LECTURE 12- GRAVITATION

Chapter 13
Professor Noronha-Hostler
Professor Montalvo
TEST

- Similar average as before
- Curve at the END (depends on the final)
- Extra credit possibility (make sure you’re in the recitation this week)
- FINAL: Notes allowed!
CLASS SURVEY

Post Class Survey DUE DECEMBER 5th
https://www.physics.rutgers.edu/ugrad/123/
ICLICKERS

Being worked on, we are aware there are missing grades, we ask for your patience.
Physics 123 – Analytical Physics I

Lecture 12: Universal Gravitation

Guest Lecturer: Dr. Sievert

Lecturer: Dr. Noronha-Hostler

Administrator: Dr. Montalvo

Fri. 11/30/2018
Today’s Objectives

1.) Wrap up Momentum
   - Recap of Lecture 11
   - Impulse
   - Totally Inelastic Collisions

2.) New material: Gravitation
   - Newton’s Law of Gravitation
   - Gravitational Potential Energy
   - Celestial Mechanics
   - General Relativity

3.) My Perspective
   - A Physicist’s Toolkit
Previously...

- **Center of Mass**
  \[ x_{COM} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots} \]

- **Momentum**
  \[ \vec{p} = m \vec{v} \]
  \[ \vec{F}_{net} = m \vec{a} = \frac{d\vec{p}}{dt} \]

- **Conservation of Momentum:**
  If \( \vec{F}_{net, ext} = 0 \) then \( \vec{P}_{tot, i} = \vec{P}_{tot, f} \)

**PHY123**
**L12 – Gravitation**
iClicker #1: Momentum

Which of the following objects possesses the largest momentum?

a.) An electron \( m = 10^{-30} \text{kg} \quad v = 10^8 \text{m/s} \)
b.) The Hubble Telescope \( m = 10^4 \text{kg} \quad v = 10^4 \text{m/s} \)
c.) A snail \( m = 10^{-2} \text{kg} \quad v = 10^{-4} \text{m/s} \)
d.) An oil supertanker \( m = 10^8 \text{kg} \quad v = 2 \text{ m/s} \)
e.) A falling raindrop \( m = 10^{-4} \text{kg} \quad v = 10 \text{ m/s} \)
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Darrin, m=45 kg, runs and jumps off a stationary, 168 kg floating platform on a lake. Darrin’s horizontal velocity as he leaps is +2.7 m/s. What is the recoil velocity of the platform?

a.) -2.7 m/s
b.) +0.72 m/s
c.) -1.4 m/s
d.) -0.72 m/s
e.) +2.7 m/s
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Collisions

- Interplay between kinetic energy and momentum

\[ K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \]

\[ \vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 \]

- Internal forces may dissipate energy into heat / etc.
  - Kinetic energy may not be conserved.

- But internal forces cannot change the center of mass
  - Momentum must always be conserved.
Elastic Collisions

- In a smooth, springy impact, the colliding objects **squeeze and bounce back**
  - Both **kinetic energy** and **momentum** are conserved

\[
\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2
\]

\[
m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}
\]

(2 Equations, 2 Unknowns)
Partially Inelastic Collisions

- In real-world collisions, **some percentage of the energy is dissipated** into heat, sound, and the destruction of the objects.
  
  - The objects **bounce back**, but with **less kinetic energy** than they started.
  
  - **Momentum** is conserved, but **kinetic energy is not**.

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]

*Partially Inelastic Collisions*
Totally Inelastic Collisions

• In the extreme case, the colliding objects **stick together without bouncing back** at all

  ➢ Since they can’t rebound, the **maximum kinetic energy is lost**

  ➢ **Only momentum** is conserved

\[
m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f
\]

**Totally Inelastic Collisions**

*(1 Equation, 1 Unknown)*
Visualizing the Energy and Momentum

\[ K_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \]

\[ P_f = m_1 v_{1f} + m_2 v_{2f} \]

**Energy:** \[ x^2 + y^2 = R^2 \] circle

**Momentum:** \[ y = -mx + b \] line

(Plot: equal masses)
From Elastic to Inelastic

Elastic:
• All kinetic energy conserved

Partially Inelastic:
• Some kinetic energy lost

Totally Inelastic:
• Maximum kinetic energy lost

Kinetic Energy = Radius
From Elastic to Inelastic

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Greatest loss of kinetic energy when the objects stick together.

