# Lattice QCD

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# Summary

- Introduction
- Hadron spectrum
  - What are the low energy degrees of freedom of QCD
- Hadronic structure
  - Moments of structure functions
  - Form factors
  - Generalized Parton Distributions
- Nucleon-Nucleon interactions
  - Scattering lengths  and phase shifts
  - Existence of 2 body bound states
- Resources needed
- Outlook

# Lattice QCD

- Lattice QCD: QCD on a discrete space time
- QCD: The continuum limit of Lattice QCD
- High energy regime:
  - Asymptotic freedom: Weak coupling
  - Perturbative calculations
- Low energy: Lattice QCD powerful tool for calculations
  - Lots of very important physics can be explored theoretically
  - Requires significant resources
- Recent theoretical and computer technology advances together with investment in computer resources and personnel make lattice QCD a tool available to us today.

# Physics we can do

- Hadronic spectrum and quark masses
- Low energy constants for EFTs $f_\pi,\ f_K,\ g_A\ \cdots$
- Hadronic structure
    - Form factors and moments of GPDs
- Hadronic interactions
    - Scattering lengths and phase shifts
- Thermodynamics and finite density QCD
    - RHIC physics
- Weak matrix elements and new physics
    - Aid experimental effort to discover new physics

# Computation break up

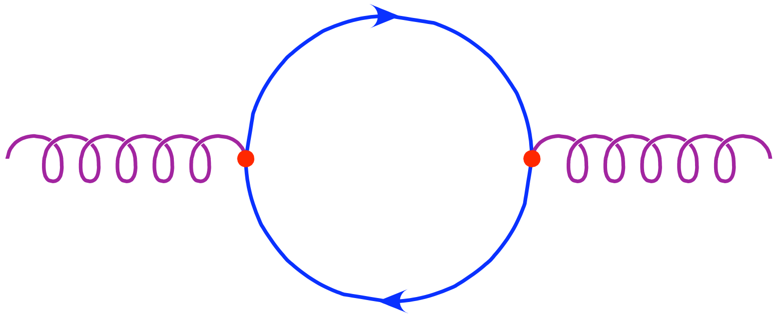
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \prod_{\mu,x} dU_\mu(x) \; \mathcal{O}[U, D(U)^{-1}] \; \det\left(D(U)^\dagger D(U)\right)^{n_f/2} \; e^{-S_g(U)}$$

- Gauge field configuration generation: The accelerator
  - Can be used by several collaborations
- Correlation function calculation: The detector
  - Used for specific calculations

# Realistic Calculations

- Include the vacuum polarization effects
  - 2 light (up down) 1 heavy (strange)
- Finite Volume
  - Compute in multiple and large volume
- Continuum Limit
  - Compute with several lattice spacings
- Chiral Limit
  - Compute with several values for the quark masses
  - Study quark mass dependence of QCD
- Need effective field theory for all the above
  - Light quark masses: $m_\pi < 400$MeV (?)

Past: Ignore vacuum polarization -- Quenched approximation

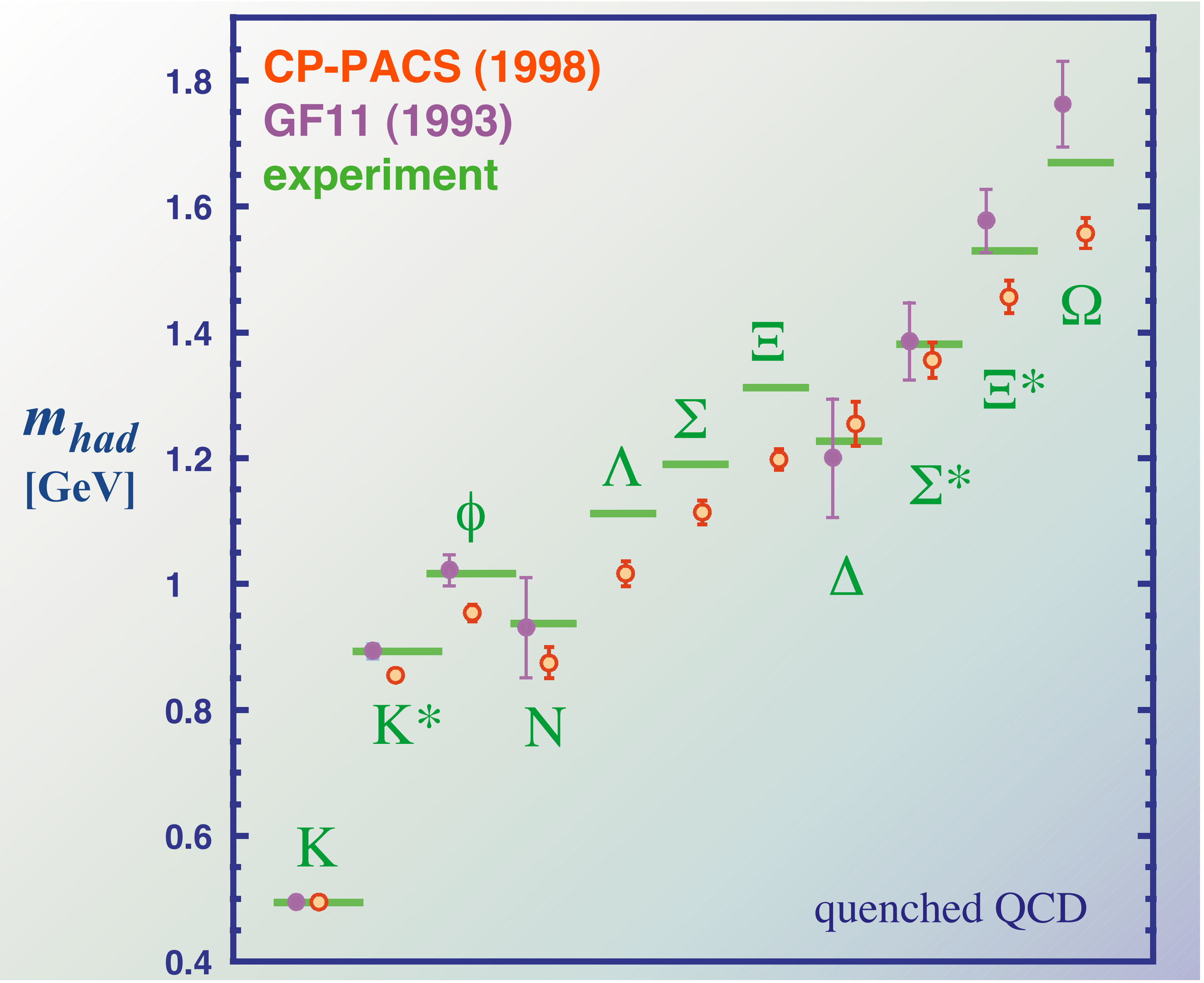


Last 5 years: Include vacuum polarization -- Dynamical calculations

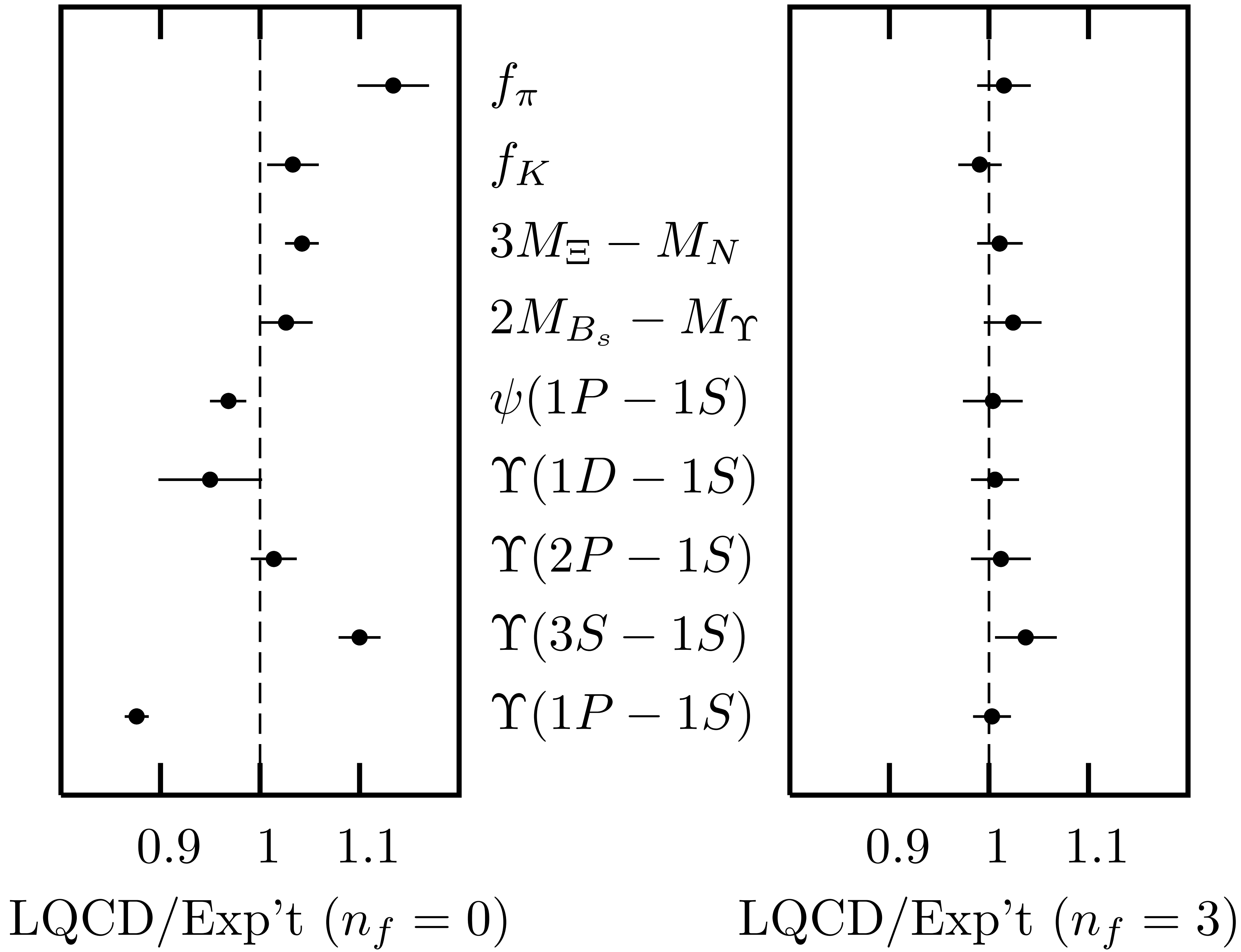
Future: Light quark masses -- Systematics -- New calculations

