I. A bucket full of rocks is being lowered at a constant speed of 1.6 m/s. At t=0 one rock slips out and takes 0.75 s to fall to the ground.

a) Describe the motion of the rock starting at t=0 using the \( v \) vs \( t \) and \( a \) vs \( t \) graphs provided below. Be sure to include minimum and maximum values of \( v \) and \( a \) on the axes.

\[
\begin{align*}
V_0 &= -1.6 \text{ m/s} \\
\text{LET DOWN : } v &= -v \\
\text{LET UP : } v &= +v \\
\alpha &= -9.8 \text{ m/s}^2 \\
V_f &= -1.6 - 9.8(0.75) = -8.95 \text{ m/s} \\
\end{align*}
\]

\[
\begin{align*}
v &= v_0 + at \\
v &= -1.6 - 9.8t \\
\end{align*}
\]

b) At what height is the bucket when the rock drops from it?

\[
\Delta y = v_0 t + \frac{1}{2} at^2 = -1.6 \left( \frac{1}{2} \right) (9.8)(0.75)^2 \Rightarrow \Delta y = -4.0 \text{ m}
\]

\[
\text{So } h = 4.0 \text{ m}
\]

c) Now redo this problem, but in this case the bucket is moving UPWARD (not down) with a constant speed of 1.6 m/s and it takes 0.75 seconds for the rock to drop to the ground. Describe the motion of the rock starting at t=0 using the \( v \) vs \( t \) and \( a \) vs \( t \) graphs provided below. Be sure to include minimum and maximum values of \( v \) and \( a \) on the axes.

\[
\begin{align*}
V_0 &= +1.6 \text{ m/s} \\
\alpha &= -9.8 \text{ m/s}^2 \\
V_f &= +1.6 - (9.8)(0.75) = -5.75 \text{ m/s} \\
\end{align*}
\]

d) At what height is the bucket when the rock drops from it in part (c)?

\[
\Delta y = (+1.6)(0.75) - \left( \frac{1}{2} \right)(9.8)(0.75)^2 = -1.6 \text{ m}
\]

\[
\text{So } h = 1.6 \text{ m}
\]
II. The "Herky Jerky" is a new amusement park ride shown to the right in which the rider travels vertically in a car as it is jerked erratically up and down.

During a portion of the ride, the velocity is described by the function (up is taken to be positive):

\[ v(t) = (15t - 0.4t^3) \text{ m/s} \]

a) Does the car ever come to a stop momentarily? If so, when? If not, why not?

\[ 0 = 15t - 0.4t^3 \]
\[ 15t = 0.4t^3 \]
\[ t^3 = \frac{15}{0.4} \]
\[ t = \frac{\sqrt[3]{15}}{\sqrt[3]{0.4}} \]

Only positive root is physically reasonable:

\[ t = 6.15 \text{ s} \]

b) Draw a free-body diagram (force diagram) of a 70 kg rider at the time \( t = 3 \text{ s} \).

\[ \alpha(t) = \frac{dv}{dt} = 15 - 1.2t^2 \]
\[ \alpha(3) = (15) - 1.2(3)^2 = +4.2 \text{ m/s}^2 \]

So \[ F_{\text{net}} = 294 \text{ N} \approx 300 \text{ N} \]

Weight \[ = 700 \text{ N} \]

And \[ F_{\text{normal}} > |F_{\text{weight}}| \]

C) Find the normal force exerted on a 70 kg rider at \( t = 3 \text{ s} \).

\[ \sum F = ma \Rightarrow F_{\text{normal}} - F_{\text{earth on rider}} = m_{\text{Rider}} \alpha \]

\[ F_{\text{normal}} = m_{\text{Rider}} \alpha + F_{\text{Earth on Rider}} \]

\[ = (70)(4.2 \text{ m/s}^2 + 9.8 \text{ m/s}^2) = 980 \text{ N} \]
III. Two pucks, initially at rest, are pushed for 2 seconds by identical forces, \( F \) (caused by an object not shown.) The white puck has a mass of 1 kg and the gray puck has a mass of 2 kg. Assume that the track is frictionless and that the pucks don’t run into each other.

\[ \Delta p = p_f - p_i = p_f \quad \text{and} \quad \Delta p = F \Delta t = 2F \]

So \( p_f = 2F \) for both pucks.

a) Which puck has the most momentum at the time \( t = 2s \)? Express each pucks’ momentum in terms of \( F \) and any other relevant numbers or constants.

b) Assuming that the forces are no longer acting before the pucks reach the 10 m mark, which puck will reach the 10 m mark first? Explain your reasoning.

