Fluids

Physics 116 2017
Tues. 2/21 and Thurs. 2/23
Air pressure

- Air surrounds us and has mass
- Due to its mass, it experiences a gravitational pull from the earth.
- The weight of the air above us creates a pressure at the surface of the earth.
- The value of air pressure at the surface of the earth is \( \sim 1 \text{ atm} = 10^5 \text{ Pa} \)
- Do we feel the air pressure weigh us down, or does it squeeze us from all directions? Think about swimming!
Observational Experiments:

Thinking before doing
How does the pressure of a fluid depend on
• the depth of the fluid – what do you expect?
• the density (mass per unit volume) of the fluid?
• the value of g?
• the pressure above the fluid?

How does the pressure exerted on the top of a submerged object compare to the pressure exerted on the bottom?
Follow-up Pressure Question

**Question:** Does fluid pressure at a given depth depend on the volume of fluid above it?

- Experimental apparatus and design – “Pascal’s Vase” – pour liquid into vase, observe resulting water heights.

- If yes, then what should happen?
- If no, then what should happen?

- How can you store this information? Reasoning by analogy - think about your own underwater experiences when swimming
Fluid Statics - Summary

• Density is the *inertial quantity* associated with fluids. It is an index that describes how much mass there is in each unit of volume of the fluid. We assume that this quantity is a constant in our mathematical descriptions that follow.

• The pressure of a static fluid increases linearly with depth. Let the origin of $y$ be at the surface of the fluid, and the positive direction be down: $P(y) = P_o + \rho g y$

• The difference in pressure on the top and bottom of an object submerged in a static fluid results in a buoyant force, which is a net force of the fluid on the object, and always points upward.

• The force that the static fluid exerts on the object is equal to the weight of the water that has been displaced by the object.

$$F_{\text{buoyancy}} = \rho_{\text{fluid}} V_{\text{submerged}} g \quad (\text{Archimedes principle})$$
Visualizing Archimedes Principle

• The claim to we want to test: “the force that the static fluid exerts on the object is equal to the weight of the water that has been displaced by the object.”

• “Weight of the water...” = $m_{\text{water}}g$

• But how can we rewrite that to match the phrase?

• $m_{\text{water}}g = \rho_{\text{fluid}}V_{\text{submerged}}g$, where we’ve written mass of the water as density of water times volume of water displaced ($V$ of object submerged).

\[ F_{\text{buoyancy}} = \rho_{\text{fluid}}V_{\text{submerged}}g \]  (Archimedes principle)

• Let’s observe, hypothesize, predict, and experiment!
Density of object > density of fluid

Buoyant force = the weight of the water displaced

\[ T = W - F_B \]
Density of object < density of fluid

- Floating: 
  - Water displaced

- Buoyant force equals weight

Density of object = density of fluid

- Floating: fully submerged
  - Water displaced
  - Some fish can remain at a fixed depth without moving by storing gas in their bladder.

- Submarines take on or discharge water into their ballast tanks.
Aluminum and steel

The cylinders have the same mass, but the aluminum cylinder is larger than the steel cylinder. Let $T_S$ and $T_A$ be the tensions in the strings attached to the steel and aluminum cylinders respectively. Also let $B_S$ and $B_A$ be the buoyant forces on the steel and aluminum cylinders.

- **a)** $T_A < T_S$ and $B_A < B_S$.
- **b)** $T_A < T_S$ and $B_A > B_S$.
- **c)** $T_A < T_S$ and $B_A = B_S$.
- **d)** $T_A > T_S$ and $B_A < B_S$.
- **e)** $T_A > T_S$ and $B_A > B_S$.
- **f)** $T_A > T_S$ and $B_A = B_S$.

**Guided Problem**

**Aluminum and steel**

(1) Limiting or extreme cases – how would tensions compare if there was no water?

(2) Now we submerge them in water. We represent the force exerted by the water on an object as $F_{\text{buoyancy}} = \rho_{\text{fluid}} V_{\text{submerged}} g$

(3) How can we use this force and what we know about the net force on the objects to compare $T_S$ and $T_A$? **Draw a force diagram!**
Guided Problem

**Immersed cubes**

Two cubes with identical shapes and volumes but different masses are immersed in oil. Both cubes are completely below the surface of the oil, but the lighter cube is farther below below the surface. Which is true?

a) The buoyant force on the 3-kg cube is triple the buoyant force on the 1-kg cube. The tensions are the same.

b) The buoyant force is greater for the 1-kg cube because it is deeper in the water. The tension force is less on the 1-kg cube than on the 3-kg cube.

c) We cannot compare the tensions unless we know the depths of the two cubes.

