Today: Projectile Motion and Relative Velocity

Review:

- **Vectors**: Quantity with **magnitude** and **direction**
  
  \[
  A \quad \quad \quad B \quad \quad \quad C \quad \quad \quad B = A \quad \quad B = A \\
  C = -A \quad \quad C = A
  \]

- **Vector addition**

  \[
  \vec{R} = \vec{A} + \vec{B}
  \]

- **Components**

  \[
  A_x i + A_y j + A_z k
  \]

- **Unit vectors** \( \hat{i}, \hat{j}, \hat{k} \)

- **2D Motion**

  \[
  \begin{align*}
  \Delta r &= r_2 - r_1 \\
  v_{av} &= \frac{\Delta r}{\Delta t} \\
  v &= \frac{dr}{dt} \\
  a_{av} &= \frac{\Delta v}{\Delta t} \\
  a &= \frac{dv}{dt}
  \end{align*}
  \]
Determine the components of the vector $K = A + B + C + D$.

A. X-component: +2  
   Y-component: +2

B. X-component: +8  
   Y-component: +6

C. X-component: -2  
   Y-component: -2

D. X-component: -8  
   Y-component: -6

E. X-component: +2  
   Y-component: +4
Two speedboats are moving on a lake. Boat A goes from traveling 15 m/s east to 20 m/s north in 10 seconds. Boat B goes from 10 m/s west to 15 m/s west in 10 seconds. Which of the following is true for the average accelerations of the boats ($a_A$ and $a_B$) in the 10-second interval described here?

A. $a_A$ has the same magnitude as $a_B$, but not the same direction.

B. $a_A$ has a greater magnitude than $a_B$.

C. $a_A$ has a smaller magnitude than $a_B$.

D. $a_A$ and $a_B$ point in the same direction.

E. $a_A$ points North and $a_B$ points West.
This illustration shows the path of a robotic vehicle, or rover. What is the direction of the rover’s average acceleration vector for the time interval from \( t = 0.0 \, \text{s} \) to \( t = 2.0 \, \text{s} \)?

A. up and to the left
B. up and to the right
C. down and to the left
D. down and to the right
E. none of the above
A truck is originally driving north. It slows down as it turns a corner, as shown in the top-view drawing. At point B, the truck is driving west. Which of the following statements is CORRECT about the average acceleration $a_{av}$ of the truck between A and B?

A. $a_{av}$ points NorthEast because of the centrifugal force.

B. $a_{av}$ points SouthWest because of the centripetal force.

C. $a_{av}$ points East because the truck slows down.

D. $a_{av}$ points South because the truck slows down.

E. We need more information to know the exact direction of $a_{av}$.
Effect of air resistance on projectiles

- Calculations are complicated.
- Acceleration is not constant.
- Effects can be very large.
- Maximum height and range decrease.

In what follows, we will ignore air resistance!
Two toy trucks roll off the ends of tables. The heights of the tables, the speeds of the trucks, and the masses of the trucks are given. Which is correct?

A. Truck A will be in the air for a LONGER amount of time than truck B.

B. Truck A will be in the air for the SAME amount of time as truck B.

C. Truck A will be in the air for a SHORTER amount of time than truck B.

D. Truck A will have the SAME velocity as truck B when the trucks hit the floor.

E. Can not be determined with the information provided.
Suppose $v_0$ is purely in $x$-direction.

What happens??

Projectile motion (no air resistance)

- Motion of object in presence of gravity with both vertical and horizontal components

**Intuition:** Path of motion (trajectory) is an arc (baseball, fireworks, cannon ball, ski jump, …)

**Assumptions:**

\[ a_x = 0 \quad a_y = -g \]

\[ \vec{a} = 0\hat{i} + (-g)\hat{j} = -g\hat{j} \]

What is the shape of the trajectory?
X and Y motion are separable—Figure 3.16

- The red ball is dropped, and the yellow ball is fired horizontally as it is dropped.

- The strobe marks equal time intervals.
Rifles are fired horizontally from platforms at various heights. The bullets fired from these rifles are identical, but they leave the rifle barrels at different speeds as shown in the diagrams. Ignore air resistance.

Rank the cases A-F according to how long the bullets are in air before they hit the ground.

a.) \( C > B > D > A > E > F \)
b.) \( B > C > D > F > E > A \)
c.) \( A > E > C > D > F > B \)
d.) \( B = C > D > A = E = F \)
e.) \( A > E > C = D = F > B \)
Projectile motion is superposition of independent $x$ - motion and $y$ - motion!