Even though \( p_w = p_g \), the lighter puck will have a greater velocity. So the white puck will reach the 10 m mark first.

c) Which puck will rise highest point on the curved portion of the track before turning around and sliding back down? Explain your reasoning.

\[ V_w = \frac{2F}{1kg} = 2F, \quad V_g = \frac{2F}{2kg} = F \]

\[ k_w = \left( \frac{1}{2} \right) (1kg)(2F)^2 = 2F^2; \quad mgh_w = k_w \Rightarrow h_w = \frac{2F^2}{g} \]

\[ k_g = \left( \frac{1}{2} \right) (2kg)(F)^2 = F^2; \quad mgh_g = k_g \Rightarrow h_g = \frac{F^2}{2g} \]

\[ h_w > h_g; \quad h_w = 4h_g \]
4. Which of the following statements is FALSE?

a) In Graph 1 the objects A and B are moving in the same direction.

b) In Graph 2 the objects A and B are moving in opposite directions at $t = 6$ s.

c) In Graph 2 object B moves faster than object A at $t = 6$ s.

d) In Graph 2 object A moves farther than object B in the first 6 s.

e) In Graph 1 both objects A and B have the same acceleration at $t = 6$ s.

5. The motion of a particle is described by

$$x(t) = (5 - 8t + 2t^2) \text{ meters}.$$ 

Which one of the following statements is TRUE?

a) There is a constant net force acting in the negative direction.

b) The particle starts from rest.

c) The velocity of the particle at $t=1$s is 4 m/s.

d) The particle always moves in the positive direction.

e) Between $t=0$s and $t=1$s the particle slows down.

6. A cannonball is fired from a cannon located at the edge of a cliff. The muzzle of the cannon is 40 meters above the water. The cannonball has an initial horizontal velocity of 80 meters per second and an initial upward vertical muzzle velocity of 45 meters per second. Which of the following changes will DECREASE the maximum height above the cannon that the cannonball reaches? (assume air resistance is negligible)

a) Use a heavier cannonball, and there is no change in the initial velocities.

b) The cannon is adjusted so the initial horizontal velocity is decreased and the initial vertical velocity is unchanged.

c) The cannon is adjusted so the initial vertical velocity is decreased and the initial horizontal velocity is unchanged.

d) The cannon is moved to a lower cliff and there is no change in the initial velocities.

e) More than one of the answers given is correct.
7. You are pulling two boxes (10 kg and 15 kg) connected with a rope on a horizontal surface. You exert a force on the 15-kg box, \( F = 80 \text{ N} \), at an angle \( \theta = 27^\circ \) by pulling on the second rope attached to it. If the boxes start from rest, how long does it take to pull them 2 m?

\[
M_{\text{TOTAL}} = 25 \text{ kg}
\]

\[
F_x = 80 \text{ N} \cos(27^\circ)
\]

\[
\alpha = \frac{F_x}{m} = \frac{80 \cos 27^\circ}{25} = 2.85 \text{ m/s}^2
\]

\[
\Delta x = \frac{1}{2}a_t^2 \Rightarrow t = \left(\frac{2\Delta x}{a_t}\right)^{\frac{1}{2}} = 1.2 \text{ s}
\]

8. Max and his friend Jess take two identical balls and throw them at \( v_0 = 4 \text{ m/s} \). Max throws his ball straight up and Jess throws hers at an angle \( \theta = 30^\circ \) above the horizontal. What is time interval, \( \Delta t \), between the landing time of the two balls? Assume that the balls are thrown from the ground level.

a) Jess’s ball lands first and \( \Delta t = 0.20 \text{ s} \)

b) Max’s ball lands first and \( \Delta t = 0.20 \text{ s} \)

(\( \text{IN BOTH CASES, } \Delta y = 0 \))

\[
0 = v_{y0}t + \frac{1}{2}at^2 \Rightarrow t = \frac{2v_{y0}}{a}
\]

\[
\Delta t = t_{\text{max}} - t_{\text{Jess}} = 0.41 \text{ s}
\]

e) \( \Delta t = 0 \) because they both land at the same time.

