Use \( g = 10 \text{ m/s}^2 \) throughout the exam (for simplicity)

I. A diver 90 meters below the surface of a lake \((\rho_{\text{water}} = 1000 \text{ kg/m}^3)\) releases a bubble of air \((\rho_{\text{air}} = 1 \text{ kg/m}^3)\) that has a volume of 7 cm\(^3\). The bubble rises to the surface through the water, which has the same temperature throughout.

(a) What is the absolute pressure in atmospheres at the diver's depth \((1 \text{ atm} = 10^5 \text{ Pa})\)?

\[
P_{\text{abs}} = P_0 + \rho g h \quad P_0 = 4 \text{ atm}
\]

\[
\rho g h = (10^3)(10)(90) = 9 \times 10^5 \quad P_a = 9 \text{ atm}
\]

\[
P_{\text{abs}} = 10 \text{ atm}
\]

(b) What is the volume of the bubble (in cm\(^3\)) when it reaches the surface (where the pressure is one atm)?

\[
T \text{ is constant so } P_1 V_1 = P_2 V_2
\]

\[
V_2 = \left(\frac{P_1}{P_2}\right) V_1 = \left(\frac{10 \text{ atm}}{1 \text{ atm}}\right)(7 \text{ cm}^3) = 70 \text{ cm}^3
\]

(c) Find the magnitude and direction of the net force acting on the diver at a depth of 90 m if he and his diving gear have a combined mass of 100 kg and a volume of 0.098 m\(^3\).

\[
\text{Let } F_B \quad F_{\text{net}} = F_B - W = (\rho_w V_0) g - mg
\]

\[
W = 980 \text{ N} - 1000 \text{ N} = -20 \text{ N}
\]

\[
F_{\text{net}} = 20 \text{ N \text{ downward}}
\]
II.

A cylinder has a base area of \( A = 0.01 \text{ m}^2 \) and is fitted with a piston that can freely move. Below the piston, the cylinder is filled with an ideal gas that has a pressure \( P_1 = 104,000 \text{ Pa} \), a volume \( V_1 = 0.0012 \text{ m}^3 \), and a temperature of \( T_1 = 0 \text{ °C} \) when in state 1. The gas is then taken through the following four-step process:

- An \( m = 22 \text{ kg} \) metal block is placed on top of the piston, compressing the gas to state 2, with the gas still at \( T_2 = 0 \text{ °C} \).
- The cylinder is then put in contact with a boiling water bath, raising the gas temperature to \( T_3 = 100 \text{ °C} \) in state 3.
- The metal block is then removed and the gas expands to state 4, still at \( T_4 = 100 \text{ °C} \).
- Finally, the cylinder is placed in contact with an ice-water bath, bringing the gas back to \( T_1 = 0 \text{ °C} \) in state 1.

a) For each process, indicate whether the process is isothermal, isobaric, or adiabatic, and explain your reasoning.

(i) The process from state 1 to state 2: circle one  
Isothermal  
Isobaric  
Adiabatic

Explain: No change in Temp.

(ii) The process from state 3 to state 4: circle one  
Isothermal  
Isobaric  
Adiabatic

Explain: Temp. is constant

(iii) The process from state 2 to state 3: circle one  
Isothermal  
Isobaric  
Adiabatic

Explain: Pressure is constant

(iv) The process from state 4 to state 1: circle one  
Isothermal  
Isobaric  
Adiabatic

Explain: Pressure is constant

b) Determine the pressure and the volume of the gas in state 2.

\[ 1 \rightarrow 2 \text{ ISOTHERMAL} \text{ SO } P_1V_1 = P_2V_2 \]

\[ P_2 = P_1 + \left( \frac{2.2 \text{ ON}}{0.01 \text{ m}^2} \right) = 1.26 \times 10^5 \text{ Pa} \]

\[ V_2 = V_1 \left( \frac{P_1}{P_2} \right) = \]

\[ P_2 = \frac{1.26 \times 10^5 \text{ Pa}}{9.9 \times 10^{-4} \text{ m}^3} \]

\[ V_2 = 9.9 \times 10^{-4} \text{ m}^3 \]

c) How much work was done on the gas going from state 1 to state 2.

\[ W_{\text{ISO/THERM}} = P_2V_2 \ln \left( \frac{V_2}{V_1} \right) = (1.04 \times 10^5 \text{ Pa})(1.2 \times 10^{-3} \text{ m}^3) \ln \left( \frac{1.2}{9.9} \right) \]

\[ = 24 \text{ J} \]
3. A fountain shoots water vertically upward with an initial speed of $v_o$. The radius of the water's exit hole is 0.016 meters. The density of water is $\rho_w$. A pipe of radius 0.032 meters that lies $h$ meters below ground feeds the fountain. Which expression describes the gauge pressure at the pumping station that pumps still water into the pipe?

