Internal energy, Temperature, and Heating
A real gas consists of a vast number of molecules, each moving randomly and undergoing millions of collisions every second.

Despite the apparent chaos, averages, such as the average number of molecules in the speed range 600 to 700 m/s, have precise, predictable values.

The “micro/macro” connection is built on the idea that the macroscopic properties of a system, such as temperature or pressure, are related to the average behavior of the atoms and molecules.
Creating a mathematical model: *How do the macroscopic quantities of pressure and temperature depend on the kinetic energies of the particles?*

The pressure arises from the particles hitting the wall and transferring momentum.

Characterize gas by its density

How many particles hit a wall in a time $\Delta t$?

Recall that force is the rate of change of the momentum:

$F = \text{total momentum change} / \Delta t$

And Pressure = Force/Area so

Now relax assumptions that all particles have same speed and move in the + direction:

$v_x^2 \rightarrow \frac{1}{2} v_{\text{average}(x)}^2$ (two directions)

Now relax assumptions that all particles move in the $x$-direction:

$v_{\text{average}(x)}^2 \rightarrow \frac{1}{3} v_{\text{rms}}^2$ (three dimensions)

Where $v_{\text{rms}}^2 = v_{\text{average}(x)}^2 + v_{\text{average}(y)}^2 + v_{\text{average}(z)}^2$

- Momentum change for each hit = $2mv_x$
- Particle density = $N/V$
- #hits in a time interval $\Delta t = (N/V)(v_x*\Delta t)$ (Area of wall)
- Total momentum change in a time interval = $(2mv_x)(N/V)(v_x*\Delta t)(\text{wall Area})$
- Pressure $= (2mv_x)^2 * (N/V)$
- Pressure $= \frac{1}{2}mv_{\text{rms}}^2 * (N/V)$
- Pressure $= \frac{4}{3}mv_{\text{rms}}^2 * (N/V)$
- Pressure $= \frac{2}{3}(KE_{\text{ave}}) * (N/V)$

From the ideal gas law: $PV = NkT$

$T = \frac{2}{3}k_B (KE_{\text{ave}})$
Macro-micro connection

- Assumptions for ideal gas:
  - # of molecules $N$ is large
  - They obey Newton’s laws
  - Short-range interactions with elastic collisions
  - Elastic collisions with walls (an impulse.....pressure)

- What we call pressure $P$ is a direct measure of the number density of molecules, and how fast they are moving ($v_{\text{rms}}$)

- What we call temperature $T$ is a direct measure of the average translational kinetic energy

\[
P = \frac{2}{3} \frac{N}{V} (KE)_{\text{avg/particle}}
\]

\[
T = \frac{2}{3k_B} (KE)_{\text{avg/particle}}
\]

\[
v_{\text{rms}} = \sqrt{\left(\frac{v^2}{m}\right)_{\text{avg}}} = \sqrt{\frac{3k_B T}{m}}
\]

\[
e_{\text{avg}} = \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} k_B T.
\]
The distribution of molecular speeds in a sample of N₂ gas

Most likely speed:
16% of the molecules have speeds between 600 m/s and 700 m/s.

N₂ molecules at 20°C

<table>
<thead>
<tr>
<th>Speed range (m/s)</th>
<th>Percentage of molecules</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 100</td>
<td>1</td>
</tr>
<tr>
<td>100 – 200</td>
<td>4</td>
</tr>
<tr>
<td>200 – 300</td>
<td>5</td>
</tr>
<tr>
<td>300 – 400</td>
<td>15</td>
</tr>
<tr>
<td>400 – 500</td>
<td>20</td>
</tr>
<tr>
<td>500 – 600</td>
<td>20</td>
</tr>
<tr>
<td>600 – 700</td>
<td>15</td>
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<tr>
<td>700 – 800</td>
<td>10</td>
</tr>
<tr>
<td>800 – 900</td>
<td>5</td>
</tr>
<tr>
<td>900 – 1000</td>
<td>4</td>
</tr>
<tr>
<td>1000 – 1200</td>
<td>1</td>
</tr>
</tbody>
</table>
Flux
Catching cigarettes

Marco and Donald are at a celebration sponsored by Phillip Morris, where cigarettes are just falling from the sky. They are discussing the quickest way to fill up their identical empty containers:

Marco: You need to hold the opening of the box horizontally so it fills fastest.

