LECTURE 10- EXAMPLE PROBLEMS

Chapter 6-8
Professor Noronha-Hostler
Professor Montalvo
Thursday November 15, 2018
9:40 – 11:00 PM

Classes on Friday Nov. 16th
NO CLASSES week of Thanksgiving (Nov. 19-23)
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TEST SUBJECTS

- Chapters 6-8 + Uniform Circular Motion (Chapter 4.5)
- Read your chapters too!
TODAY

- Inclined plane with
  - Forces, Work-Kinetic Energy Theory, Conservation of Energy
- Pendulum
  - Potential energy vs. Kinetic energy
- Loop the loop
  - Uniform Circular Motion, Conservation of energy
- Stopping a go kart with a spring
  - Conservation of Energy, Hook’s law
INCLINED PLANE

Jennifer is pushing a heavy box up a rough inclined surface at a constant speed by applying a horizontal force as shown in the drawing. The coefficient of kinetic friction for the box on the inclined surface is $\mu_k$. Which one of the following expressions correctly determines the normal force on the box?

- **A.** $F_N = \frac{F - mg \tan \theta}{\mu_k}$
- **B.** $F_N = F - \mu_k mg \tan \theta$
- **C.** $F_N = \frac{F \cos \theta - mg \sin \theta}{\mu_k}$
- **D.** $F_N = F \cos \theta - \mu_k mg \sin \theta$
- **E.** $F_N = \frac{F \sin \theta - mg \cos \theta}{\mu_k}$
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- **E.** $F_N = \frac{F \sin \theta - mg \cos \theta}{\mu_k}$
INCLINED PLANE + FRICTION

Free body diagram

25°
SPEED AT THE BOTTOM OF THE HILL?

Girl on a sled starts at rest down a hill of length L, what is her velocity at the bottom of the hill?
SPEED AT THE BOTTOM OF THE HILL?

Girl on a sled starts at rest down a hill of length L, what is her velocity at the bottom of the hill?

Draw the Free body diagram

\[ F_N \]
\[ f_k \]
\[ mg \]
SPEED AT THE BOTTOM OF THE HILL?

Girl on a sled starts at rest down a hill of length L, what is her velocity at the bottom of the hill?

Forces along axes
SPEED AT THE BOTTOM OF THE HILL?

Girl on a sled starts at rest down a hill of length \( L \), what is her velocity at the bottom of the hill?

\[
F_N = mg \cos \theta
\]

\[
mg \sin \theta - f_k = ma
\]

**Newton’s Laws**
SPEED AT THE BOTTOM OF THE HILL?

Find acceleration

\[ mg \sin \theta - f_k = ma \]
Recall

\[ f_k = \mu_k F_N \]

\[ mg \sin \theta - \mu_k F_N = ma \]
Recall

\[ F_N = mg \cos \theta \]

\[ mg \sin \theta - \mu_k mg \cos \theta = ma \]

\[ a = g \sin \theta - \mu_k g \cos \theta \]
SPEED AT THE BOTTOM OF THE HILL?

Find velocity

\[ a = g \sin \theta - \mu_k g \cos \theta \]

\[ v^2 = v_0^2 + 2ad \]

\[ v_0 = 0 \]

\[ d = L \]

\[ v^2 = 2aL \]

\[ v^2 = 2L(g \sin \theta - \mu_k g \cos \theta) \]

\[ v^2 = 2Lg(\sin \theta - \mu_k \cos \theta) \]

\[ v = \sqrt{2Lg(\sin \theta - \mu_k \cos \theta)} \]
SPEED AT THE BOTTOM OF THE HILL?

Work-Kinetic Energy Theorem
SPEED AT THE BOTTOM OF THE HILL?

Work-Kinetic Energy Theorem

\[ W = \Delta K = K_f - K_i \]

\[ K_f = \frac{1}{2}mv_f^2 \quad K_i = 0 \]

\[ W = Fd \cos \phi \]
SPEED AT THE BOTTOM OF THE HILL?

**Work-Kinetic Energy Theorem**

\[ W = \frac{1}{2} m v_f^2 \]

\[ W = Fd \cos \phi \]

Angle between F and d, NOT the same as angle of the inclined plane!

Also,

\[ d = L \]

\[ \theta \]

\[ W = F_x L \]
SPEED AT THE BOTTOM OF THE HILL?