# Spectrum (past)

- Computation of similar results with dynamical quarks is under way.
- Meson spectrum
  - See J. Dudek
- Heavy quarkonia spectrum (MILC, UKQCD HPQCD .... )

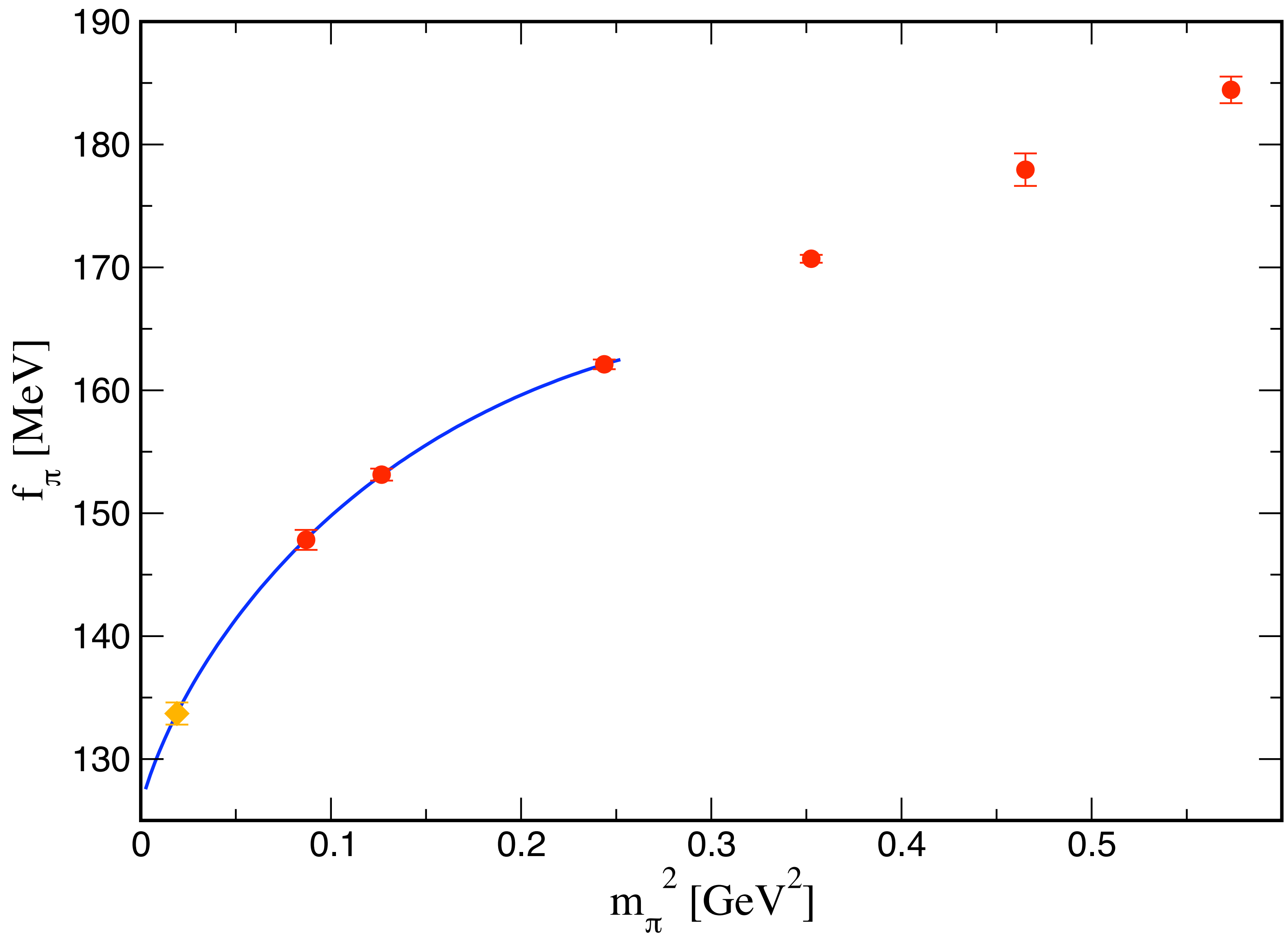


# Spectrum II




MILC, HPQCD, UKQCD

# Pion Decay constant




- Fit the lower 3 points
- One loop χPT extrapolation: 133.7(9)(3.0)MeV
- $\chi^2$/d.o.f. ~ .5

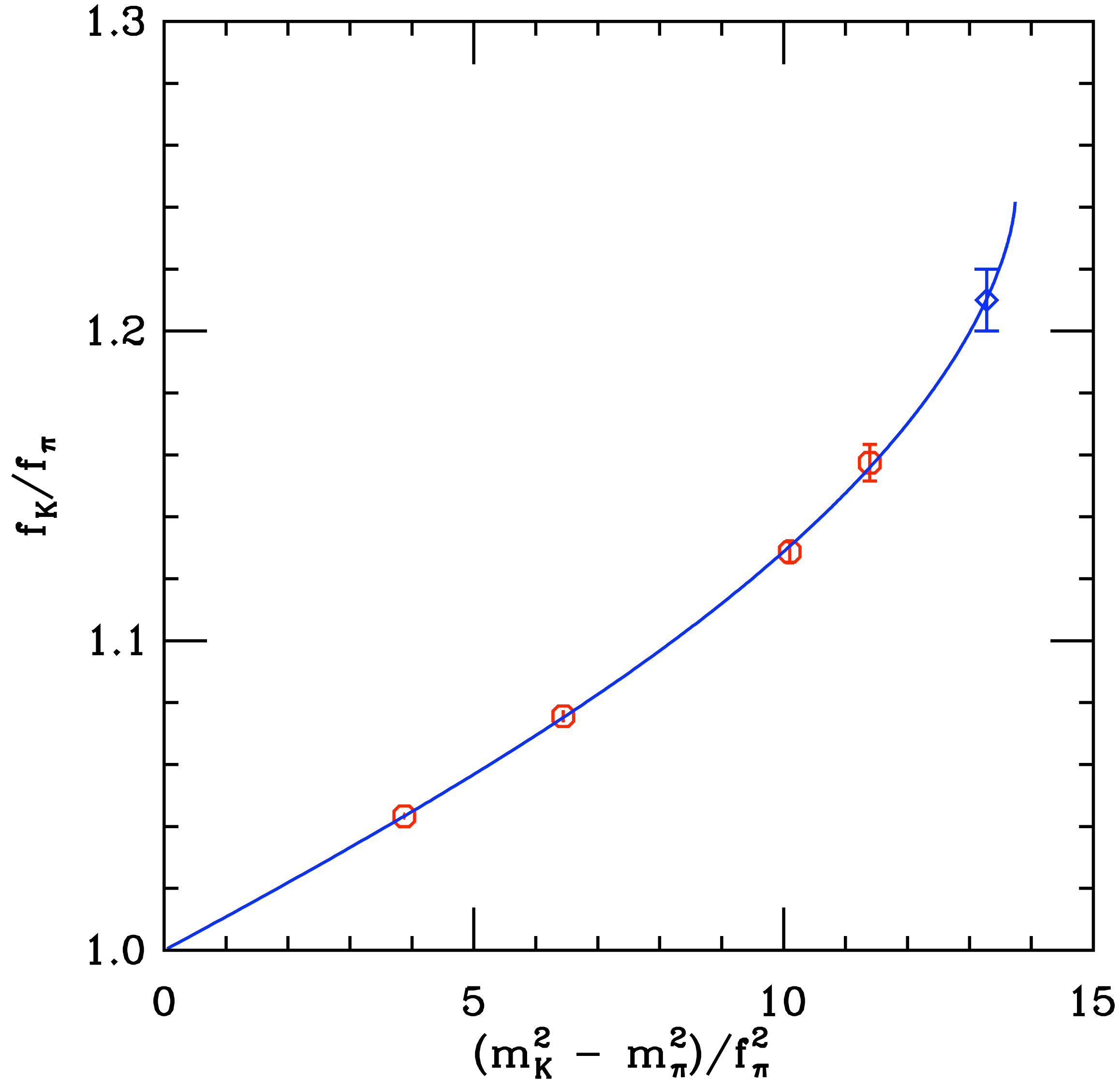
# $F_K/F_\pi$

Beane, Bedaque, KO, Savage hep-lat/0606023

Gasser-Leutwyler:

$$\frac{f_K}{f_\pi} = 1 + \frac{5}{4} l_\pi(\mu) - \frac{1}{2} l_K(\mu) - \frac{3}{4} l_\eta(\mu) + \frac{8}{f^2}\left(m_K^2 - m_\pi^2\right) L_5(\mu)$$

$$l_i(\mu) = \frac{1}{16\pi^2}\frac{m_i^2}{f^2}\log\left(\frac{m_i^2}{\mu^2}\right)$$




$$f_K/f_\pi = 1.218 \pm 0.002 \ {}^{+0.011}_{-0.024}$$

$$\left.\frac{f_K}{f_\pi}\right|_{\mathrm{exp.}} = 1.223(12)$$

Result comparable with MILC $\left.\frac{f_K}{f_\pi}\right|_{\mathrm{MILC}} = 1.210(4)(13)$

Need much higher precision to see effects of Mixed $\chi$PT Baer et.al.'05

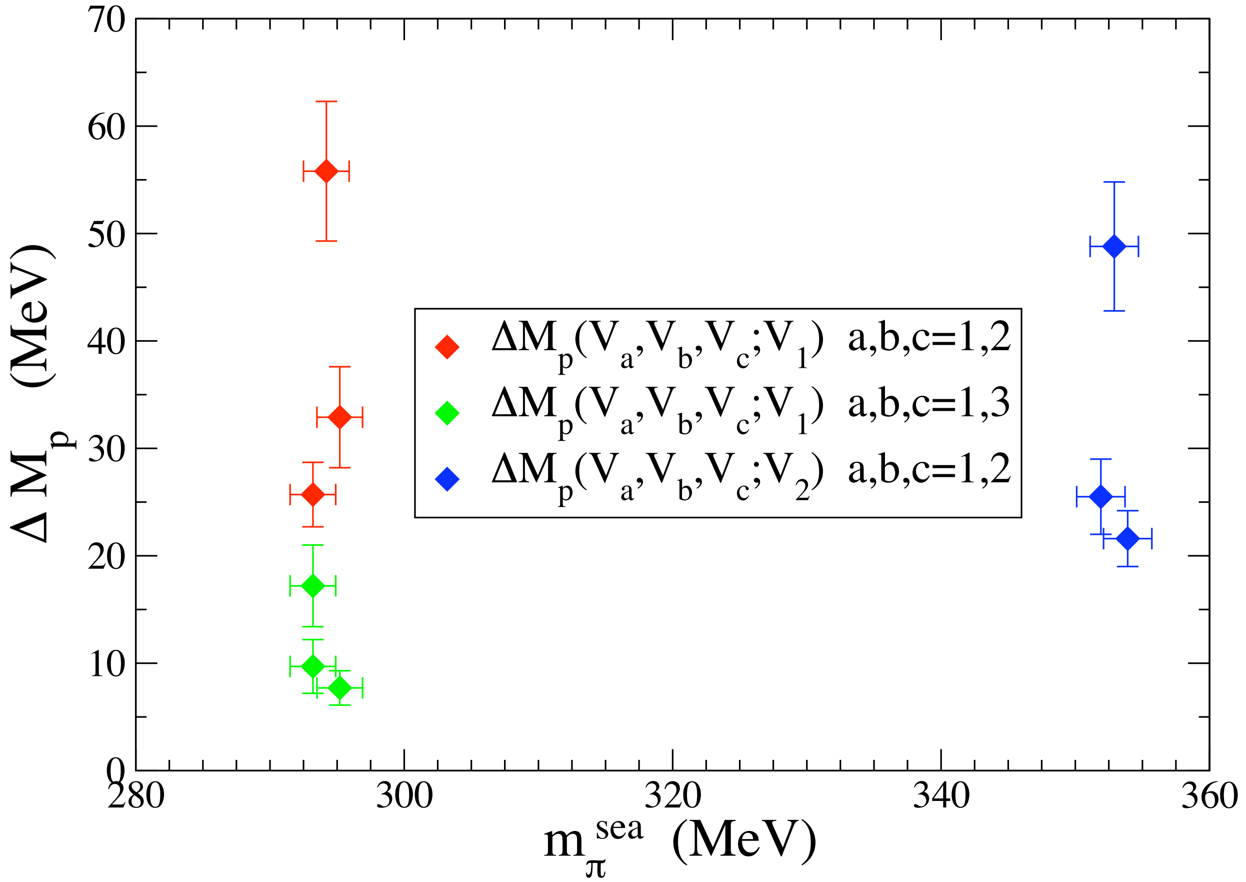
# Neutron-Proton Mass difference

Beane, KO, Savage hep-lat/0605015

$$M_n - M_p|^{d-u} = \frac{2}{3}\left(2\overline{\alpha} - \overline{\beta}\right)\left(\frac{1-\eta}{1+\eta}\right) m_\pi^2$$

MILC: $\eta = m_u/m_d = 0.43(1)(8)$

| Extraction | $M_n - M_p|^{d-u}$ (MeV) at $m_\pi^{\mathrm{phys.}}$ |
|---|---|
| LO $\mathcal{O}(m_q)$ | $1.96 \pm 0.92 \pm 0.37$ |
| NLO $\mathcal{O}(m_q^{3/2})$ | $2.26 \pm 0.57 \pm 0.42$ |




| Extraction | $\frac{1}{3}\left(2\overline{\alpha} - \overline{\beta}\right)$ (l.u.) | $\overline{\alpha} + \overline{\beta}$ (l.u.) | $g_1$ | $|g_{\Delta N}|$ | $\chi^2$/dof |
|---|---|---|---|---|---|
| LO $\mathcal{O}(m_q)$ | $0.198 \pm 0.093$ | $2.07 \pm 0.08$ | $--$ | $--$ | 0.56 |
| NLO $\mathcal{O}(m_q^{3/2})$ | $0.229 \pm 0.058$ | $3.4 \pm 1.1$ | $-0.10 \pm 0.35$ | $0.60 \pm 0.66$ | 0.21 |