Kinetic Energy = \text{Radius}
A bullet with a mass of 10g is fired with a speed of 100 m/s into a wooden block of mass 1.0 kg initially hanging at rest.

➢ Assuming the bullet lodges in the wooden block, what is their recoil speed after the collision?

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f \]

\[ (0.01 \text{ kg})(+100 \text{ m/s} \hat{i}) + (1.0 \text{ kg})(0) = (1.01 \text{ kg}) \vec{v}_{2f} \]

\[ \vec{v}_f = \left( \frac{0.01 \text{ kg}}{1.01 \text{ kg}} \right)(+100 \text{ m/s} \hat{i}) = +0.99 \text{ m/s} \hat{i} \]
The Ballistic Pendulum: Conservation of Energy

\[ m_1 = 0.01 \text{ kg} \]
\[ m_2 = 1.0 \text{ kg} \]
\[ v_f = +0.99 \text{ m/s} \]

What is the maximum height the pendulum reaches after being struck?

\[
\frac{1}{2} (m_1 + m_2) v_f^2 = (m_1 + m_2) g h
\]

\[
h = \frac{v_f^2}{2g} = \frac{(0.99 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.05 \text{ m}
\]

(See Sample Problem 9.07)
The same pendulum is used to measure the firing speed of an unknown projectile. If the pendulum rises 12 cm, then the speed of the projectile was...

a.) 155 m/s
b.) 240 m/s
c.) 100 m/s
d.) 630 m/s
e.) 41.7 m/s
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(See Sample Problem 9.07)
Impulse

- Forces cause a **change in momentum** over time

\[
\vec{F} = \frac{d\vec{p}}{dt} \quad \vec{F}_{avg} \Delta t = \vec{p}_f - \vec{p}_i
\]

- The impact in collisions often yields **very strong forces**, but only over **very short times**

- The **impulse** is the total effect of these forces to **change momentum** in the collision

\[
\vec{J} = \vec{F}_{avg} \Delta t \quad \vec{J} = \vec{p}_f - \vec{p}_i
\]

---

**Units:** \( N \cdot s = \text{kg} \cdot \frac{m}{s} \)
Example: Impulse

A person with a mass of 90 kg jumps to the ground with a speed of 6 m/s just before impact. What is the average force experienced in coming to rest if:

a.) Their knees are held stiff, coming to rest in 0.1 s?

b.) Their knees bend, coming to rest in 0.5 s?

\[
\vec{J} = \vec{p}_f - \vec{p}_i \\
\vec{J} = F_{avg} \Delta t
\]

\[
\vec{J} = (0) - (90 \text{ kg})(-6 \text{ m/s} \hat{j}) = +540 \text{ kg m/s} \hat{j}
\]

a.) \[
F_{avg} = \frac{\vec{J}}{\Delta t} = \frac{+540 \text{ kg m/s} \hat{j}}{0.1 \text{ s}} = +5400 \text{ N} \hat{j}
\]

b.) \[
F_{avg} = \frac{\vec{J}}{\Delta t} = \frac{+540 \text{ kg m/s} \hat{j}}{0.5 \text{ s}} = +1080 \text{ N} \hat{j}
\]

~ 4000 N will break bones!
Newton’s revelation about gravity:

- **Not** just the Earth
- **Not** just $g = 9.8 \, m/s^2$
- The **same force** that pulls the apple to Earth also holds the moon in Earth orbit and the Earth in solar orbit

---

Every object that has mass exerts an attractive gravitational force on **every other one in the universe**
Newton’s Law of Gravitation

• The force of gravity between two masses:
  ➢ Is always **attractive**
  ➢ Increases **linearly** with both **masses**
  ➢ Decreases with **distance** as the **inverse squared**

\[
F_g = G \frac{m_1 m_2}{r^2}
\]

\[G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2\]
• Imagine a **broken paintball gun**, shooting paintballs **uniformly in 3D space**

  ➢ A total of 100 / minute, spreading out in a **sphere**
  ➢ The **surface area of the sphere** increases: \( A = 4\pi r^2 \)
  ➢ The **density** of paintballs **decreases** like \( 1/r^2 \)

\[
\text{Intensity} = \frac{\#}{\text{Area}} = \frac{\#}{4\pi r^2}
\]

• Lots of effects follow **inverse-square laws**:
  ➢ Brightness from a lightbulb, volume from a stereo, ...
  ➢ Electric force, magnetic force, ...
Sample Problem 13.01:
Determine the net gravitational force on mass $m_1$.