Work-Kinetic Energy Theorem

\[ F_x L = \frac{1}{2} mv_f^2 \]

Recall, the force is in the x direction is

\[ F_x = mg \sin \theta - f_k \]

\[ L(mg \sin \theta - f_k) = \frac{1}{2} mv_f^2 \]

\[ L(mg \sin \theta - \mu_k mg \cos \theta) = \frac{1}{2} mv_f^2 \]

\[ v_f = \sqrt{2Lg(\sin \theta - \mu_k \cos \theta)} \]
SPEED AT THE BOTTOM OF THE HILL?

Conservation of Energy

Diagram with angles and distances labeled.
SPEED AT THE BOTTOM OF THE HILL?

Conservation of Energy for **Isolated System**

\[
\Delta E = \Delta E_{mec} + \Delta E_{th} = 0
\]

\[
\Delta E_{mec} = (U_f + K_f) - (U_i + K_i)
\]

\[
\Delta E_{th} = f_k L = \mu_k LF_N = \mu_k L mg \cos \theta
\]
SPEED AT THE BOTTOM OF THE HILL?

Mechanical Energy

\[ \Delta E_{mec} = (U_f + K_f) - (U_i + K_i) \]

Initial

\[ U_i = mgh \]
\[ K_i = 0 \]

Final

\[ U_f = 0 \]
\[ K_f = \frac{1}{2}mv_f^2 \]

\[ \Delta E_{mec} = \frac{1}{2}mv_f^2 - mgh \]

Need to find \( h \)
SPEED AT THE BOTTOM OF THE HILL?

\[
\sin \theta = \frac{h}{L} \quad \quad \quad \quad h = L \sin \theta
\]
SPEED AT THE BOTTOM OF THE HILL?

Mechanical energy

$$\Delta E_{mec} = \frac{1}{2}mv_f^2 - mgh \quad h = L \sin \theta$$

$$\Delta E_{mec} = \frac{1}{2}mv_f^2 - mgL \sin \theta$$
**SPEED AT THE BOTTOM OF THE HILL?**

Conservation of Energy for **Isolated System**

\[
\Delta E = \Delta E_{mec} + \Delta E_{th} = 0
\]

\[
\Delta E_{mec} = \frac{1}{2}mv_f^2 - mgL \sin \theta
\]

\[
\Delta E_{th} = \mu_k Lmg \cos \theta
\]

\[
\frac{1}{2}mv_f^2 - mgL \sin \theta + \mu_k Lmg \cos \theta = 0
\]

\[
v_f = \sqrt{2Lg(\sin \theta - \mu_k \cos \theta)}
\]
A steel ball is whirled on the end of a chain in a horizontal circle of radius $R$ with a constant period $T$. If the radius of the circle is then reduced to $0.75R$, while the period remains $T$, what happens to the centripetal acceleration of the ball?

A.) The centripetal acceleration increases to 1.33 times its initial value.

b) The centripetal acceleration increases to 1.78 times its initial value.

c) The centripetal acceleration decreases to 0.75 times its initial value.

d) The centripetal acceleration decreases to 0.56 times its initial value.

e) The centripetal acceleration does not change.
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FIRST, FIND VELOCITY

\[
T = \frac{2\pi R}{v_1}
\]

\[
v_1 = \frac{2\pi R}{T}
\]

\[
T = \frac{2\pi (0.75R)}{v_2}
\]

\[
v_2 = 0.75 \frac{2\pi R}{T}
\]

\[
v_2 = 0.75v_1
\]
CENTRIPETAL ACCELERATION

Original

\[ a_1 = \frac{v_1^2}{R} \]

Smaller Radius

\[ a_2 = \frac{v_2^2}{0.75R} \]

\[ a_2 = \frac{(0.75v_1)^2}{0.75R} \]

\[ a_2 = 0.75 \frac{v_1^2}{R} \]

Recall

\[ v_2 = 0.75v_1 \]

\[ a_2 = 0.75a_1 \]
PENDULUM
SPEED AT LOWEST POINT?

A Pendulum of Length L is released with an angle of $\theta$. What is its speed at the lowest point?
SPEED AT LOWEST POINT?

\[ \cos \theta = \frac{L - h}{L} \]

\[ h = L - L \cos \theta \]
SPEED AT LOWEST POINT?