Where: 
\[ a_x = 0 \; ; \; a_y = -g \]

Let’s do the math …

Simplify! Let:
\[ x_0 = y_0 = 0 \; \text{at} \; t = 0 \]
\[ a_x = 0 \; ; \; a_y = -g \]
\[ v_0 \; \text{at angle} \; \alpha_0 \]

Initial Conditions: 
\[ v_{0x} = v_0 \cos \alpha_0 \; ; \; v_{0y} = v_0 \sin \alpha_0 \]

Trajectory: 
\[ y = f(x) \; ; \; \text{range} = R \; ; \; \text{height} = h \]

Solve $x$ and $y$ motion independently:

1. \[ v_x = v_{0x} = v_0 \cos \alpha_0 \]
2. \[ x = v_{0x} t = (v_0 \cos \alpha_0) t \]
3. \[ v_y = v_{0y} - gt = v_0 \sin \alpha_0 - gt \]
4. \[ y = v_{0y} t - \frac{1}{2} gt^2 = (v_0 \sin \alpha_0) t - \frac{1}{2} gt^2 \]
$$y = (\tan \alpha_0) x - \left( \frac{g}{2v_0^2 \cos^2 \alpha_0} \right) x^2$$

This has the form: $y = bx - cx^2 = \text{PARABOLA!!!}$

- To calculate $h$ (max height)

Occurs when:
$$v_y = 0 \implies 0 = v_0 \sin \alpha_0 - gt \implies t_1 = \frac{v_0 \sin \alpha_0}{g}$$

Plug into

$$y_{\text{max}} = h = \frac{v_0^2 \sin^2 \alpha_0}{2g}$$

- To calculate the range $R$:

Occurs when: $y = 0$

$$0 = x \left[ \tan \alpha_0 - \left( \frac{g}{2v_0^2 \cos^2 \alpha_0} \right) x \right] \implies R = \frac{v_0^2 \sin 2\alpha_0}{g}$$
So, range max when:

$$\alpha_o = 45 \left( = \frac{\pi}{4} \right)$$

\[
R = \frac{v_0^2 \sin 2\alpha_0}{g}
\]

**Example:**
Secret Agent 007 skis off a 100m high cliff at \( v_{0x} = 20 \text{ m/s} \). He intends to land in the car at point D.

Where should “Q” park the car?

\( x_0 = y_0 = 0 \) at \( t = 0 \)

\( x = v_{0x} t \Rightarrow x = (20 \text{ m/s})t \)

\( y = v_{0y} t - \frac{1}{2} gt^2 \Rightarrow (-100 \text{ m}) = -\frac{1}{2} gt^2 \)

\[ t = \sqrt{\frac{-100 \text{ m}}{-4.90 \text{ m/s}^2}} = 4.52 \text{ s} \]

\[ \Rightarrow D = (20 \text{ m/s})(4.52 \text{ s}) = 90.4 \text{ m} = D \]
A baseball is thrown from point S in right field to home plate. The dashed line in the diagram shows the path of the ball. For this exercise, use a coordinate system with up as the positive vertical direction and to the left as the positive horizontal direction.

Select the best choice from the answers below.

A) The horizontal velocity vs. time graph is B, and the vertical velocity vs. time graph is I.
B) The horizontal velocity vs. time graph is A, and the vertical velocity vs. time graph is H.
C) The horizontal acceleration vs. time graph is C, and the vertical acceleration vs. time graph is B.
D) The horizontal acceleration vs. time graph is C, and the vertical acceleration vs. time graph is I.
E) None of the answers above are correct.
Tranquilizing a falling monkey

• Where should the blow-hard aim his blowgun?

Dashed arrows show how far the dart and monkey have fallen at specific times relative to where they would be without gravity. At any time, they have fallen by the same amount.

**Without gravity**
• The monkey remains in its initial position.
• The dart travels straight to the monkey.
• Therefore, the dart hits the monkey.