Donald: It doesn't matter if you tilt your box, the cigarettes fall at the same rate either way.

They hold their boxes out, the opening to Marco's box is horizontal, and Donald's is tilted at θ degrees to the horizontal. Whose box fills faster?

A. Donald

B. Marco

C. They fill at the same rate.
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A. Donald  
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Limiting cases

What would be a “limiting case” argument that supports the outcome of this experiment?
How many cigarettes go through each unit of area of the opening in each unit of time?

• It depends on the rate at which the cigarettes fall – higher rate will result in more cigarettes
How many cigarettes go through each unit of area of the opening in each unit of time?

- It depends on the rate at which the cigarettes fall – higher rate will result in more cigarettes
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How many cigarettes go through each unit of area of the opening in each unit of time?

• It depends on the rate at which the cigarettes fall – higher rate will result in more cigarettes
• It depends on the size of the opening – a bigger opening will fill faster
• It depends on the orientation – horizontal will fill fastest and vertical won’t fill at all – how to quantify?
Flux: How many cigarettes go through each unit of area of the opening in each unit of time?
**Flux:** How many cigarettes go through each unit of area of the opening in each unit of time?

<table>
<thead>
<tr>
<th>Number that falls in each time interval</th>
<th>X</th>
<th>Area of plane it falls through</th>
<th>X</th>
<th>Cosine of “tilting” angle</th>
</tr>
</thead>
</table>

Flux: How many cigarettes go through each unit of area of the opening in each unit of time?

The flux characterizes the flow by combining the rate and the crowdedness.
How might you characterize rainfall onto an umbrella using the idea of flux?

Consider Pressure to be a “momentum change” flux as the particles hit a wall

• Total momentum change in a time interval = $(2mv_x) (N/V) (v_x \Delta t) (\text{wall Area })$, so

• Pressure = total momentum change per unit area

• Pressure $= (2mv_x) * (N/V) (v_x)$
  $= (2mv_x^2) * (N/V)$

• Pressure $= \left( \frac{1}{3}mv_{rms}^2 \right) * (N/V)$ (considering two directions and 3 dimensions)
  $= (2/3) \left( \frac{1}{2}mv_{rms}^2 \right) * (N/V)$
  $= (2/3)(KE_{ave}) * (N/V)$

From the ideal gas law: \( PV = NkT \)

• \( T = \frac{2}{3k_B} (KE_{ave}) \) \hspace{1cm} (k_B=1.38 \times 10^{-23} \text{ J/K}) \text{ Boltzmann constant}
Internal energy and the 1\textsuperscript{st} law of thermodynamics

- The average kinetic energy of each particle is:

$$ (KE)_{\text{avg/particle}} = \frac{3}{2} kT $$

- Summing up the energy of the particles is the same as multiplying the average energy per particl times the total number of particles. This total energy is called the internal energy of the system:

$$ U_{\text{internal}} = \frac{3}{2} NkT $$

- Energy can be added to/removed from the system either by doing work on/by the system or heating/cooling the system. This is the first law of thermodynamics,

$$ \Delta U_{\text{internal}} = W + Q $$
Changing the system energy

\[ \Delta U_{\text{int}} = W + Q \]
Heating and Temperature

Q: What happens to matter when it is heated or cooled?

“heating” and “cooling” are processes which add or take away energy from the system, so they increase or decrease the kinetic energy of the particles in an ideal gas at a constant volume. What about other matter, like solids and liquids?

*There is electrical potential energy involved between particles, and so the response to heating is more complicated. Back to the macroscopic world...*
In real matter, it depends...