d) The buoyant force is the same for the two cubes. The tension is greater on the 3-kg mass than on the 1-kg mass.
You observe two cubes floating at rest on the surface of a fluid as shown. What can you reason about the properties of these two cubes?

a) Cube $A$ must be less dense than cube $B$.
b) Cube $A$ must have less mass than cube $B$.
c) Cube $A$ must have less volume than cube $B$.
d) The net force on cube $A$ is less than the net force on cube $B$. 
**Guided Problem**

**Floating cubes**

Two cubes are floating at rest on the surface of a fluid as shown. Which of the following statements is true?

a) Cube A must be less dense than cube B.

b) Cube A must have less mass than cube B.

c) Cube A must have less volume than cube B.

d) The net force on cube A is less than the net force on cube B.

\[ m_{\text{block}} g = \rho_{\text{fluid}} V_{\text{sub}} g \]

\[ \rho_{\text{block}} V_{\text{block}} g = \rho_{\text{fluid}} V_{\text{sub}} g \]

\[ \rho_{\text{block}} = \rho_{\text{fluid}} \frac{V_{\text{sub}}}{V_{\text{block}}} \]

- We don’t know anything about \( V_{\text{sub}} \), what if A is a shipping crate and B is a box of tissues? So we can’t say anything about mass.

- We don’t know their absolute volumes, we just know that B has a higher fraction of its volume submerged than A.

\[ \frac{V_{B,\text{sub}}}{V_B} > \frac{V_{A,\text{sub}}}{V_A} \Rightarrow \rho_B > \rho_A \]
Three cubes, each one 10 centimeters on a side, are submerged in a fluid. Cubes A and B are made of steel \((\rho = 7 \text{ g/cm}^3)\) and cube C is made of aluminum \((\rho = 2.7 \text{ g/cm}^3)\). Cubes A and C are at the same depth. Which cube has the greatest buoyant force acting on it?

(a) Cube A.
(b) Cube B.
(c) Cube C.
(d) A & B are equal and are greater than C.
(e) All buoyant forces are the same.
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(a) Cube A.
(b) Cube B.
(c) Cube C.
(d) A & B are equal and are greater than C.
(e) All buoyant forces are the same.
FLUID DYNAMICS

Consider an IDEAL FLUID

Fluid motion is very complicated. However, by making some assumptions, we can develop a useful model of fluid behavior.

An ideal fluid is:

**Incompressible** – the density is constant
**Irrotational** – the flow is smooth, no turbulence
**Nonviscous** – fluid has no internal friction
**Steady flow** – the velocity of the fluid at each point is constant in time.
EQUATION OF CONTINUITY

(conservation of mass)

mass flowing into a pipe = mass flowing out of a pipe

\[ m_1 = m_2 \]
\[ \rho V_1 = \rho V_2 \]

Since \( V = A \Delta x \) and \( v = \Delta x / \Delta t \) then \( V = A v \Delta t \)

(\text{where} \ V \text{ represents volume and} \ v \text{ represents speed})

\[ \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t \]

\( A_1 v_1 = A_2 v_2 \) = volume flow rate = Volume/unit of time = constant
Conservation of energy – Bernoulli’s Equation

Consider a pressurized fluid that flows through an infinite pipe. The fluid has kinetic energy associated with it because it is moving with respect to the pipe. In order to get the fluid moving from rest, there had to have been a pressure difference. And if the pipe increases/decreases elevation, then there will also be a change in the gravitational potential energy of the fluid.
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Assuming: uniform density, no internal friction or turbulence – then the total mechanical energy density of the system is constant.

Describe a situation that is represented by each bar chart

\[
P_1 + KE_1 + U_{g1} = P_2 + KE_2 + U_{g2}
\]

\[
P_1 + KE_1 + U_{g1} = P_2 + KE_2 + U_{g2}
\]
Change in kinetic energy
\[ \Delta K = \frac{1}{2}m v_2^2 - \frac{1}{2}m v_1^2 = \frac{1}{2}(\rho V) v_2^2 - \frac{1}{2}(\rho V) v_1^2 \]

Change in potential energy
\[ \Delta U = m g y_2 - m g y_1 = (\rho V) g y_2 - (\rho V) g y_1 \]

Work done
(Recall that work=force*displacement and force = pressure * area)
\[ W_{\text{net}} = F_1 \Delta x_1 - F_2 \Delta x_2 = (P_1 A_1) \Delta x_1 - (P_2 A_2) \Delta x_2 = P_1 V - P_2 V \]

Work done = change in kinetic + potential energy
\[ W_{\text{net}} = P_1 V - P_2 V = \Delta K + \Delta U \]
\[ P_1 V - P_2 V = \left\{ \frac{1}{2}(\rho V) v_2^2 - \frac{1}{2}(\rho V) v_1^2 \right\} + \left\{ (\rho V) g y_2 - (\rho V) g y_1 \right\} \]