**Trajectory of dart with gravity**

**With gravity**
• The monkey falls straight down.
• At any time $t$, the dart has fallen by the same amount as the monkey relative to where either would be in the absence of gravity: $\Delta y_{\text{dart}} = \Delta y_{\text{monkey}} = -\frac{1}{2}gt^2$.
• Therefore, the dart always hits the monkey.
A zoo keeper fires a tranquilizer dart directly at a monkey. The monkey lets go at the same instant that the dart leaves the gun barrel. The dart reaches a maximum height $P$ before striking the monkey. Ignore air resistance.

When the dart is at $P$, the monkey

A. is at $A$ (higher than $P$).

B. is at $B$ (at the same height as $P$).

C. is at $C$ (lower than $P$).

D. not enough information given to decide
Relative Velocity (Ch 3.5)

**SO FAR:** Motion described in FIXED reference frame (coordinate system)

**BUT:** Nothing prevents us from choosing another frame moving at constant velocity: \( v = \text{const.} \).

Relative Motion in 1D:

![Diagram showing relative motion in 1D](image)

- **Fixed frame A:** person on station platform
- **Moving frame B:** train moving at constant speed
- **Moving object P:** person walking on train

What is velocity of person w/r/t train?

\[ x_{P/A} = x_{P/B} + x_{B/A} \]
VELOCITIES

\[
\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt}
\]

Relative velocity of reference frames (train w/r/t station)

Velocity of passenger in stationary frame

\[ \rightarrow v_{P/A} = v_{P/B} + v_{B/A} \]

Velocity of passenger measured in moving frame

[ Subscripts help keep order: \( P/A = P/B \times B/A \) ]

EXAMPLE: You drive east on College Ave. at 15 m/s. Your roommate is driving west on College Ave. at 20 m/s. What is roommate’s velocity w/r/t you?

\[
v_{R/E} = v_{R/Y} + v_{Y/E}
\]

\[ \Rightarrow v_{R/Y} = v_{R/E} - v_{Y/E} \]

\[ v_{R/Y} = -20 \text{ m/s} - (+15 \text{ m/s}) = -35 \text{ m/s} \]

(Roommate is moving in \(-x\) direction in your frame.)
Now suppose roommate: 15 m/s East
You: 20 m/s East
What is roommate’s velocity w/r/t you?

\[ \mathbf{v}_{R/E} = \mathbf{v}_{R/Y} + \mathbf{v}_{Y/E} \Rightarrow \mathbf{v}_{R/Y} = \mathbf{v}_{R/E} - \mathbf{v}_{Y/E} \]

\[ \mathbf{v}_{R/Y} = 15 \text{ m/s} - 20 \text{ m/s} = -5 \text{ m/s} \]

(Roommate is STILL moving in \(-x\) direction in your frame.)

Relative Motion in 2- or 3-D

\[ \mathbf{r}_{P/A} = \mathbf{r}_{P/B} + \mathbf{r}_{B/A} \]

\[ \frac{d}{dt} \Rightarrow \mathbf{v}_{P/A} = \mathbf{v}_{P/B} + \mathbf{v}_{B/A} \]

(P/A = P/B \times B/A)
Flying in a crosswind

- A crosswind affects the motion of an airplane.
EXAMPLE: A boat heading North crosses a river with velocity 8.00 km/h with respect to the water. The river flows 4.00 km/h East. (a) What is the boat’s velocity relative to an observer on either bank?

E = earth (fixed frame); R = river (moving frame); B = boat

\[ v_{B/E} = v_{B/R} + v_{R/E} \]

\[ v_{B/E} = (4.00i + 8.00j) \text{ km/h} \]

\[ v_{BE} = \sqrt{(8.00)^2 + (4.00)^2} = 8.94 \text{ km/h} \]

\[ \tan \theta = \frac{4.00}{8.00} \implies \theta = 26.6^\circ \]

(b) In what direction should the boat head to reach the north bank directly opposite the starting point?

\[ v_{B/E} = v_{B/R} + v_{R/E} \]

\[ \sin(\alpha) = \frac{4.0}{8.0} = \frac{1}{2} \]

\[ \alpha = \sin^{-1}(1/2) = 30^\circ \]
The pilot of a plane with airspeed of 200 km/h is able to fly directly west, even though a strong wind blows from the north at 120 km/h.

Fixed frame – earth (E)
Moving frame – air (W)
Moving object – plane (P)

$v_{P/W} = 200 \text{ km/h}

v_{W/E} = 120 \text{ km/h}

Which vector diagram depicts $v_{P/E}$ correctly?