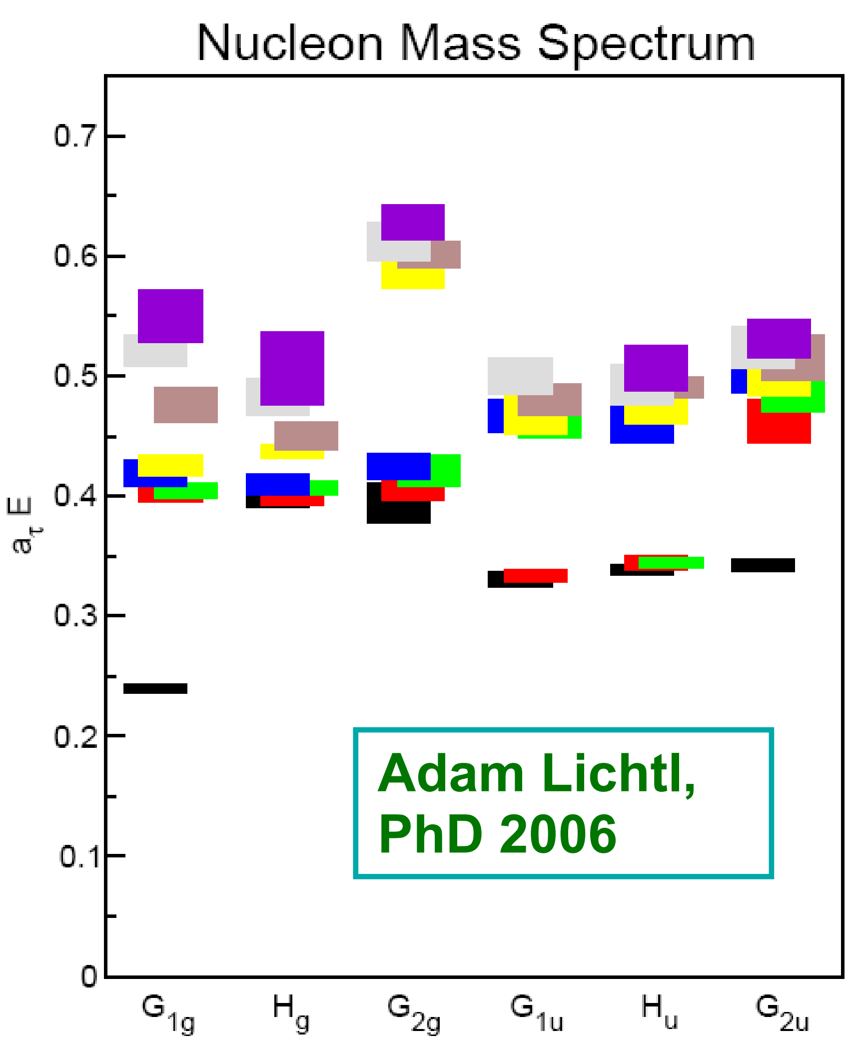
minus EM part

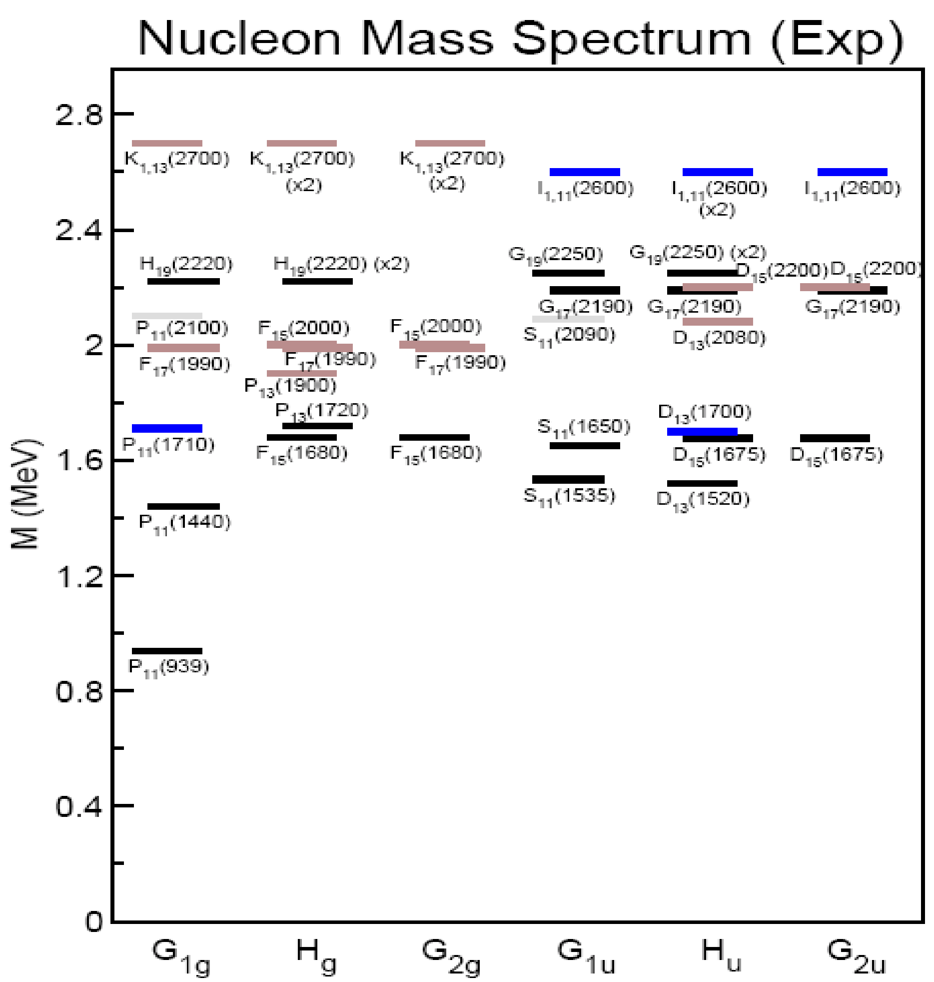
Exp. value: $M_n - M_p = 1.2933317(5)$ MeV → $M_n - M_p = 2.05(30)$ MeV

Gasser Leutwyler '82

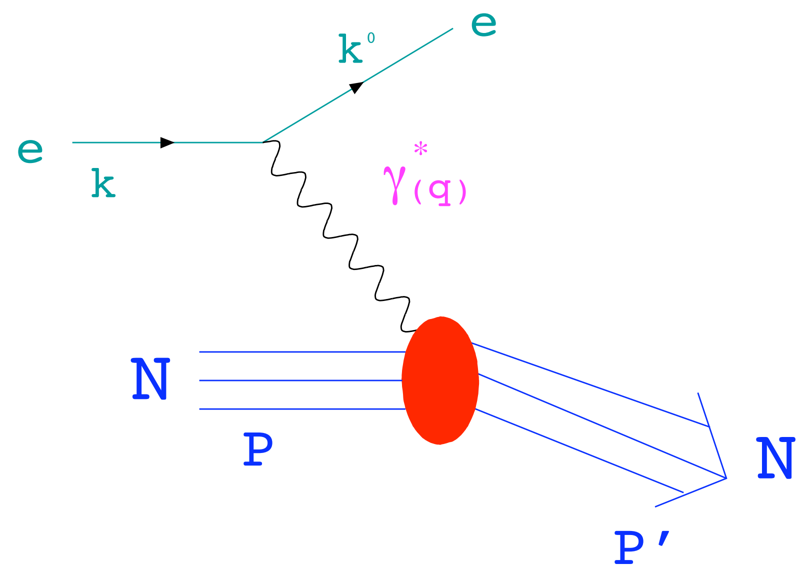
# Spectrum (Future)

- Variational methods
- Classify states according to lattice representations
- Several excited states should be extracted

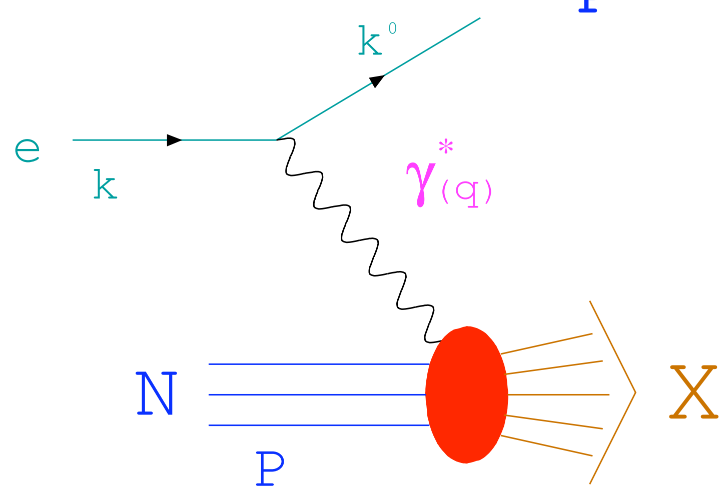


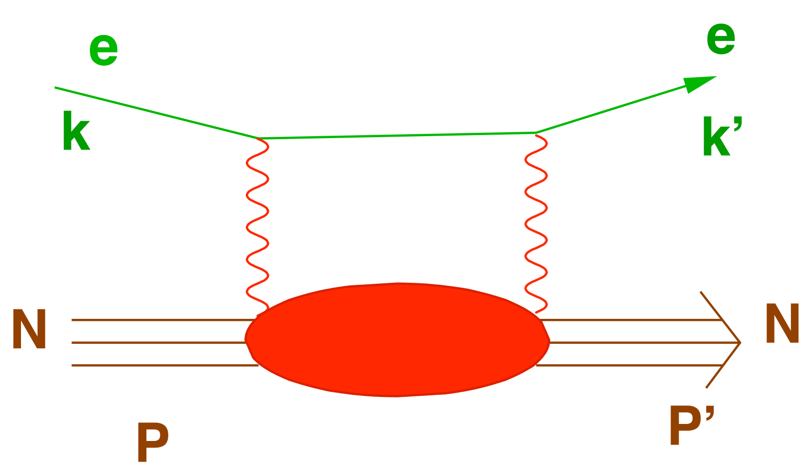
# Hadron Structure




Elastic scattering
Form Factors

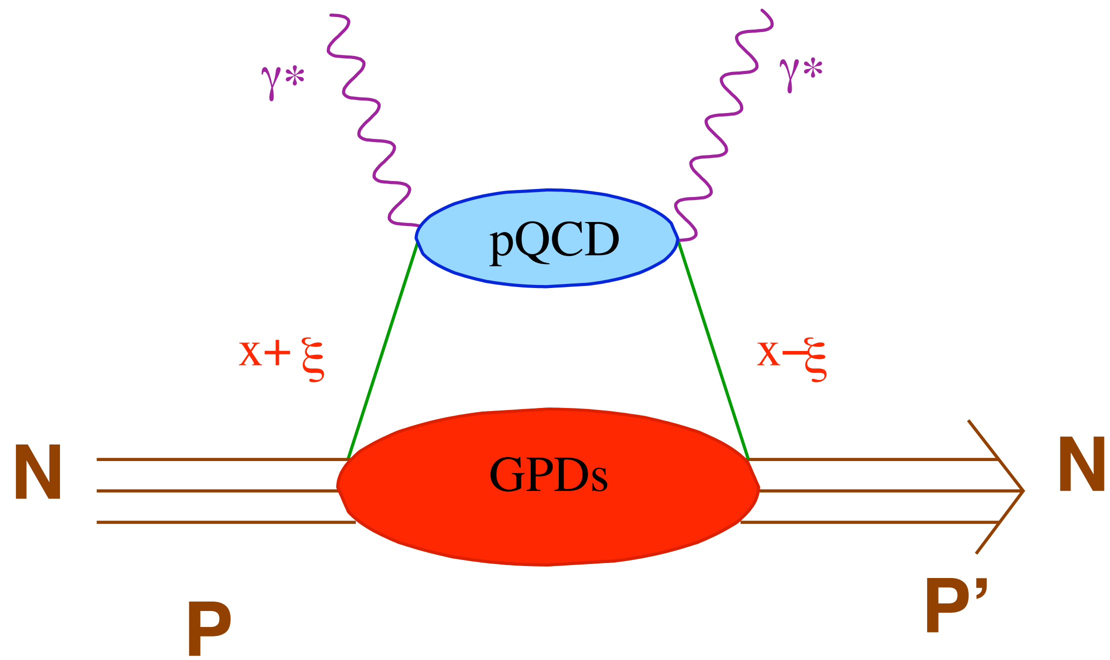



Deep Inelastic Scattering
Structure functions




Deeply Virtual Compton Scattering
Generalized Parton Distribution functions

# Generalized Parton Distributions




$$\mathcal{O}_\Gamma(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{q}(\frac{-\lambda n}{2}) \mathbf{\Gamma}\, \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} q(\frac{\lambda n}{2})$$

