{C} = \vec{B} + \vec{A} \]

(c) We can also add them by constructing a parallelogram.

\[ \vec{C} = \vec{A} + \vec{B} \]

(a) The sum of two parallel vectors

\[ \vec{C} = \vec{A} + \vec{B} \]

(b) The sum of two antiparallel vectors

\[ \vec{C} = \vec{A} + \vec{B} \]

[animation]
Subtracting vectors

Subtracting $\vec{B}$ from $\vec{A}$ ...

... is equivalent to adding $-\vec{B}$ to $\vec{A}$.

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{A} + (-\vec{B})$

With $\vec{A}$ and $-\vec{B}$ head to tail, $\vec{A} - \vec{B}$ is the vector from the tail of $\vec{A}$ to the head of $-\vec{B}$.

animation
Multiplying a vector by a scalar

• If $c$ is a scalar, the product $cA$ has magnitude $|c|A$. 

(a) Multiplying a vector by a positive scalar changes the magnitude (length) of the vector, but not its direction.

\[ \vec{A} \quad \rightarrow \quad 2\vec{A} \]

$2\vec{A}$ is twice as long as $\vec{A}$.

(b) Multiplying a vector by a negative scalar changes its magnitude and reverses its direction.

\[ \vec{A} \quad \rightarrow \quad -3\vec{A} \]

$-3\vec{A}$ is three times as long as $\vec{A}$ and points in the opposite direction.
Components of a vector

- Adding vectors graphically provides limited accuracy. Vector components provide a general method for adding vectors.

- Any vector can be represented by an $x$-component $A_x$ and a $y$-component $A_y$.

- Use trigonometry to find the components of a vector: $A_x = A \cos \theta$ and $A_y = A \sin \theta$, where $\theta$ is measured from the $+x$-axis toward the $+y$-axis.
Positive and negative components

- The components of a vector can be positive or negative numbers.

(a) $B_y$ is positive: Its component vector points in the $+y$-direction.
   $B_x$ is negative: Its component vector points in the $-x$-direction.

(b) Both components of $\vec{C}$ are negative.
COMPONENTS AND \textit{COMPONENT VECTORS}

Clearly:

\[ \vec{A} = \vec{A}_x + \vec{A}_y \]

\( \vec{A}_x, \vec{A}_y \) are component vectors of \( \vec{A} \)

\( A_x, A_y \) are components of \( \vec{A} \)

Notice:

\[
\frac{A_y}{A} = \sin \theta \implies A_y = A \sin \theta
\]

\[
\frac{A_x}{A} = \cos \theta \implies A_x = A \cos \theta
\]

and

\[
\frac{A_y}{A_x} = \frac{A \sin \theta}{A \cos \theta} = \tan \theta ; \quad A_x^2 + A_y^2 = A^2
\]

\textbf{EXAMPLE}: You are driving east on Canal Street. The Empire State building is 5km at 53° N of E. How many km \( E \) and then \( N \) must you drive?

\[
A_E = A \cos(53^\circ) = (5 \text{ km})(0.6) = 3 \text{ km}
\]

\[
A_N = A \sin(53^\circ) = (5 \text{ km})(0.8) = 4 \text{ km}
\]
Consider the vectors shown. Which is a correct statement about \( \vec{A} + \vec{B} \)?

A. \( x\)-component > 0, \( y\)-component > 0
B. \( x\)-component > 0, \( y\)-component < 0
C. \( x\)-component < 0, \( y\)-component > 0
D. \( x\)-component < 0, \( y\)-component < 0
E. \( x\)-component = 0, \( y\)-component > 0
Unit vectors

- A unit vector has a magnitude of 1 with no units.
- The unit vector $\hat{i}$ points in the $+x$-direction, $\hat{j}$ points in the $+y$-direction, and $\hat{k}$ points in the $+z$-direction.
- Any vector can be expressed in terms of its components as $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$.
UNIT VECTORS

Consider vectors of unit length in \( x \)- and \( y \)-directions
\[
\hat{i} ; \quad |\hat{i}| = 1 \\
\hat{j} ; \quad |\hat{j}| = 1
\]

\( \vec{A}_x = A_x \hat{i} ; \quad \vec{A}_y = A_y \hat{j} \Rightarrow \vec{A} = A_x \hat{i} + A_y \hat{j} \)

Unit vectors make vector addition very easy!

\[
\vec{C}_y = A_y + B_y \\
\vec{C}_x = A_x + B_x
\]

\[
\vec{C} = \vec{A} + \vec{B} = C_x \hat{i} + C_y \hat{j} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}
\]

3D:
\[
\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}
\]
\[ \vec{r}_1 + \Delta \vec{r} = \vec{r}_2 \]

\[ \Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \]

\( \Delta \vec{r} \) is displacement

Suppose this displacement occurred in interval \((t_2 - t_1)\)

\[ \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \vec{v}_{av} \quad \text{Average velocity} \]

\[ \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \]

\( \vec{v} \) is tangent to particle path

\[ \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \]

\[ \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \]
**EXAMPLE:** In one play during the RU football game last Saturday, the ball was snapped to the quarterback who dropped back 1 yd in 0.5 s, he then ran 5 yds parallel to the line of scrimmage in 1 s, then he threw the ball 5 yds at an angle of 37° from the forward direction in 0.5 s.

