FIRST SCIENCE ANSWERS.

Chapter 4.

Guided Review.

2. \[ F = Ma = (2000 \text{ kg})(2.5 \text{ m/s}^2) = 5000\text{ kg m/s}^2 = 5000 \text{ N} = 5 \times 10^3 \text{ N} \]

3. 120 lb = \( (4.445 \text{ N/lb })(120 \text{ lb}) = 533 \text{ N} \)

\[ M_g = 533\text{ N} \text{ so that } M = 533\text{ N}/(9.8\text{ N/kg}) = 54.4 \text{ kg} \]

6. \( F_y = 0 \)

\[ F_{1x} = 60 \cos 45 \quad F_{2x} = 60 \cos 45 \quad F_{3x} = 60 \quad F_1 = 120 \cos 45 + 60 = 145 \text{ N} \]

8. (a) There are four forces: two horizontal: rope-on-box \( F_{RB} \), friction \( F_{FBx} \) (floor-on-box)

Two vertical: weight \( F_{EB} \), earth-on-box, ‘normal’ force, also floor-on-box \( F_{FBy} \)

(b) The two vertical forces add up to zero.

The two horizontal forces add up to 120 N – 32 N = 88 N. This is the net force, in the direction in which the rope pulls. \( A = 88/23 = 3.83 \text{ m/s}^2 \), in the direction of the net force.

11. (a) The angle is about 26.6 deg. \( \text{friction} = M_g \sin 26.6 = 219 \text{ N} \), normal force = \( N = M_g \cos 26.6 = 438 \text{ N} \). Ratio is \( \tan 26.6 = .50 \)

(b) \( f = M_g \sin 10 = 85.1 \text{ N} \) This is less than \( \mu N \), which gives only the maximum force of friction.

(c) The file cabinet moves, so that the coefficient of sliding friction (0.2) needs to be used, and the maximum value of the force of friction, \( f \), which is \( \mu_s N \).

\( N \) is perpendicular to the ramp surface and equal to \( M_g \cos 20 = 460 \text{ N} \). \( f \) is along the surface and downward. Its magnitude is \( 0.2 \text{ N} = 92 \text{ Newton} \). The applied force is given as 700 N and is upward along the ramp. The weight is vertically down and is equal to \( M_g = 490 \text{ N} \).

12. \( F_y = 12 \sin 55 = 9.83 \text{ N} \). \( F_x = -12 \cos 55 = -6.88 \text{ N} \).

14. There are three forces: the force of friction, \( f \) along the hill and upward, the weight, \( M_g \), vertically down, and the normal force, perpendicular to the hill. There is no acceleration perpendicular to the hill, and therefore no net force in that direction. Hence \( N = M_g \cos \theta \). We need to know the mass to calculate \( N \) and the weight. You also need to know either the force of friction or the coefficient of friction.

It is possible to find \( \mu \), but that is a little harder: the force down is \( M_g \sin 20 - f \), or \( M_g \sin 20 \mu M_g \cos 20 \). This is equal to \( M_a \). We can cancel \( M \) from each term to get \( g \sin 20 - \mu g \cos 20 = 1.5 \) and solve for \( \mu \), which turns out to be 0.20.

16. (a) If we neglect air resistance there is only the weight, vertically down.

(b) \( v \) is horizontal. (c) the acceleration is \( g = 9.8 \text{ m/s}^2 \) straight down.
18. (a) The initial momentum is \( (2000)(30) = 6 \times 10^4 \) kg m/s. This is also the final momentum, equal to \( 2M v_f \). Hence the final velocity, \( v_f = 6 \times 10^4/4 \times 10^3 = 15 \) m/s.
(b) The main part is to draw the vector diagram. To find the numerical values is then only a mathematical exercise that takes time. (I would skip the numerical part.) The vector diagram is a triangle. One side is the total momentum \( 6 \times 10^4 \) kg m/s. The two momenta of the two cars after the collision have to add up to that. One is given, \( 2 \times 10^4 \) at 35 deg. The other one completes the triangle. The calculation is somewhat tedious and leads to \( v = 22.5 \) m/s at 14.8 deg.

22. (a) For \( R = 2.7 \) m, \( a_c + v^2/R = R \omega^2 = 24.3 \) m/s² or about 24 m/s².
(b) A diagram shows that \( a_x = -a_c \cos \theta \) and \( a_y = -a_c \sin \theta \). The two graphs are similar but start at different times. \( a_x \) starts at -1, going up, and \( a_y \) starts at zero going down. The two are ‘out of phase’.