Isolated System, no friction

\[ \Delta E_{mec} = (U_f + K_f) - (U_i + K_i) = 0 \]

\[ K_i = 0 \quad K_f = \frac{1}{2}mv_f^2 \quad \frac{1}{2}mv_f^2 = mgh \]

\[ U_i = mgh \quad U_f = 0 \quad \frac{1}{2}mv_f^2 = mg(L - L\cos \theta) \]

\[ v_f = \sqrt{2g(L - L\cos \theta)} \]
A ball is attached to a string and whirled in a horizontal circle. The ball is moving in uniform circular motion when the string separates from the ball (the knot wasn’t very tight). Which one of the following statements best describes the subsequent motion of the ball?

a) The ball immediately flies in the direction radially outward from the center of the circular path the ball had been following.

b) The ball continues to follow the circular path for a short time, but then it gradually falls away.

c) The ball gradually curves away from the circular path it had been following.

d) The ball immediately follows a linear path away from, but not tangent to the circular path it had been following.

e) The ball immediately follows a line that is tangent to the circular path the ball had been following.
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LOOP THE LOOP
MINIMUM HEIGHT TO SURVIVE THE LOOP?

\[ U_1 = mgh \quad K_1 = 0 \]
\[ U_2 = mg(2R) \quad K_2 = \frac{1}{2}mv_2^2 \]
\[ U_3 = 0 \quad K_3 = \frac{1}{2}mv_3^2 \]

\[ mgh = 2mgR + \frac{1}{2}mv_2^2 \]
MINIMUM HEIGHT TO SURVIVE THE LOOP?

For the minimum h, that means that the normal force from the loop is approaching zero.

\[ F_N \to 0 \]

\[ F_N + F_g = ma = m \frac{v^2}{R} \]

\[ F_g = m \frac{v^2}{R} \]

Since \( F_g = mg \)

\[ mg = m \frac{v^2}{R} \]

\[ v^2 = gR \]
MINIMUM HEIGHT TO SURVIVE THE LOOP?

Collecting the Equations

\[ mgh = 2mgR + \frac{1}{2}mv^2 \]

And

\[ v^2 = gR \]

\[ h = 2R + \frac{1}{2g}v^2 \]

\[ h = 2R + \frac{1}{2}R = \frac{5}{2}R \]
MINIMUM HEIGHT TO SURVIVE THE LOOP?

\[ h = \frac{5}{2}R \]

\[ R = 11.75 \text{ cm} \]

\[ h = \frac{5}{2}(11.75 \text{ cm}) = 30 \text{ cm} \]
A block is in contact with a rough surface as shown in the drawing. The block has a rope attached to one side. Someone pulls the rope with a force, which is represented by the vector in the drawing. The force is directed at an angle $\theta$ with respect to the horizontal direction. The magnitude of the force is equal to two times the magnitude of the frictional force, which is designated $f$. For what value of $\theta$ is the net work on the block equal to zero joules?

a) $0^\circ$

b) $30^\circ$

c) $45^\circ$

d) $60^\circ$

e) Net work will be done in the object for all values of $\theta$. 
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a) 0°
b) 30°
c) 45°
d) 60°
e) Net work will be done in the object for all values of $\theta$. 
GO KART + SPRING
HOW MUCH DOES THE SPRING COMPRESS?

\[ K_i = \frac{1}{2}mv_0^2 \quad K_f = 0 \]

\[ W = \Delta K = \frac{1}{2}mv_0^2 \]

\[ W = \frac{1}{2}kx^2 \]

\[ \frac{1}{2}kx^2 = \frac{1}{2}mv_0^2 \]

\[ m = 100\text{kg} \quad k = 10,000\text{N/m} \]
HOW MUCH DOES THE SPRING COMPRESS?

\[
\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2 \quad \Rightarrow \quad x = \sqrt{\frac{mv_0^2}{k}}
\]

\(m = 100\text{kg}\) \quad \(k = 10,000\text{N/m}\) \quad \(v = 10\text{m/s}\)

\[
x = \sqrt{\frac{100\text{kg}(10\text{m/s})^2}{10,000\text{N/m}}} = 1\text{m}
\]
Jamal’s gravitational potential energy is 1870 J as he sits 2.20 m above the ground in a sky diving airplane. What is his gravitational potential energy when he begins to jump from the airplane at an altitude of 923 m?

a) $3.29 \times 10^4$ J

b) $9.36 \times 10^2$ J

c) $4.22 \times 10^6$ J

d) $1.87 \times 10^3$ J

e) $7.85 \times 10^5$ J
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GOOD LUCK ON THE TEST!