Physics 116

Lecture: Rotational Dynamics
2017
Tues. 1/31 and Thurs 2/2
Recap

\[ x \leftrightarrow \theta \quad v_x \leftrightarrow \omega \quad a_x \leftrightarrow \alpha \quad m \leftrightarrow I \]
\[ t \leftrightarrow t \quad K \leftrightarrow K \quad \vec{F} \leftrightarrow \vec{\tau} \]

**TRANSLATION**

\[ v_x = v_{x0} + a_x t \]
\[ x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \]
\[ v_x^2 = v_{x0}^2 + 2a_x \Delta x \]
\[ K = \frac{1}{2} m v^2 \]
\[ \sum \vec{F} = m \vec{a} \]

**ROTATION**

\[ \omega = \omega_0 + \alpha t \]
\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \]
\[ \omega^2 = \omega_0^2 + 2 \alpha \Delta \theta \]
\[ K = \frac{1}{2} I \omega^2 \]
\[ \sum \vec{\tau} = I \vec{\alpha} \]
Torque is a VECTOR PRODUCT

\[ \vec{\tau} = \vec{r} \times \vec{F} \]
\[ \tau = rF\sin\theta \]

RHR:
• Counterclockwise rotation is considered OUT OF THE PAGE ⬤
• Clockwise rotation is considered INTO THE PAGE. ⬤

*DEMO TIME (Torque stick!)*
Newton’s 2\textsuperscript{nd} Law

Recall that linear acceleration is caused by unbalanced external forces, and is inversely proportional to the object’s mass:

$$a = \frac{F_{\text{net}}}{m}$$

Analogously, angular acceleration is caused by unbalanced external torques and is inversely proportional to the object’s moment of inertia:

$$\alpha = \frac{\tau_{\text{net}}}{I}$$
Newton’s 2\textsuperscript{nd} Law

Angular acceleration is caused by \textit{unbalanced} external torques and is inversely proportional to the object’s moment of inertia:

\[ \alpha = \frac{\tau_{\text{net}}}{I} \]

\textit{DEMO TIME (Torque on Meter Stick – are there torques? A net torque?)}

\textit{DEMO TIME (Inertia Rods – if I supply (roughly) equal torques, how does angular acceleration relate to moment of inertia?)}
A disk is free to rotate about an axis. A force applied at a distance d from the axis causes an angular acceleration $\alpha$. What angular acceleration is produced if the same force is applied at a distance 2d from the axis?

1. $\alpha$
2. $2\alpha$
3. $\alpha/2$
4. $4\alpha$
5. $\alpha/4$
A wheel of radius $R_1$ has an axle of radius $R_2 = \frac{1}{4}R_1$. If a force $F_1$ is applied tangent to the wheel, a force $F_2$, applied tangent to the axle that will keep the wheel from turning, is equal to

1. $F_1/4$
2. $F_1$
3. $4F_1$
4. $16F_1$
5. $F_1/16$
To start the playground merry-go-round rotating (radius = 2m), a rope is wrapped around it and pulled. A force of 200 N is exerted on the rope for 10 seconds. During this time the merry-go-round makes one complete revolution.

1. Find the torque exerted by the rope on the merry-go-round.

2. Find the angular speed of the merry-go-round after 10 seconds.

3. For the moment of inertia of the merry-go-round.

Let’s work through this together
In each figure below, the jet engine is slowing down due to the application of a constant torque. All of the engines are identical, but they start with different angular speeds and have torques of different magnitudes applied to the rotating shafts within the engines. Magnitudes of the initial angular speeds and torques are given in the figures.

Rank these situations on the basis of the magnitude of the angular acceleration of the engines as they slow down, from GREATEST to LEAST.

1. \( B > D > E > F > A > C \)
2. \( C > A > F > E > D > B \)
3. \( C = D = F > A = B = E \)
4. \( B = D > E = F > A = C \)
5. \( A = B = C = D = E = F \)
A weight is tied to a rope that is wrapped around a pulley. The pulley is initially rotating counterclockwise and is pulling the weight up. The tension in the rope creates a torque on the pulley that opposes this rotation. Consider the following statements which MIGHT be true at the instant the pulley stops rotating counterclockwise before starting to rotate clockwise:

A. The torque on the pulley about its axis is equal to zero.
B. The angular acceleration of the pulley is equal to zero.
C. The angular momentum of the pulley about the axis is equal to zero.