$$\Gamma = \not{n} \text{ or } \Gamma = \not{n}\gamma_5 \text{ or } \Gamma = n_\mu \sigma^{\mu\nu}\gamma_5$$

$$\Delta = P' - P \qquad \xi = -n\cdot\Delta/2 \qquad t = \Delta^2$$

**Vector:** $$\langle P, s|\, \mathcal{O}_{\not{n}}(x)\, |P', s'\rangle = \bar{u}(p,s)\left[\not{n}\, H(x,\xi,t) + \frac{n_\mu \Delta_\nu}{2m} i\sigma^{\mu\nu}\, E(x,\xi,t)\right] u(p',s')$$

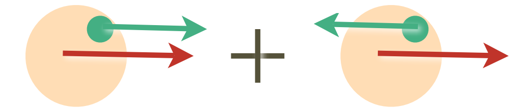
**Axial Vector:** $$\langle P, s|\, \mathcal{O}_{\not{n}\gamma_5}(x)\, |P', s'\rangle = \bar{u}(p,s)\left[\not{n}\gamma_5\, \tilde{H}(x,\xi,t) + \frac{n\cdot\Delta}{2m}\gamma_5\, \tilde{E}(x,\xi,t)\right] u(p',s')$$

**Tensor:**

$$\begin{aligned}\langle P, s|\, \mathcal{O}_{5T}(x)\, |P', s'\rangle \quad &= \quad \bar{u}(p,s)\Big[n_\mu \sigma^{\mu k}\gamma_5 \left(H_T(x,\xi,t) - \frac{t}{2m^2}\tilde{H}_T\right) + \frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\left(E_T(x,\xi,t) + 2\tilde{H}_T(x,\xi,t)\right) \\ &+ \quad \frac{n_\mu \Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5\Delta_\alpha}{2m^2}\, \tilde{H}_T(x,\xi,t) + \frac{\epsilon^{\mu\nu\alpha\beta}P_\alpha\gamma_\beta}{m}\, \tilde{E}_T(x,\xi,t)\Big] u(p',s')\end{aligned}$$

# GPDs: Unified picture of hadronic structure

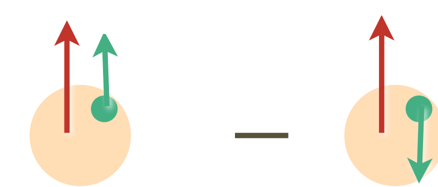
$$H(x,0,0) = q(x)$$



**Forward limit: Δ=0**

$$\tilde{H}(x,0,0) = \Delta q(x)$$

$$H_T(x,0,0) = \delta q(x)$$



**Local limit**

Vector

$$\int dx H(x,\xi,t) = F_1(t) \qquad \int dx E(x,\xi,t) = F_2(t)$$

Axial Vector

$$\int dx \tilde{H}(x,\xi,t) = g_A(t) \qquad \int dx \tilde{E}(x,\xi,t) = g_P(t)$$

Tensor

$$\int dx H_T(x,\xi,t) = g_T(t)$$

# Moments of GPDs

## Operator Product Expansion

Off forward Matrix elements of local operators $\langle P, S|\mathcal{O}|P', S'\rangle$

Unpolarized $$\mathcal{O}^{q}_{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overset{\leftrightarrow}{D}_{\mu_2}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - trace\right]q$$

Polarized $$\mathcal{O}^{5q}_{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_5\gamma_{\mu_1}\overset{\leftrightarrow}{D}_{\mu_2}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - trace\right]q$$

Transversity $$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overset{\leftrightarrow}{D}_{\mu_1}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - traces]q$$

- Generalized form factors: $A_{nk}(t)$ $B_{nk}(t)$ $C_{nk}(t)$
- Moments of GPDs are polynomials in $\xi$ with coefficients the generalized form factors.

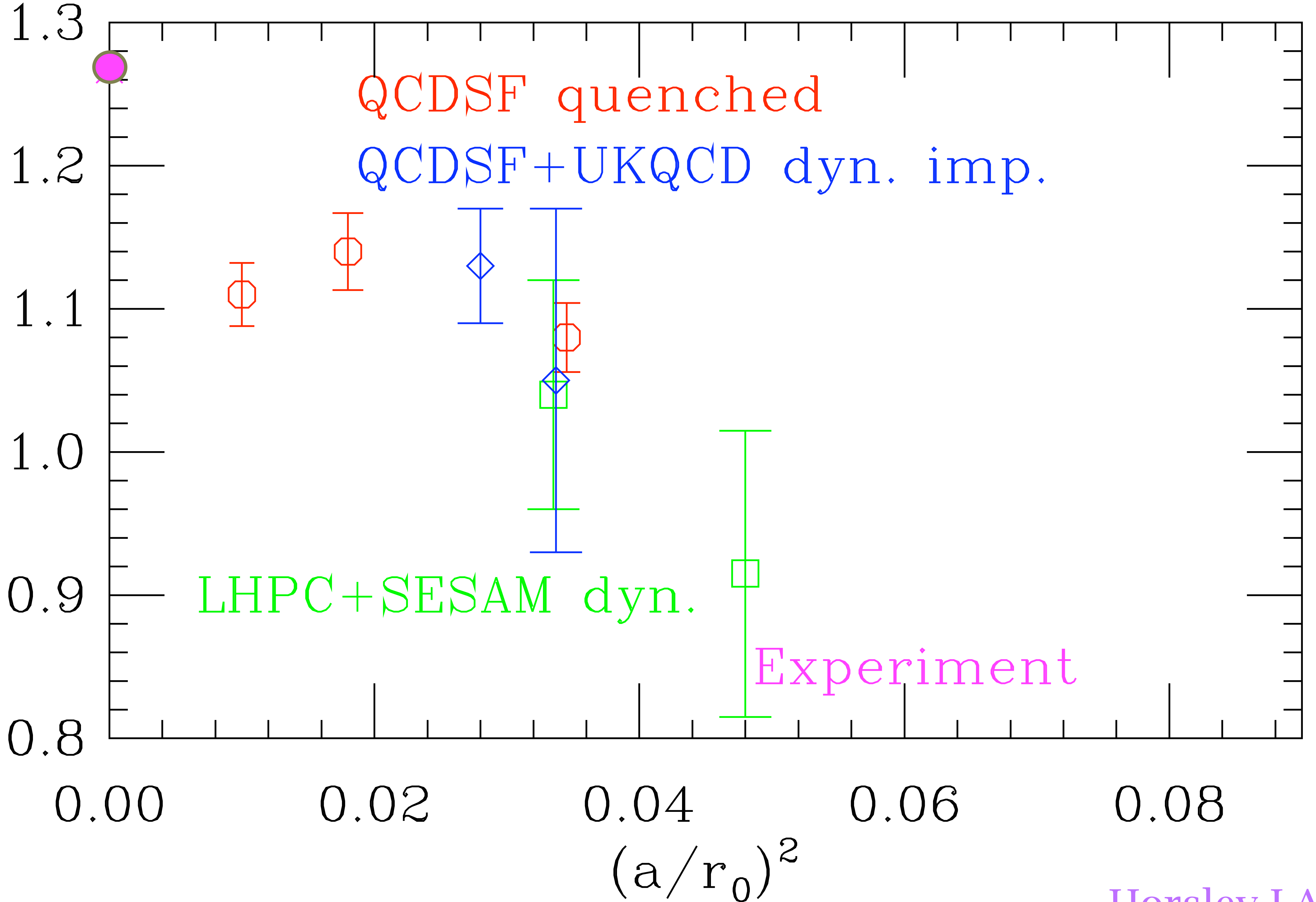
# The Axial coupling

- Accurately known (neutron beta decay)

$$g_A / g_V = 1.2695(29)$$ [PDG2006]