23. (a) \( v^2/r = 25^2/20 = 31.25 \) m/s².
(b) The force causing the car to turn is the force of friction between the tires and the road.

26. The initial angular momentum \( I \omega \) is \( (3)(8) = 24 \) kg m²/s. (The radian is a distance divided by a distance and therefore does not contribute a dimension.)
The final angular momentum is the same and equal to \( (3 + 3) (kg m^2) (\omega) = 24 \) kg m²/s so that \( \omega = 24/5 = 4.8 \) radian/s.

Problems and reasoning skill building.

2. (a) In general, the force on the car when it is at the top is equal to the force of the track on the car, \( F_{TC} \), plus the weight, \( Mg \), both vertically down.
(b) As the car’s speed is reduced, \( F_{TC} \) becomes smaller. The minimum speed (without the car’s falling down) is that for which \( F_{TC} \) is zero and only the weight is left. With that value the centripetal force is equal to \( Mg \), so that \( Mg = Mv^2/R \) or \( v^2 = gR \) and \( v = \sqrt{gR} \). All larger speeds are possible.

6. (a) The net force, \( F_{net} \) is \( (10 - 2) = 8 \) N. \( a = F_{net}/M = 8/4 = 2 \) m/s².
\( x = v_0 t + \frac{1}{2} at^2 \) or \( 0 + \frac{1}{2} (2) (3^2) = 9 \) m.
(b) When the rope breaks, friction (= 2 N) will be the net force, and will bring the car to a stop with an acceleration of -2/4 or 0.5 m/s², in a direction opposite to the direction of motion.

7*. When it starts there will be two forces, the thrust of 15 N up, and the weight, down, with a net force up of 10.1 N. After 8 s only the weight, 4.9 N down, is left. For the first 8s the acceleration is 20.2 m/s² up. After that it is equal to \( g \), and equal to 9.8 m/s², The velocity goes up for 8 s, and from then on goes down at the lower rate. To find the highest point is harder: (This only for extra points!) First find how far the rocket has gone in 8 s and how fast it is then going. Then answers are 646.4 m and 161.6
m/s. This is the starting point for the second segment. Here \( v = 161.6 - 9.8 \ t \). This is zero when \( t = 16.5 \ \text{s} \). That happens at the highest point. (After that the velocity becomes negative as the rocket returns to earth.) From the usual kinematic equations this is at a height of 1979 m.

10. (a) \( F_x = F_2 \cos \theta \)
    \( F_y = F_1 - F_3 + F_2 \sin \theta \)
    \( F = \sqrt{(F_x^2 + F_y^2)} \) at an angle such that \( \tan \alpha = F_y/F_x \)
(b) \( F_x = F_2 \cos \theta - F_1 \cos \theta \)
    \( F_y = F_1 \sin \theta + F_2 \sin \theta - F_3 \)
    And again \( F = \sqrt{(F_x^2 + F_y^2)} \) etc., as before.

12. (a) \( a = F_{SP}/M \). The weight of a puck is about 6 oz, so that its mass is about .17 kg. This acceleration, \( a \), is the rate at which the speed changes while the stick is in contact with the puck.
(b) It is hard to guess at the magnitude of the force. For a force of 200 N, \( a \) would be \( 1.18 \times 10^3 \ \text{m/s}^2 \).

13. The forces are \( T = 22 \ \text{N} \) up and \( Mg \) down. The acceleration is given by \( 22 - Mg = Ma \), so that
    \( M = 22/(a + g) = 22/(1.2 + 9.8) = 2.0 \ \text{kg} \).

15. In each case the forces are \( Mg \) down and \( F_{SC} \) (scale-on-child) up. \( F_{SC} \) is the reading on the scale. (If the metric system is used, the readings are likely to be in kg. The reading is then equal to the mass that would have to be put on the scale such that its weight is \( F_{SC} \).)
(a) \( F_{SC} = Mg = 392 \ \text{N} \). (On a kg-scale the reading would be this value divided by \( g \), or 40 kg.)
(b) \( Mg - F_{SC} = Ma \) so that \( F_{SC} M((g - a) = 272 \ \text{N} \).
(c) Same as (a).
(d) \( F_{SC} - Mg = Ma, F_{SC} = M(g + a) = 792 \ \text{N} \).
(e) Heaviest for the greatest \( F_{SC} \), i.e., for (d), lightest for the smallest \( F_{SC} \), i.e., for (b).