Dividing both sides by \( V \) we define “energy density”, or the energy per unit of volume, which represents the effect of a pressure difference on the total energy density in the system for a fluid with a uniform density
\[ P_1 - P_2 = \left\{ \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \right\} + \{\rho g y_2 - \rho g y_1\} \]

called “Bernoulli’s Equation”

How does this compare with conservation of energy (energy density) equation?
**Venturi Effect**

\[ P_1 - P_2 = \{ \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 \} + \{ \rho g y_2 - \rho g y_1 \} \]

Start with Bernoulli’s Eq and look at two points in a flowing fluid that are at the same elevation, \( y_1 = y_2 \)

Since the location 1 has a wider diameter than the location 2, then by the continuity equation \( A_1 v_1 = A_2 v_2 \), \( v_1 < v_2 \). This means that the pressure at 2 has to be SMALLER than the pressure at 1, and we get the counter-intuitive result that in regions where a fluid flows faster, the pressure is LOWER. This is known as the **Venturi Effect**, and is a foundational idea in air flight.
Below is a cross section of an airplane wing. The air moves faster across the top of the wing. **Lift** on an airplane wing is due to the different air speeds (and therefore pressures) on the upper and lower surfaces of the wing.

https://www.youtube.com/watch?v=UqBmdZ-BNig
In a storm how does a house lose its roof?

Air flow is disturbed by the house. The "streamlines" crowd around the top of the roof.

⇒ faster flow above house

⇒ reduced pressure above roof to that inside the house

⇒ roof lifted off because of pressure difference.
Why do rabbits not suffocate in the burrows?

Air must circulate. The burrows must have two entrances. Air flows across the two holes is usually slightly different ⇒ slight pressure difference ⇒ forces flow of air through burrow. One hole is usually higher than the other and the small mound is built around the holes to increase the pressure difference.
Example problem: Water Tower

A water tower is made up of a reservoir and scaffolding (height $H$). The reservoir is filled to the top (a height $h$ from its bottom). At the bottom of the reservoir is a valve that can be opened. Assume that it is raining, and the reservoir is kept full while the valve is open.

a) PREDICT the speed of the water as it leaves the reservoir.

b) PREDICT where the water will land when it hits the ground.

c) Attach a hose to the reservoir valve and shoot the water straight up from the ground level
   • Use energy considerations to PREDICT how high the water will shoot
   • Use Bernoulli’s equation to validate your prediction.
   • Test your prediction.
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a) PREDICT the speed of the water as it leaves the reservoir.

\[ P_1 + \rho gh = P_2 + \frac{1}{2} \rho v_f^2 \]

then, since $P_1 = P_2 = P_{\text{atm}}$,

\[ \rho gh = \frac{1}{2} \rho v_f^2 \quad \Rightarrow \quad v_f = (2gh)^{\frac{1}{2}} \]

Test your prediction!

P1 at top of water is open to air, so $P_1 = P_{\text{atm}}$.
P2 is the pressure of the water at the bottom opening of the basin, which is also open to air, so $P_2 = P_{\text{atm}}$
Water basin starts at 15m high and fills to about 24.5m high, so $h = 9.5m$
Then $v = (2\times9.8\times9.5)^{\frac{1}{2}} = 13.6m/s$
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\[
\rho gh = \frac{1}{2} \rho v_f^2 \implies v_f = (2gh)^{\frac{1}{2}}
\]

From projectile motion:

\[
\Delta x = v_f t \quad \text{and} \quad H = \frac{1}{2} gt^2 \implies \Delta x = v_f \left( \frac{2H}{g} \right)^{\frac{1}{2}}
\]

Test your prediction!

Height of bottom of basin from ground is 15m, and the exit speed from the basin is 13.6m/s

\[
\Delta x = v_f \left( \frac{2H}{g} \right)^{\frac{1}{2}} = 13.6 \times \left( \frac{2 \times 15}{9.8} \right)^{\frac{1}{2}} = 24 \text{m}
\]
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Choose state 1 as water on top of reservoir, state 2 as water reaching highest point after shot out of hose:

\[ P_1 - P_2 = \{ \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 \} + \{ \rho g y_2 - \rho g y_1 \} \]

\[ P_1 = P_2 = \text{atmospheric pressure} \]
\[ v_1 = v_2 = 0 \]

So:

\[ P_1 - P_2 = \{ \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 \} + \{ \rho g y_2 - \rho g y_1 \} = 0 \]
\[ \rho g y_2 = \rho g y_1 \Rightarrow y_2 = y_1 \]

it should shoot up to the same height as the water level.
Test your prediction!