Review of last lecture:

- Momentum:
  \[ p = mv \quad \Sigma F = ma = \frac{dp}{dt} \]

- Impulse of a Force:
  \[ J = \int_{t_1}^{t_2} F \, dt \]
  Impulse-Momentum Theorem:
  \[ J = p_2 - p_1 \]

- Conservation of Momentum:
  If \( \Sigma F_{ext} = 0 \) then \( P_F = P_I \Rightarrow [p_{A_f} + p_{B_f}] = [p_{A_f} + p_{B_f}] \)

- Types of Collisions:
  Elastic: \( P & KE \) conserved
  Inelastic: \( P \) only is conserved
  Completely Inelastic: \( P \) only conserved & objects stick
A 3.00-kg rifle fires a 0.00500-kg bullet at a speed of 300 m/s. Which force is greater in magnitude:
(i) the force that the rifle exerts on the bullet; or
(ii) the force that the bullet exerts on the rifle?

A. the force that the rifle exerts on the bullet
B. the force that the bullet exerts on the rifle
C. both forces have the same magnitude
D. not enough information given to decide

Newton’s 3rd Law

\[ F_{B \text{on} A} = -F_{A \text{on} B} \]
ROCKET PROPULSION

Conservation of momentum of rocket and propellant !!

No external forces:

\[ \Rightarrow P_i = P_f \]

\[ (M + m)v = Mv_f + m(v + v_{ex}) \]

\[ Mv = Mv_f + mv_{ex} \]

\[ M(v_f - v) = -mv_{ex} \]

Rocket is propelled by recoil !
Rocket Propulsion Demo
**CENTER OF MASS**

Point in an extended mechanical system that moves as though all the mass were concentrated at that point.

Consider a collection of (different) masses distributed on x-axis.

Define center of mass:

\[ x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} \]

i.e.:

\[ x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\sum_i m_i x_i}{M} \]

Similar expression for \( y_{cm} \) and \( z_{cm} \)

\[ r_{cm} = \frac{\sum_i m_i r_i}{M} \]

If mass is a continuous distribution:

\[ r_{cm} = \frac{1}{M} \int r \, dm \]
The center of mass is the mass-weighted average position of the particles.

**EXAMPLE:**

\[
x_{cm} = \frac{m(4) + 3m(-2)}{m + 3m} = \frac{(4 + (-6))}{4} = \frac{-2}{4} = -\frac{1}{2}
\]
LOCATING CM

• If body is homogeneous and has a geometric center (e.g., uniform sphere or cube) then CM is geometric center.

• CM of symmetric body is along axis of symmetry (cylinder, wheel, dumbbell).

• For collection of extended bodies, find CM of each body, then CM of collection is CM of those point masses.

**EXAMPLE:** Two meter sticks, each with a mass of 0.25 kg, form a “T”. Where is the CM?

\[
\begin{align*}
\text{\textbf{EXAMPLE:}} & \quad \text{Two meter sticks, each with a mass of} \\
& \quad \text{0.25 kg, form a “T”. Where is the CM?}
\end{align*}
\]

\[
\begin{align*}
x_{cm} &= 0 \\
y_{cm} &= \frac{(0.25 \text{ kg})0 + (0.25 \text{ kg})(-0.5 \text{ m})}{(0.5 \text{ kg})} = -0.25 \text{ m}
\end{align*}
\]
A wire is bent into a U-shape. Where is the CM?
An L-shaped profile is cut from sheet metal. Where is the CM?
The right half of a meter stick has twice the density of the left half. Where is the CM?
MOTION OF CM

\[ r_{cm} = \frac{\sum_i m_i r_i}{M} \Rightarrow M r_{cm} = \sum_i m_i r_i \]

\( \left( \frac{d}{dt} \right) \Rightarrow M v_{cm} = \sum_i m_i v_i = \sum_i p_i = P \)

again \( \left( \frac{d}{dt} \right) \Rightarrow M a_{cm} = \frac{dP}{dt} = \sum F_{ext} \)

\[ \sum F_{ext} = M a_{cm} = \frac{dP}{dt} \]

• Under influence of external force CM moves like an imaginary particle of mass M.

• Total momentum \( P \) of collection of masses can only be changed by an external force.
**EXAMPLE:** Mass $m$ moves along $x$-axis with speed $v$ towards mass $2m$ which is at rest. They collide and stick. What is the motion of the CM before and after the collision?

Before collision:

After the collision CM moves with the combined mass:

$$v_{cm} = \frac{v}{3}$$

Before collision:

$$x_{cm} = \frac{mx_1 + 2mx_2}{3m} = \frac{x_1 + 2x_2}{3}$$

$$v_{cm} = \frac{dx_{cm}}{dt} = \frac{dx_1}{dt} + 2 \frac{dx_2}{dt} = \frac{v_i + 2(0)}{3} = \frac{v_i}{3}$$

Why does the velocity of the CM not change?
A rocket ship moves in a gravity-free region of space with constant velocity. It fires a short burst of gas from the rear engine. Afterwards, the CM of the rocket and gas system has:

\[ \sum F_{ext} = M a_{cm} \]

A) Sped up

B) Slowed down

C) Has same constant velocity

D) Changed but can’t tell how

E) Insufficient info to tell anything
Isaac Newton recognized that the moon’s motion can be explained if the force of gravity is inversely proportional to the square of the distance from the Earth’s center.
Newton’s Law of Gravitation

- We also know that the force of gravity is proportional to the object’s mass \( m \).
- Because of Newton’s 3rd law, it must also be proportional to the Earth’s mass \( M \).
- Thus:

\[
F \propto \frac{Mm}{r^2}
\]

The proportionality constant is called \( G \), the “gravitational constant”:

\[
G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}
\]

- **Newton’s law of gravitation**: Any two bodies in the universe, separated by distance \( r \), exert an attractive force on each other (A on B, B on A).

\[
F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}
\]

- **Magnitude** given by the formula.
- **Direction**: Toward the other body.
Vector Nature of Gravity Force

Write law of gravitation as a vector equation:

\[ F = -G \frac{Mm}{r^2} \hat{r} \]

The direction of the force is given by the unit vector \( \hat{r} \) pointing away from the other body.

Gravitational forces add as vectors:

\[ \vec{F} = \vec{F}_{10} + \vec{F}_{20} + \vec{F}_{30} + \cdots \]

The “weight” of an object can be defined as the vector sum of the gravitational forces of all the other masses in the universe!
Shown below are snapshots of a space rock (meteorite) of mass $m$ in the vicinity of planets. **How does the net force on the meteorite in Case A compare to the net force on the meteorite in Case B?**

A. The net force on the meteorite is greater in Case A.
B. The net force on the meteorite is the same in both cases.
C. The net force on the meteorite is greater in Case B.
D. The net force has the same magnitude, but different directions in the two cases.
E. The net force has the same direction, but different magnitudes in the two cases.