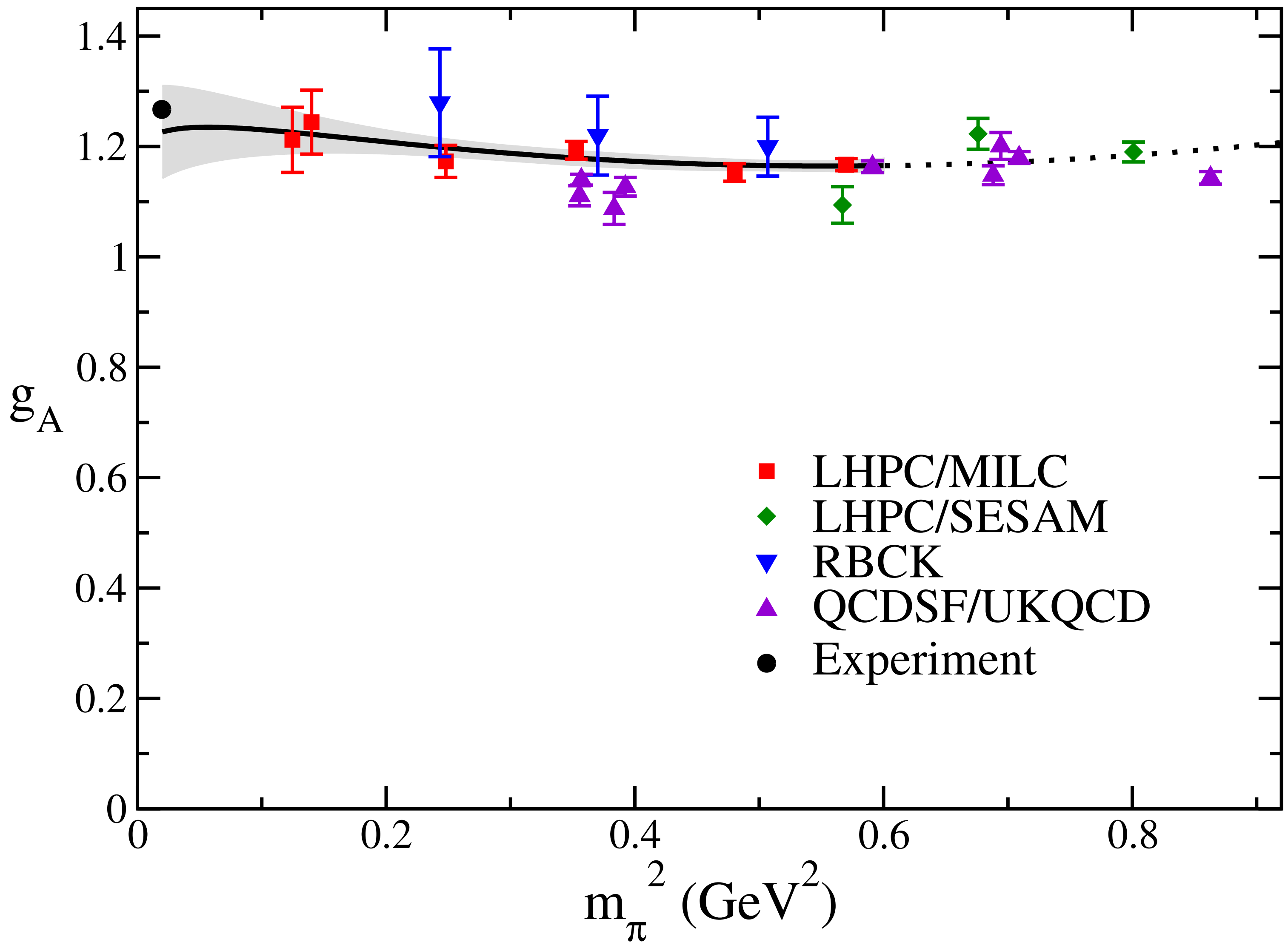
- Lattice calculations had trouble computing it
- Charge symmetry breaking effects not taken into account
- ChiPT results:
    - Finite volume: [Bean, Savage '04]
    - Small Scale expansion: [T. R. Hemmert, M. Procura, and W. Weise '03]
    - Partial NNLO: [Bernard, Veronique and Meissner' 06]

# Axial Coupling (past)




Horsley LAT02

# Axial Coupling (present)




- Large volumes
- Lighter pion masses
- $g_A(m_\pi$=140MeV) = 1.23(8)

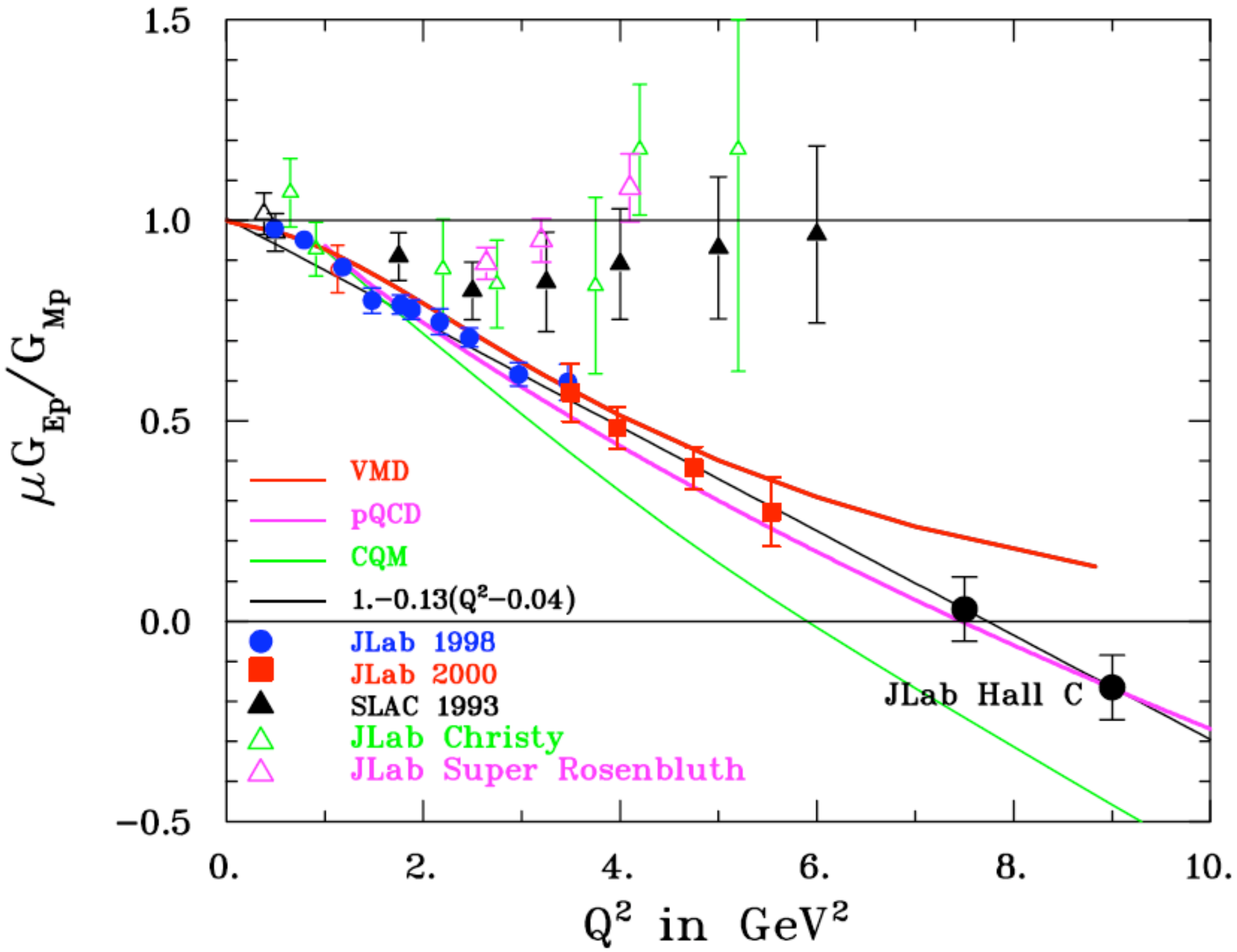
Cost: ~ 1Tflop-year

# Axial Coupling (future)

- Continuum extrapolation
- Lighter pion masses
- Study the behavior of chiral extrapolation and reduce systematics from the extrapolation
- Include charge symmetry breaking effects
- Future: Expect few percent accuracy both statistical and systematic