$$\frac{1}{2} \rho_w (v_o)^2 + gh$$

a) $\frac{1}{2} \rho_w (v_o)^2 / 4 + \rho_w gh$

b) $\frac{1}{2} \rho_w (v_o)^2 + \rho_w gh$

c) $\frac{1}{2} \rho_w (v_o)^2 / 2 + \rho_w gh$

d) $\rho_w gh$

e) $\frac{1}{2} \rho_w (v_o)^2 / 16 + \rho_w gh$

4. Two spheres are completely submerged in water. The smaller sphere has a radius $r$ and experiences a buoyancy force of 5 N. The larger sphere has a radius $R = 4r$. What is the buoyancy force acting on the larger sphere?

$$F_B = \rho_w V_{sp} g \implies F_B \propto V_{sp}$$

a) 10 N

b) 20 N

c) 40 N

d) 80 N

e) 320 N

$$\frac{F_{B \text{ LARGE}}}{F_{B \text{ SMALL}}} = \left(\frac{r_{\text{ LARGE}}}{r_{\text{ SMALL}}}\right)^3 = 64 \implies F_{B \text{ LARGE}} = 64 \times 5 N = 320 N$$

5. A 100 g piece of hot charcoal (220 °C; $c_c=1.0 \text{ J/kg}\cdot\text{K}$) is dropped into 1 kg of water that is at 40 °C ($c_w=4.2 \text{ J/kg}\cdot\text{K}$). When the charcoal and the water come to equilibrium the final temperature will be:

$$M_{CC}c_c \Delta T_c = (M_w c_w) \Delta T_w$$

a) Between 40 °C and 70 °C

b) Between 71 °C and 100 °C

c) Between 101 °C and 130 °C

d) Between 131 °C and 160 °C

e) Between 161 °C and 220 °C

So for every 1 degree of temperature change of the water the charcoal will cool by 42 °C. The water can only warm then by < 5 °C
6. A gas is initially in state $a$ with a pressure of 100,000 Pa and a volume of 0.001 m$^3$. The gas goes through the cycle $adcba$ shown on the $PV$-diagram, where $P = 300,000$ Pa, $V = 0.003$ m$^3$. What is the value of $Q$ for the entire cycle?

a) 800 J 

b) 100 J 

c) 600 J 

d) 400 J 

e) 900 J

\[ \Delta U_{\text{cycle}} = 0 \]

\[ Q = W \]

\[ Q_{\text{cycle}} = W_{\text{cycle}} \]

\[ W_{\text{cycle}} = \text{AREA ENCLOSED} = \Delta P \Delta V \]

\[ = (2 \times 10^{-3} \text{ m}^3) \times (2 \times 10^5 \text{ Pa}) = 400 \text{ J} \]

7. A real heat engine takes in 1000 J at a 600K hot reservoir and does 250 J of work. Which of the following is true?

a) The temperature of the cold reservoir must be 450 K.

b) The temperature of the cold reservoir could be lower than 400 K.

c) The temperature of the cold reservoir could be higher than 500 K.

d) The temperature of the cold reservoir must be lower than 275 K.

e) There is not enough information to determine whether any of these statements is true.

\[ \varepsilon_{\text{real}} = \frac{250 \text{ J}}{1000 \text{ J}} = 25\% \]

\[ 25\% \leq 1 - \frac{T_c}{T_H} \Rightarrow \frac{T_c}{T_H} \leq 75\% \]

8. During one cycle, a heat engine exhausts 130 J of thermal energy for every 200 J of thermal energy it absorbs. What is the efficiency of the engine?

a) 45% 

b) 55% 

c) 35% 

d) 40% 

e) 65%

\[ Q_H = 200 \text{ J} \]

\[ Q_c = 130 \text{ J} \]

\[ W = 70 \text{ J} \]

\[ \frac{W}{Q_H} = \frac{70}{200} = 35\% \]
Use $g = 10 \text{ m/s}^2$ throughout the exam (for simplicity)

1. A diver 90 meters below the surface of a lake ($\rho_{\text{water}} = 1000 \text{ kg/m}^3$) releases a bubble of air ($\rho_{\text{air}} = 1 \text{ kg/m}^3$) that has a volume of 5 cm$^3$. The bubble rises to the surface through the water, which has the same temperature throughout.