9. A 1.0-kg block is pushed up a rough \( 22^\circ \) inclined plane by a force of 6.7 N acting parallel to the incline. The acceleration of the block is 1.4 m/s\(^2\) up the incline. Determine the magnitude of the force of friction acting on the block.

a) 1.9 N

b) 2.2 N

(\( \text{IN BOTH CASES, }\theta = 22^\circ, a = 1.4 \text{ m/s}^2 \))

\[
F_x = ma \Rightarrow F - mg \sin \theta = ma
\]

\[
F_x = 6.7 - (1)(9.8)(\sin 22^\circ) = 4.4\text{ N}
\]

c) 1.6 N

d) 1.3 N

e) 3.3 N

\[
\Rightarrow F_f = 1.63 \text{ N}
\]
10. A stone falls from rest and is moving with a speed \( v \) just before it hits the ground. How fast is it moving when it passes a point that is \( \frac{3}{4} \) of the way to the ground?

\[
\begin{align*}
\text{a) } & \frac{3}{4} v \\
\text{b) } & \frac{1}{4} v \\
\Rightarrow & \frac{3}{4} m g h = \frac{1}{2} m v' \quad \Rightarrow \quad v' = \left( \frac{3}{4} \right)^{\frac{1}{2}} v \\
\end{align*}
\]

11. A football is kicked from ground level with a velocity of 15 m/s at some unknown angle above the horizontal. The football barely clears the 3-m tall goalposts and the team scores! What is the speed of the football as it clears the goalposts? Disregard air resistance and assume the ground is level.

\[
\begin{align*}
\text{a) } & 18.5 \text{ m/s} \\
\text{b) } & 21.4 \text{ m/s} \\
\text{c) } & 12.9 \text{ m/s} \\
\Rightarrow & \frac{1}{2} m v^2 = \frac{1}{2} m v_f^2 + m g (3) \\
\text{d) } & 7.67 \text{ m/s} \\
\text{e) } & 16.8 \text{ m/s} \\
\Rightarrow & v_f = \sqrt{15^2 - 2 \cdot 9 \cdot 3} = 12.9 \text{ m/s} \\
\end{align*}
\]

12. The 2.0 kg mass in the figure is attached to a spring in its equilibrium position, which is attached to the wall as shown. The masses are released from rest. After the 3.0-kg mass has fallen 1.5 m, it is moving with a speed of 3.8 m/s. Find the spring constant of the attached spring.

\[
\begin{align*}
\text{a) } & \frac{11}{9} = \frac{11}{5} \frac{16}{5} = 7.1 \text{ N/m} \\
\text{b) } & 2.4 \text{ N/m} \\
\text{c) } & 20 \text{ N/m} \\
\text{d) } & 16 \text{ N/m} \\
\Rightarrow & k = \left[ \frac{3 \cdot (1.5) (1.5) - \left( \frac{1}{2} \right) (1.5) (3.8)^2}{(1.5)^2} \right] \frac{2}{15} \\
\text{e) } & 10 \text{ N/m} \\
\frac{k}{k} = & 7.1 \text{ N/m} \\
\end{align*}
\]
13. A toy car of mass $m$ enters a loop-the-loop track of radius $r$. What is the minimum amount of kinetic energy necessary in order for the car to make it completely around the loop without falling down?

a) $(5/2)^{1/2} \, mg r$

b) $(3/2)^{1/2} \, mg r$

c) $2 \, mg r$

d) $(3/2) \, mg r$

c) $(5/2) \, mg r$

**Solution:**

At the top:

$$mg = F_{net} = \frac{mv^2}{r}$$

So:

$$mg = \frac{mv^2}{r} \Rightarrow v^2 = gr$$

$$\Rightarrow \frac{1}{2}mv_{top}^2 = \frac{1}{2}mgr$$

So at the bottom:

$$k_{bottom} = k_{top} + mg(2r)$$

$$= \frac{1}{2}mgr + 2mgr = \frac{5}{2}mgr$$

14. A 1.6-kg block is attached to the end of a 2.0-m string to form a pendulum. The pendulum is released from rest when the string is horizontal. When the block is at the lowest point of its swing and moving horizontally a 40-g bullet, also moving horizontally but in the opposite direction, hits the block. The bullet remains in the block and causes the block to come to rest at the low point of its swing. What was the magnitude of the bullet's velocity just before hitting the block?

a) 56 m/s

b) 350 m/s

c) 40 m/s

d) 250 m/s

c) 400 m/s

**Solution:**

Conservation of Energy:

$$mg h = \frac{1}{2}mv_p^2$$

Conservation of Momentum:

$$m_p V_p = m_b V_b$$

$$V_b = \frac{m_p}{m_b} \sqrt{2gL} = 250 \, m/s$$