# Proton ElectroMagnetic Form Factors

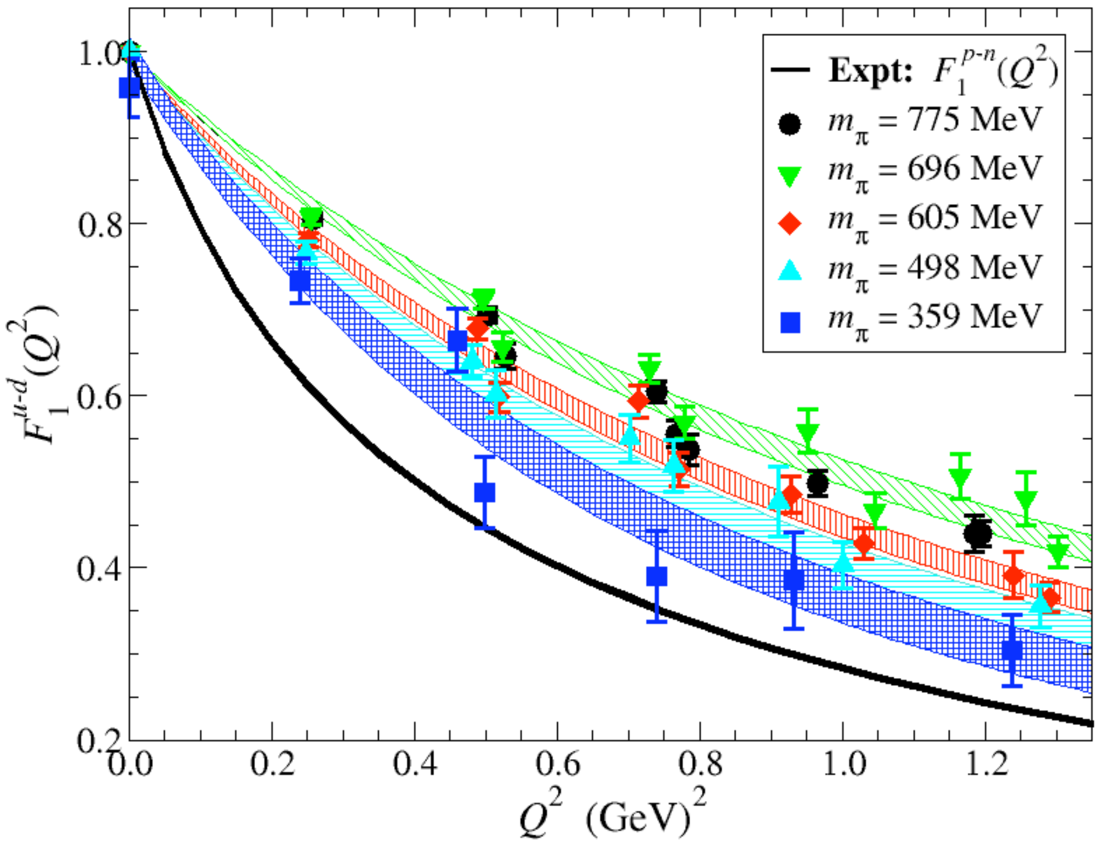
- Rosenbluth separation disagrees with Polarization transfer
- Two photon effects
- What does the lattice predict for the ratio?

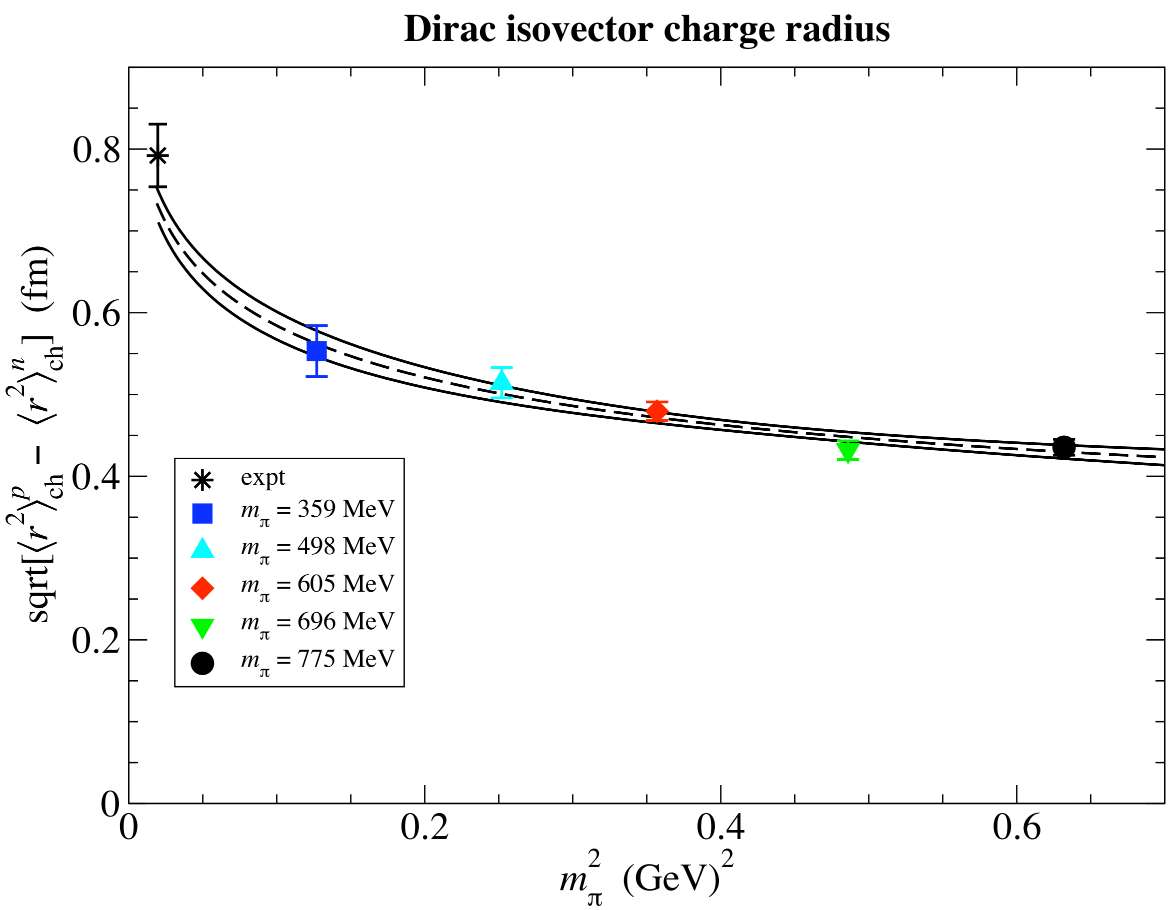



Perdristat: Jlab Users Meeting 2005

# Vector Isovector form factor

[LHPC]



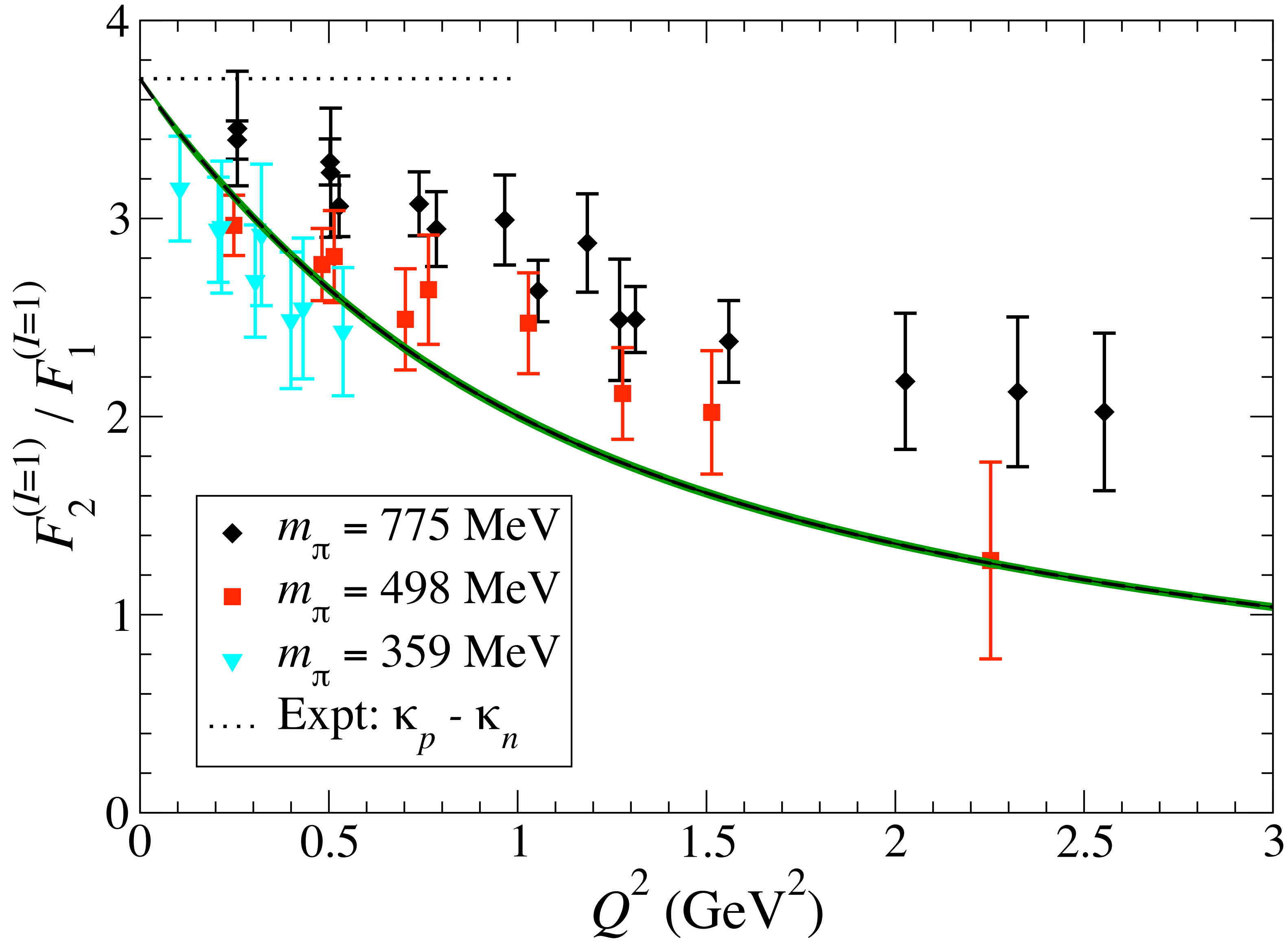




- Light pion masses: Lattice approaches experiment
- Chiral fit using leading order ChiPT [Leinweber et.al. '01 ]
- Nucleon size grows as quarks become lighter