What was the average velocity, \( \bar{v}_{av} \), of the ball during this play?

\[
\bar{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}
\]

\[
\vec{r}_1 = 0 \quad \vec{r}_2 = ?
\]

\[
\vec{r}_2 = (-1 \text{ yd})\hat{i} + (-5 \text{ yd})\hat{j} + \vec{C}
\]

\[
\vec{r}_2 = (-1 \text{ yd})\hat{i} + (-5 \text{ yd})\hat{j} + (4 \text{ yd})\hat{i} + (3 \text{ yd})\hat{j}
\]

\[
\vec{r}_2 = (3 \text{ yd})\hat{i} + (-2 \text{ yd})\hat{j}
\]

\[
\Delta t = 2 \text{ s}
\]

\[
\bar{v}_{av} = \frac{(3 \text{ yd})\hat{i} - (2 \text{ yd})\hat{j}}{2 \text{ s}} = (1.5 \frac{\text{yd}}{\text{s}})\hat{i} - (1 \frac{\text{yd}}{\text{s}})\hat{j} = \bar{v}_{av}
\]
EXAMPLE: Suppose you drive on a curved road. Your position is given by: \( x = v_o t + ct^2; \quad y = kt^3 \)
(Note: car at origin when \( t = 0 \))

- What is \( \vec{v}(t = 0) \)?
- What is \( \vec{v}(t = 3s) \)?

\[
\vec{r} = (v_o t + ct^2) \hat{i} + (kt^3) \hat{j}
\]

\[
\vec{v} = (v_o + 2ct) \hat{i} + (3kt^2) \hat{j}
\]

=> \( \vec{v}(t = 0) = v_o \hat{i} \)

\[
\vec{v}(t = 3) = (v_o + 6c) \hat{i} + (27k) \hat{j}
\]

- What is \( \vec{v}_{av} \) for the interval \( 0 \rightarrow 3s \)?

\[
\vec{V}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{\Delta t}; \quad \vec{r}(t=0)=0
\]

\[
\vec{r}(t=3)= (3v_o + 9c) \hat{i} + (27k) \hat{j}
\]

=> \( \vec{v}_{av} = (v_o + 3c) \hat{i} + (9k) \hat{j} \)
At a certain instant in time, the location airplane is given by:

\[ \vec{r}_1 = 10 \text{ km} \hat{i} + 10 \text{ km} \hat{j} + 10 \text{ km} \hat{k} \]

One hour later, the plane is located at:

\[ \vec{r}_2 = -10 \text{ km} \hat{i} + 20 \text{ km} \hat{j} + 10 \text{ km} \hat{k} \]

The average velocity of the plane during that hour is:

A: \[ \vec{v}_{av} = (0 \hat{i} + 30 \hat{j} + 20 \hat{k}) \text{ km/hr} \]

B: \[ \vec{v}_{av} = (0 \hat{i} - 30 \hat{j} - 20 \hat{k}) \text{ km/hr} \]

C: Cannot tell. Need to know position during that hour.

D: \[ \vec{v}_{av} = (-20 \hat{i} + 10 \hat{j} + 0 \hat{k}) \text{ km/hr} \]

E: \[ \vec{v}_{av} = (20 \hat{i} - 10 \hat{j} + 0 \hat{k}) \text{ km/hr} \]

\[ \vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \]
Addition of velocity vectors: airplane in crosswind

\[ \vec{v}_{A/E} = 100 \text{ km/h, east} \]

\[ \vec{v}_{P/A} = 240 \text{ km/h, north} \]

\[ \alpha \]
ACCELERATION VECTOR

Consider previous example:

\( \vec{v} \) changes direction

\( \vec{v} \) changes magnitude

- **Average** acceleration:

\[
\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}
\]

(\( \vec{a}_{av} \parallel \Delta \vec{v} \))

- **Instantaneous** acceleration

\[
\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}
\]

\( \vec{a} \neq 0 \) for any curved path

\( \vec{a} \neq 0 \) if \( \vec{v} \) changes: magnitude or direction or both
\[
\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}
\]
\[
\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}
\]
\[
\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}
\]

**EXAMPLE:** Consider car again:

\[
\vec{r} = x\hat{i} + y\hat{j} = (v_o t + ct^2)\hat{i} + (kt^3)\hat{j}
\]
\[
\vec{v} = (v_o + 2ct)\hat{i} + (3kt^2)\hat{j}
\]
\[
\vec{a} = (2c)\hat{i} + (6kt)\hat{j}
\]

- Parallel and perpendicular components of $\vec{a}$

$\vec{a}_\parallel$ : changes magnitude, $v$

$\vec{a}_\perp$ : changes direction of $\vec{v}$

If: $a_\parallel = 0 \rightarrow \vec{v}$ only changes direction

$\vec{a}_\parallel$ same direction as $\vec{v} \rightarrow v$ increases

$\vec{a}_\parallel$ opposite (anti parallel) to $\vec{v} \rightarrow v$ decreases
Show Phet Maze Game?