(a) What is the absolute pressure in atmospheres at the diver's depth (1 atm $= 10^5$ Pa)?

\[ P_0 = 1 \text{ atm} \]
\[ P_{\text{abs}} = P_0 + \rho g h \]
\[ \rho g h = \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \left(10 \frac{\text{m}}{\text{s}^2}\right) (90 \text{ m}) \]
\[ = 9 \times 10^5 \text{ Pa} = 9 \text{ atm} \]

(b) What is the volume of the bubble (in cm$^3$) when it reaches the surface (where the pressure is one atm)?

Since $T$ is constant

\[ P_1 V_1 = P_2 V_2 \Rightarrow V_2 = V_1 \left(\frac{P_1}{P_2}\right) \]
\[ = (5 \text{ cm}^3) \left(\frac{10 \text{ atm}}{1 \text{ atm}}\right) = 50 \text{ cm}^3 \]

(c) Find the magnitude and direction of the net force acting on the diver at a depth of 90 m if he and his diving gear have a combined mass of 100 kg and a volume of 0.097 m$^3$.

\[ \uparrow \quad F_B \quad \text{W} = mg = 1000 \text{ N} \]
\[ \downarrow \quad \text{W} \quad F_B = (\rho_{\text{water}} V_0) g = 970 \text{ N} \]
\[ \sum F = 970 \text{ N} - 1000 \text{ N} = -30 \text{ N} \]
\[ F_{\text{net}} = 30 \text{ N} \quad \text{pointing downward} \]
A cylinder has a base area of \( A = 0.01 \text{ m}^2 \) and is fitted with a piston that can freely move. Below the piston, the cylinder is filled with an ideal gas that has a pressure \( P_1 = 104,000 \text{ Pa} \), a volume \( V_1 = 0.0012 \text{ m}^3 \), and a temperature of \( T_1 = 0 ^\circ \text{C} \) when in state 1. The gas is then taken through the following four-step process:

- An \( m = 42 \text{ kg} \) metal block is placed on top of the piston, compressing the gas to state 2, with the gas still at \( T_2 = 0 ^\circ \text{C} \).
- The cylinder is then put in contact with a boiling water bath, raising the gas temperature to \( T_3 = 100 ^\circ \text{C} \) in state 3.
- The metal block is then removed and the gas expands to state 4, still at \( T_4 = 100 ^\circ \text{C} \).
- Finally, the cylinder is placed in contact with an ice-water bath, bringing the gas back to \( T_1 = 0 ^\circ \text{C} \) in state 1.

a) For each process, indicate whether the process is isothermal, isobaric, or adiabatic, and explain your reasoning.

(i) The process from state 2 to state 3: circle one isothermal isobaric adiabatic

Explain: **PRESSURE DOESN'T CHANGE**

(ii) The process from state 4 to state 1: circle one isothermal isobaric adiabatic

Explain: **PRESSURE DOESN'T CHANGE**

(iii) The process from state 1 to state 2: circle one isothermal isobaric adiabatic

Explain: **TEMP IS CONSTANT**

(iv) The process from state 3 to state 4: circle one isothermal isobaric adiabatic

Explain: **TEMP IS CONSTANT**

b) Determine the pressure and the volume of the gas in state 2.

\[
\begin{align*}
1 & \rightarrow 2 \hspace{1cm} \text{ISOTHERMAL} \hspace{0.5cm} \text{SO} \hspace{0.5cm} P_1 V_1 &= P_2 V_2 \\
\implies P_2 &= P_1 + \left(\frac{42 \text{ kg}}{0.01 \text{ m}^2}\right) = 146,000 \text{ Pa} \\
&= 1.46 \times 10^5 \text{ Pa} \\
V_2 &= \frac{P_1 V_1}{P_2} = \left(\frac{1.04 \times 10^5}{1.46}\right)(0.0012) = 8.5 \times 10^{-4} \text{ m}^3
\end{align*}
\]

\[ p_2 = 1.46 \times 10^5 \text{ Pa} \]
\[ V_2 = 8.5 \times 10^{-4} \text{ m}^3 \]

(c) How much work was done on the gas going from state 1 to state 2.

\[ W = \int_{V_1}^{V_2} P \, dV = P_1 \ln \left( \frac{V_2}{V_1} \right) = (1.04 \times 10^5)(1.2 \times 10^{-3} \text{ m}^3) \ln \left( \frac{1.04 \times 10^5}{0.85} \right) \]

\[ = 43 \text{ J} \]
3. A fountain shoots water vertically upward with an initial speed of $v_0$. The radius of the water's exit hole is 0.016 meters. The density of water is $\rho_w$. A pipe of radius 0.032 meters that lies $h$ meters below ground feeds the fountain. Which expression describes the gauge pressure at the pumping station that pumps still water into the pipe?