$m_1 = 6.0 \text{ kg}$
$m_2 = 4.0 \text{ kg}$
$m_3 = 4.0 \text{ kg}$
$a = 2.0 \text{ cm}$

\[ F_{13} = \frac{1}{4} F_{12} \]

Which force is bigger?
- Equal masses
- Twice the distance

Which directions?
+ $F_{12} \hat{j}$
− $F_{13} \hat{i}$
Example: Superposition of Gravitational Forces

\[ F_{g} = G \frac{m_1 m_2}{r^2} \]

\[ G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]

\[
F_{12} = G \frac{m_1 m_2}{a^2} = \left(6.67 \times 10^{-11} \frac{N \text{ m}^2}{\text{kg}^2}\right) \frac{(6.0 \text{ kg})(4.0 \text{ kg})}{(0.02 \text{ m})^2} = 4.0 \times 10^{-6} N
\]

\[
F_{13} = G \frac{m_1 m_2}{(2a)^2} = \left(6.67 \times 10^{-11} \frac{N \text{ m}^2}{\text{kg}^2}\right) \frac{(6.0 \text{ kg})(4.0 \text{ kg})}{(0.04 \text{ m})^2} = 1.0 \times 10^{-6} N
\]

4x smaller!
Example: Superposition of Gravitational Forces

\[
\vec{F}_{1,\text{net}} = -1.0 \ \mu N \ \hat{i} + 4.0 \ \mu N \ \hat{j}
\]

\[
\vec{F}_{13} = -1.0 \times 10^{-6} N \ \hat{i}
\]

\[
\left| \vec{F}_{1,\text{net}} \right| = \sqrt{(-1.0 \ \mu N)^2 + (4.0 \ \mu N)^2} = 4.1 \ \mu N
\]

\[
\theta = \tan^{-1} \left( \frac{4.0 \ \mu N}{-1.0 \ \mu N} \right) = -76^\circ \text{ or } +104^\circ
\]

Careful about the quadrant of \( \tan^{-1} \)!
Four identical particles are arranged as shown, with three placed on a circle and the fourth one placed at the center. Rank the arrangements according to the magnitude of the net gravitational force on the central particle, from greatest to least.

a.) #1 > #2 > #3
b.) #2 > #3 > #1
c.) #2 > #1 > #3
d.) #3 > #1 > #2
e.) #3 > #2 > #1
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The work needed to escape the pull of gravity gives the gravitational potential energy of the interaction.

\[
U_G = \int_{r}^{\infty} \frac{G m_1 m_2}{x^2} \, dx = -\frac{G m_1 m_2}{r}
\]

- Negative potential energy! Takes work to escape to zero.
• The **kinetic energy** needed to escape an object’s gravity determines the minimum **escape speed**

\[
K_i + U_G = 0
\]

\[
\frac{1}{2}mv^2 - G\frac{mM}{R} = 0
\]

\[
v = \sqrt{\frac{2GM}{R}}
\]
On Monday, Nov. 26, 2018 the NASA InSight lander arrived safely on Mars.
• **Newton’s law of gravitation** determines how stars, planets, and spacecraft travel through space.

• **Kepler’s laws** of planetary motion describe planets orbiting the sun, satellites orbiting the Earth, and so on.
  
  ➢ Planetary orbits do not have to be circles; in general, they are **ellipses**.

  ➢ Planets move **quicker when nearby** their stars and **slower when farther away**.  

  ➢ The **radius** of a planet’s orbit is directly related to the **period** it takes to complete one orbit.
Kepler’s “Law of Periods”

- For a planet moving in a circular orbit, its \textit{centripetal acceleration} is given by the \textit{gravitational force}

\[ \vec{F}_G = \vec{F}_c \]

- Recall \textit{uniform circular motion}: centripetal acceleration and the \textit{period} of orbit

\[ v = \frac{2\pi r}{T} \quad F_c = m \frac{v^2}{r} \]

\[ G \frac{mM}{r^2} = m \frac{v^2}{r} \]

\[ T^2 = \frac{(2\pi)^2}{GM} r^3 \]

\textbf{Law of Periods}
An Earth-like planet has been discovered in circular orbit around a nearby star. Its orbital radius is observed to be 150 million km ($1.5 \times 10^8 \text{m}$), and its orbital period is 250 Earth days ($2.16 \times 10^7 \text{s}$). What is the mass of the newly-discovered planet?

a.) $1.7 \times 10^{22} \text{ kg}$

b.) $4.3 \times 10^{21} \text{ kg}$

c.) $4.1 \times 10^{12} \text{ kg}$

d.) $3.1 \times 10^9 \text{ kg}$

e.) $1.3 \times 10^6 \text{ kg}$
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"…That one body may act upon another at a distance [through] a Vacuum, without the Mediation of any thing else ... is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it."