16. For both boxes considered as one system, the only horizontal force is the force of the rope of 3 N. The acceleration is \( 3/(10 + 5) \) or 0.2 m/s\(^2\). This is the acceleration of each box.
For the 10 kg box the only horizontal force is that of the string, \( T. T = Ma, or T = 10 \ a = 2 \ \text{N} \).
The 5 kg box is acted on by the force of 3 N to the right and \( T = 2 \ \text{N} \) to the left. The acceleration is the net force, 1 N, divided by the mass, 5 kg, or 0.2 m/s\(^2\), as before.

18. (a) diagram (similar to #15). (b) The acceleration is the same for each box. For both boxes together, \( a = 3/12 = 0.25 \ \text{m/s}^2 \).
(c) \( v^2 = v_0^2 + 2 \ a \ x = 0 + (2)(.25)(2) = 1 \), so that \( v = 1 \ \text{m/s} \).
\( v = v_0 + at = 0 + 0.25 \ t \) and \( t = 4 \ \text{s} \).
21. (a) \(a_c = \frac{v^2}{R} = \omega^2 R\). \(R = 0.09\) m, \(\omega = (40,000/60)(2\pi) = 4189\) radians/s. \(a_c = 1.58 \times 10^6\) m/s\(^2\). (b) \(F_c = Ma_c = 7.9 \times 10^3\) N (or the weight of a body whose mass is 806 kg).

24. Orbital radius of earth = 1.5 \(\times 10^{11}\) m.
\(v = 1\) circumference/year = \(2\pi R/y\). \(1 y = (365)(24)(3600)\) s so that \(v = 3.0 \times 10^4\) ( = 30 km/s or \(~18\) miles/s). \(a_c = \frac{v^2}{R} = 6 \times 10^{-3}\) m/s\(^2\). The fraction of the weight is \(v^2/Rg = 6.1 \times 10^{-4}\) or 0.06 %.

25. \(I_1 \omega_1 = (40 \text{ kg m}^2)(70 \text{ rev/min})\). \(I_2 = 10 \text{ kgm}^2\). \(I_1 \omega_1 = I_2 \omega_2\), \(\omega_2 = \omega_1(I_1/I_2) = 4 \omega_1 = 280\) rpm.

28. \(\tau = 30\) Nm. \(I = 8 \text{ kg m}^2\). \(\alpha = \tau/I = 30/8\) rad/s\(^2\).
\(\omega = \omega_0 + \alpha t = 0 + (30/8)(10) = 37.5\) rad/s.

30. \(v\) and \(a\) are the same, the two forces are different.

33. The force relation is \(GMe/R^2 = m v^2/R\), or \(GMe/R^3 = v^2/R^2\).
Since \(vT = 2\pi R\) (where \(T\) is the time for one revolution, or the ‘period’ of the motion, here 24 h), we can also write this relation as \(GMe/R^3 = 4\pi^2/T^2\), which we can solve for \(R^3\) and hence \(R\), to get \(R = 4.27 \times 10^7\) m.

35. (a) The circumference of a wheel is \(2\pi R = 3.46\) m. At 6.7 m/s the wheels rotate at 6.7/\(2\pi R\) revolutions/s or 6.7/R radians/s. I.e., \(\omega = 6.7/55 = 12.2\) rad/s.
(b) \(\omega = \omega_0 + \alpha t = 0 + 5\alpha\), so that \(\alpha = \omega/t = 12.2/5 = 2.44\) rad/s\(^2\)

Multiple choice questions.
1. \(v^2 = v_0^2 + 2ax\), where \(v = 0\), \(v_0 = 4\) m/s, and \(x = 0.4\) m, so that \(a = 16/.8 = 20\) m/s\(^2\). \(F = ma = 1000\) N
The answer is (e).

2. (b)

3. The forces are the weight of the children, the normal force, and the frictional force up the slide. There is no force down the slide. The answer is (c)

4. \(x\)-direction: 150 cos 20 = 45. a.
\(y\) – direction: 150 sin 20 + N = Mg.
Hence lots of mistakes: b, d. In addition, ax should be just a.

5. \(F_y = 5\sin 45 – 3.5 = 3.53 – 3.5 \sim 0\). Therefore the answer is east, (a).

6. d.

7. c.
Synthesis problems and projects.

3. (a) friction. (b) $mv^2/R = f$. $F_{max} = \mu N = \mu mg$. At the point where $f = f_{max}$, the mouse begins to slide, i.e. $mv^2/R = \mu mg$, or $v^2/R = \mu g$.
(c) $v^2/R$ or $\omega^2 R$ toward the center. (d) $\mu g = \omega^2 R$
(e) $\omega^2 = \mu g/R$ or $\omega = \sqrt{(\mu g/R)}$, so that the answer is (- ½).