   a) $\frac{1}{2} \rho_w (v_0)^2 / 16 + \rho_w gh$
   b) $\frac{1}{2} \rho_w (v_0)^2 / 4 + \rho_w gh$
   c) $\frac{1}{2} \rho_w (v_0)^2 + \rho_w gh$
   d) $\frac{1}{2} \rho_w (v_0)^2 / 2 + \rho_w gh$
   e) $\rho_w gh$

$$P_{\text{gauge}} = \frac{\Delta U_{\text{vol}}}{\rho_w} + \Delta P_{\text{vol}}$$

$$V_{\text{in}} A_{\text{in}} = V_{\text{out}} A_{\text{out}} \Rightarrow V_{\text{in}} = \left(\frac{r_{\text{in}}}{r_{\text{out}}^2}\right) V_{\text{out}} = \left(\frac{1}{4}\right) V_{0}$$

4. Two spheres are completely submerged in water. The smaller sphere has a radius $r$ and experiences a buoyancy force of 5 N. The larger sphere has a radius $R = 2r$. What is the buoyancy force acting on the larger sphere?

   a) 10 N
   b) 20 N
   c) 40 N
   d) 80 N
   e) 320 N

$$F_B = \rho_w V_{\text{sub}} g \quad \text{so} \quad V_{\text{sub}}$$

$$\frac{F_B_{\text{large}}}{F_B_{\text{small}}} = \left(\frac{r_{\text{large}}}{r_{\text{small}}}\right)^3 = 8$$

$$F_{B_{\text{large}}} = 8 F_{B_{\text{small}}} = 40 N$$

5. A 100 g piece of hot charcoal (220 °C; $c_c = 1.0 \text{ J/kg} \cdot \text{K}$) is dropped into 1 kg of water that is at 40 °C ($c_w = 4.2 \text{ J/kg} \cdot \text{K}$). When the charcoal and the water come to equilibrium the final temperature will be:

   a) Between 161 °C and 220 °C
   b) Between 131 °C and 160 °C
   c) Between 101 °C and 130 °C
   d) Between 71 °C and 100 °C
   e) Between 40 °C and 70 °C

$$(M_c C_c) \Delta T_c = (M_w C_w) \Delta T_w$$

$$\Delta T_c = \frac{(42)}{2} \Delta T_w$$

$$\Rightarrow \Delta T_c = 42 \Delta T_w$$

So for every 1 degree of temp. change of the water, the charcoal will cool by 42 °C. The water will warm by < 5 °C.
6. A gas is initially in state $a$ with a pressure of 100,000 Pa and a volume of 0.001 m$^3$. The gas goes through the cycle $adcbad$ shown on the $PV$-diagram, where $P = 300,000$ Pa and $V = 0.003$ m$^3$. What is the value of $Q$ for the entire cycle?

- a) 900 J
- b) 400 J
- c) 100 J
- d) 600 J
- e) 800 J

\[ \Delta U = Q - W \]
\[ \Delta U_{\text{cycle}} = 0 \quad \Rightarrow \quad Q = W \]
\[ W = \text{AREA ENCLOSED} = \Delta P \Delta V \]
\[ W = (2 \times 10^5 \text{ Pa})(2 \times 10^{-3} \text{ m}^3) = 400 \text{ J} \]

7. A real heat engine takes in 1000 J at a 600K hot reservoir and does 250 J of work. Which of the following is true?

- a) The temperature of the cold reservoir must be lower than 275 K.
- b) The temperature of the cold reservoir must be 450 K.
- c) The temperature of the cold reservoir could be lower than 400 K.
- d) The temperature of the cold reservoir could be higher than 500 K.
- e) There is not enough information to determine whether any of these statements is true.

\[ Q_H = 1000 \text{ J} \quad W = 250 \text{ J} \quad Q_C = 750 \text{ J} \quad \varepsilon_{\text{REAL}} = 25\% \]
\[ T_H = 600 \text{ K} \quad \varepsilon_{\text{REAL}} \leq \varepsilon_{\text{CARACT}} = 1 - \frac{T_C}{T_H} \geq 25\% \quad \Rightarrow \quad \frac{T_C}{T_H} \leq 75\% \]

8. During one cycle, a heat engine exhausts 110 J of thermal energy for every 200 J of thermal energy it absorbs. What is the efficiency of the engine?

- a) 45%  
- b) 55%  
- c) 35%  
- d) 40%  
- e) 65%  

\[ Q_H = 200 \text{ J} \]
\[ Q_C = 110 \text{ J} \]
\[ W = 90 \text{ J} \]
\[ \varepsilon_{\text{REAL}} = \frac{W}{Q_H} = \frac{90 \text{ J}}{200 \text{ J}} = 45\% \]