— Isaac Newton

• Is gravity instantaneous?

➢ If our sun went supernova right now, would the Earth fall out of its orbit immediately?

➢ Or would there be a time delay for the “information” to reach us?

➢ What really transmits the gravitational force?
Einstein’s Equivalence Principle

Could you tell the difference between gravity and external acceleration?

...What if there was no difference between them?
A beam of light would appear to "bend" if the floor were accelerating upward.

...Would light also "bend" in the presence of gravity?
• Even though **light doesn’t have any mass**, it is still affected by the **pull of gravity**!

➢ We can observe this “**gravitational lensing**” of light from distance stars

➢ Bent into an “**Einstein ring**” by passing nearby a massive galaxy
Einstein’s **Theory of Relativity** reimagines gravity as the result of the curved geometry of space.

- Two boats sailing to the south pole converge...
  - **Not because of a “force”** between them
  - Because they are moving across the curved surface of the Earth
• One consequence of relativity is that violent collisions of stars and black holes can emit gravitational waves

➢ Gravitational waves were directly observed at the LIGO and VIRGO observatories in 2016

➢ Awarded the 2017 Nobel Prize in Physics
A Parting Thought...
What Have You Learned This Semester?

**Kinematics**

A description of how motion occurs

If $\ddot{a} = \text{const.}$ then

- $\vec{v}_f = \vec{v}_i + \ddot{a}t$
- $\Delta \vec{r} = \vec{v}_i t + \frac{1}{2} \ddot{a} t^2$
- $v_f^2 = v_i^2 + 2\ddot{a} \cdot \Delta \vec{r}$

**Dynamics**

A theory about what causes motion to occur

Newton’s Laws:

- **N2L:** $\vec{F}_{net} = m\ddot{a}$
- **N3L:** $\vec{F}_{12} = -\vec{F}_{21}$

---

Work – Kinetic Energy Theorem!
Your Physicist’s Toolkit

- You have developed a broad toolkit to **view any physical problem in different ways**.
  - Each perspective **highlights some features and obscures others**
  - **Use them all** to understand the **same physics principles from many angles**.

<table>
<thead>
<tr>
<th>Perspective</th>
<th>Variable</th>
<th>Agent of Change</th>
<th>N2L</th>
<th>N3L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forces</td>
<td>$\vec{a}$</td>
<td>$\vec{F}$</td>
<td>$\vec{F} = m\vec{a}$</td>
<td>$\vec{F}<em>{12} = -\vec{F}</em>{21}$</td>
</tr>
<tr>
<td>Energy</td>
<td>$K = \frac{1}{2}mv^2, U$</td>
<td>$W = \vec{F} \cdot \Delta\vec{r}$</td>
<td>$\vec{F} \cdot \Delta\vec{r} = K_f - K_i$</td>
<td>$W_{12} = -W_{21}$</td>
</tr>
<tr>
<td>Momentum</td>
<td>$p = mv$</td>
<td>$\vec{J} = \vec{F}_{avg} \Delta t$</td>
<td>$\vec{F}_{avg} \Delta t = \vec{p}_f - \vec{p}_i$</td>
<td>$\vec{J}<em>{12} = -\vec{J}</em>{21}$</td>
</tr>
<tr>
<td>Angular Momentum</td>
<td>$\vec{L} = \vec{r} \times \vec{p}$</td>
<td>$\vec{\tau} = \vec{r} \times \vec{F}$</td>
<td>$\vec{\tau}_{avg} \Delta t = \vec{L}_f - \vec{L}_i$</td>
<td>(Next Semester!)</td>
</tr>
</tbody>
</table>

**PHY123**

**L12 – Gravitation**

39 / 40
We know that Einstein’s Theory of Relativity is correct because...

a.) Einstein said so, and he was a pretty smart guy.

b.) The Theory of Relativity uses complicated math, so it must be right.

c.) Newton was a jerk, so his Law of Gravity was bound to be wrong.

d.) Newton’s Law of Gravity was no good at predicting the motion of planets.

e.) Relativity makes testable predictions that have been confirmed by experiment.
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