Which of the following is(are) TRUE?

1. A only
2. B only
3. C only
4. A and B
5. A, B, and C

**DEMO TIME (Flywheels and energy storage: how can those possibly be related?)**
Angular momentum

Recall that **linear momentum** is the fundamental quantity of linear motion. It is the product of the mass and the velocity:

\[
\text{Linear momentum} = m \mathbf{v}
\]

Analogously, **angular momentum** is the fundamental quantity of rotational motion. It is the product of the moment of inertia and the angular velocity:

\[
\text{Angular momentum} \quad \mathbf{L} = I \mathbf{\omega}
\]
Direction of angular momentum

For an object spinning around an axis of symmetry, the angular momentum vector and the angular velocity vector are parallel.
Four identical small cylinders rest on a circular horizontal turntable at the various positions shown in the top-view diagram below. The turntable is rotating clockwise at a constant angular speed.

**Rank the angular momentum of the cylinders about the axis of rotation of the turntable.**

1. \( R > P = S = T \)
2. \( P = S = T > R \)
3. \( P = R = S = T \)
4. \( P = R = S = T = 0 \)
5. Can not be determined without unknown angular velocity
A bicycle wheel has a diameter of 64.0 cm and a moment of inertia of .5 kg m$^2$. The bicycle is placed spinning with a positive angular velocity on a stationary stand, and a resistive braking force of 60 N is applied to the rim of the tire. What is the angular acceleration of the wheel?

1. 38 rad/s$^2$
2. 19 rad/s$^2$
3. -38 rad/s$^2$
4. -19 rad/s$^2$
5. none of the above
Conservation of angular momentum

Recall that **linear momentum** is conserved if there are no unbalanced external forces acting on the system.

If \( \Sigma F_{\text{net}} = 0 \), then the total momentum of the system is constant.

\[ \Sigma mv \text{ is constant} \]

Analogously, **angular momentum** is also conserved if there are no unbalanced external torques acting on the system.

If \( \Sigma \tau_{\text{net}} = 0 \), then the total momentum of the system is constant.

\[ L = \Sigma I\omega \text{ is constant} \]
Rotational collisions

Recall that **linear momentum** is conserved in all collisions, while kinetic energy is conserved only in perfectly elastic collisions.

\[
\Sigma Mv_{\text{before}} = \Sigma m v_{\text{after}}
\]

But \( K_i \neq K_f \) unless the collision is perfectly elastic.

Analogously, **angular momentum** is also conserved in all collisions, while kinetic energy is conserved only in perfectly elastic ones. For 2-d objects that rotate about an axis of symmetry:

\[
\Sigma I \omega_{\text{before}} = \Sigma I \omega_{\text{after}}
\]
A bullet of mass $m$ is shot at a hinged rod and hits the rod a distance $d$ from the hinge. The rod was initially at rest and has a moment of inertia of $I_0$ about an axis through its hinge. The bullet is fired at an angle $\theta$ to the rod, as shown in the top view, with an initial velocity of $v$ at a distance $d$ from the hinge. The angular speed of the rod with the bullet embedded right after the collision is $\omega_f$. Assume there is no friction in the hinge and that the rod is free to rotate about the hinge axis. Consider the following two statements:

- **A.** The angular momentum about the hinge of the rod and bullet together after the bullet is embedded is the same as the initial angular momentum of the bullet about the hinge.
- **B.** The total kinetic energy of the rod and bullet after the bullet is embedded is the same as the initial kinetic energy of the bullet.

1. A only
2. B only
3. both A and B
4. There is not enough information to determine.
A disk, for which the moment of inertia is $I_1$, rotates about a vertical, frictionless axle with angular velocity $\omega_0$. A second disk, which has moment of inertia $I_2$ and is not rotating, drops onto the first disk, as illustrated below. The disks stick together, which results in their both spinning with an angular velocity $\omega_f$.

1. What objects go into your system?

2. Is the linear momentum of the system conserved? Why or why not?

3. Is the angular momentum of the system conserved? Why or why not?

4. Calculate $\omega_f$

5. Is the change in the kinetic energy of the system greater than, less than or equal to zero?

6. If the change in kinetic energy is not zero, where did the energy come from (or go to)?