Heating is a way of adding energy to a system, so it can:

• cause the particles to become more energetic.
• cause them to change their average positions relative to each other, which will change the potential energy of the system and result in a phase change.
Phases (states) of Matter

**Solid**
- Holds Shape
- Fixed Volume

**Liquid**
- Shape of Container
- Free Surface
- Fixed Volume

**Gas**
- Shape of Container
- Volume of Container
Phase Transitions

Heating: Add energy to system

Cooling: Remove energy from system

Solid -> Plasma

Solid -> Gas

Solid -> Liquid

Gas -> Plasma

Gas -> Liquid

Liquid -> Plasma

Recombination

Ionization

Vaporization

Condensation

Freezing

Melting

Deposition

Sublimation
Heating matter

- When matter is heated, its temperature changes OR its phase changes, but they don’t happen simultaneously.

- *For example*: a solid is heated, its temperature changes until it reaches the melting temperature for that substance. Continued heating melts the substance and the temperature stays constant at the melting temperature until the entire solid is melted. Once the matter is in a liquid form entirely then continued heating results in an increase in temperature, until the liquid reaches the vaporization temperature, etc.
Heating Matter

The diagram illustrates the heating process of matter, showing the changes in temperature and the arrangement of molecules as the matter transitions from solid to liquid and then to gas. The x-axis represents time, and the y-axis represents temperature. The diagram includes different states of matter: solid, solid-liquid mixture, liquid, liquid-gas mixture, and gas. The arrangement of the molecules visually demonstrates the changes in density and movement as the matter heats up.
Heating Matter

Diagram showing the relationship between temperature and time with phases of matter (solid, solid/liquid, liquid, liquid/gas, gas) and their corresponding molecular arrangements.

Arrangement of the molecules:
- Solid: Dense arrangement
- Solid/liquid: Dense to less dense
- Liquid: Less dense to more liquid-like
- Liquid/gas: More liquid-like to gas-like
- Gas: Sparse and free-moving
Matter’s *resistance to temperature change* is characterized by an index, called the **specific heat**. The symbol lower case “c” is commonly used and its value gives the energy required for every 1 kg of mass to increase in temperature by 1 K. (Units: J / K•kg)

\[ Q = M c \Delta T \]
The specific heat capacity of aluminum is about \textbf{twice} that of iron. Consider two blocks of equal mass, one made of aluminum and the other one made of iron, initially in thermal equilibrium.

Both blocks are heated at the same constant rate until reaching a temperature of 500 K. Which of the following statements is true?

a) The iron takes less time than the aluminum to reach 500 K

b) The aluminum takes less time than the iron to reach 500 K

c) The two blocks take the same amount of time to reach 500 K
Matter’s *resistance to temperature change* is characterized by an index, called the **specific heat**. The symbol lower case “c” is commonly used and its value gives the energy required for every 1 kg of mass to increase in temperature by 1 K. (Units: J / K•kg)

\[ Q = M c \Delta T \]

When matter changes phase (e.g., between solid and liquid or between liquid and gas) that phase change has energy associated with it. The *phase change index*, or **latent heat of transformation** is the energy required to complete the phase change for every kg of the substance. The symbol upper case “L” is commonly used and its units are (J / kg)

\[ Q = \pm M L \]
40 g of water at 100 °C and 60 g of water at 0 °C are mixed together. When the mixture reaches thermal equilibrium, the final temperature will be:

a) greater than 50 °C.
b) equal to 50 °C.
c) less than 50 °C.
50 g of steam at 100 °C and 50 g of liquid water at 80 °C are mixed together in an insulated container. When the combination reaches thermal equilibrium, the final temperature will be:

a) greater than 90 °C.

b) equal to 90 °C.

c) less than 90 °C.
60 g of ice at 0 °C and 40 g of liquid water at 20 °C are mixed together in an insulated container. When the combination reaches thermal equilibrium, the final temperature will be:

a) greater than 10 °C .
b) equal to 10 °C .
c) less than 10 °C .