**[G. T. Fleming]**

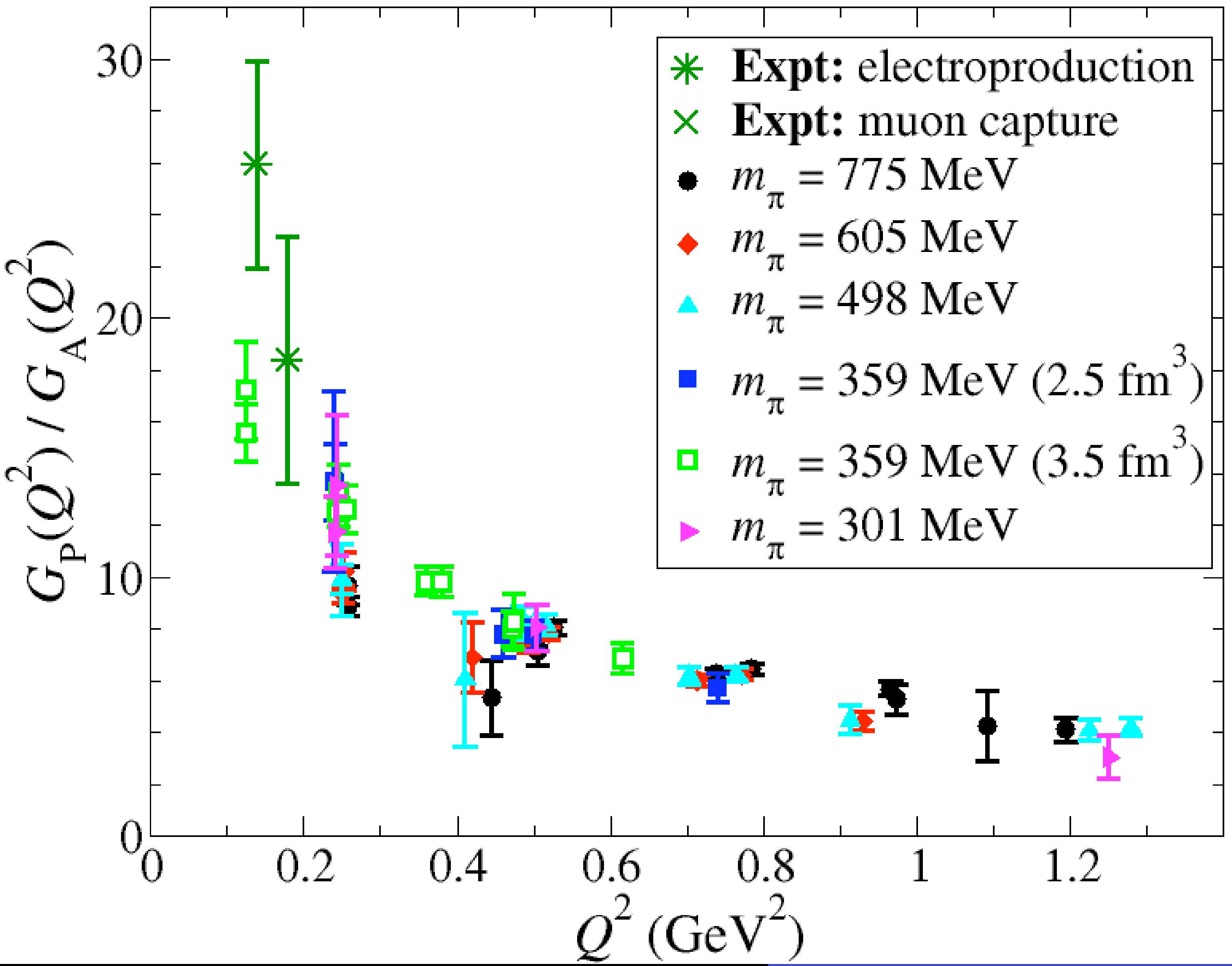
# Vector Isovector form factor ratio

[LHPC]




- Lattice results approach experiment as quark mass is lowered

# Axial Isovector Form Factor



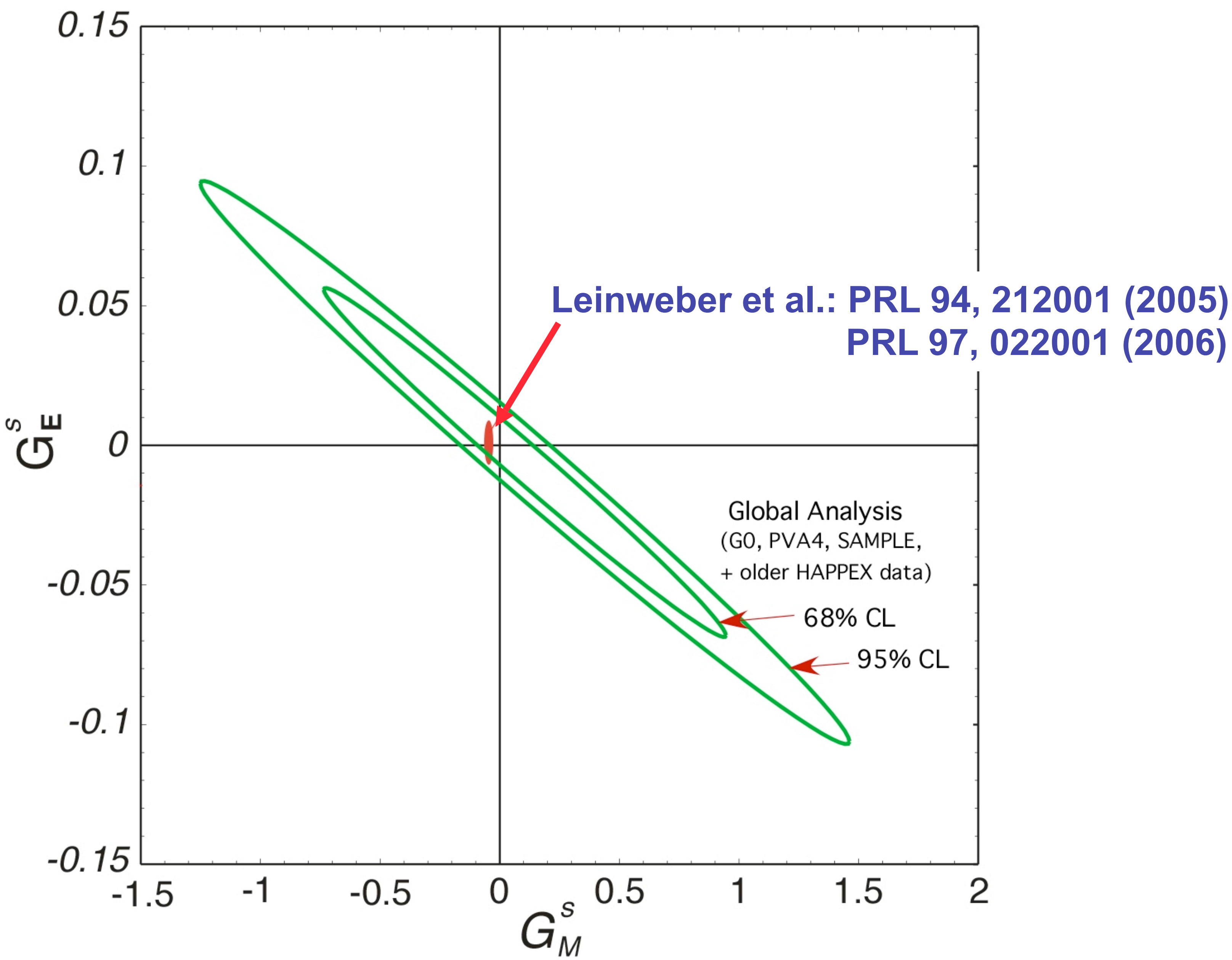

[G. T. Fleming]

# Electromagnetic Structure of Octet Baryons

- Magnetic moments and charge radii of the octet baryons in the quenched approximation [Boinepalli et. al. '06]

- Magnetic moments of baryon octet and decuplet using the background field method in the quenched approximation [Lee et. al. '05].

# Strange Form Factors




Slide contributed by A. Thomas (Young et.al. Phys.Rev.Lett. 97, '06)

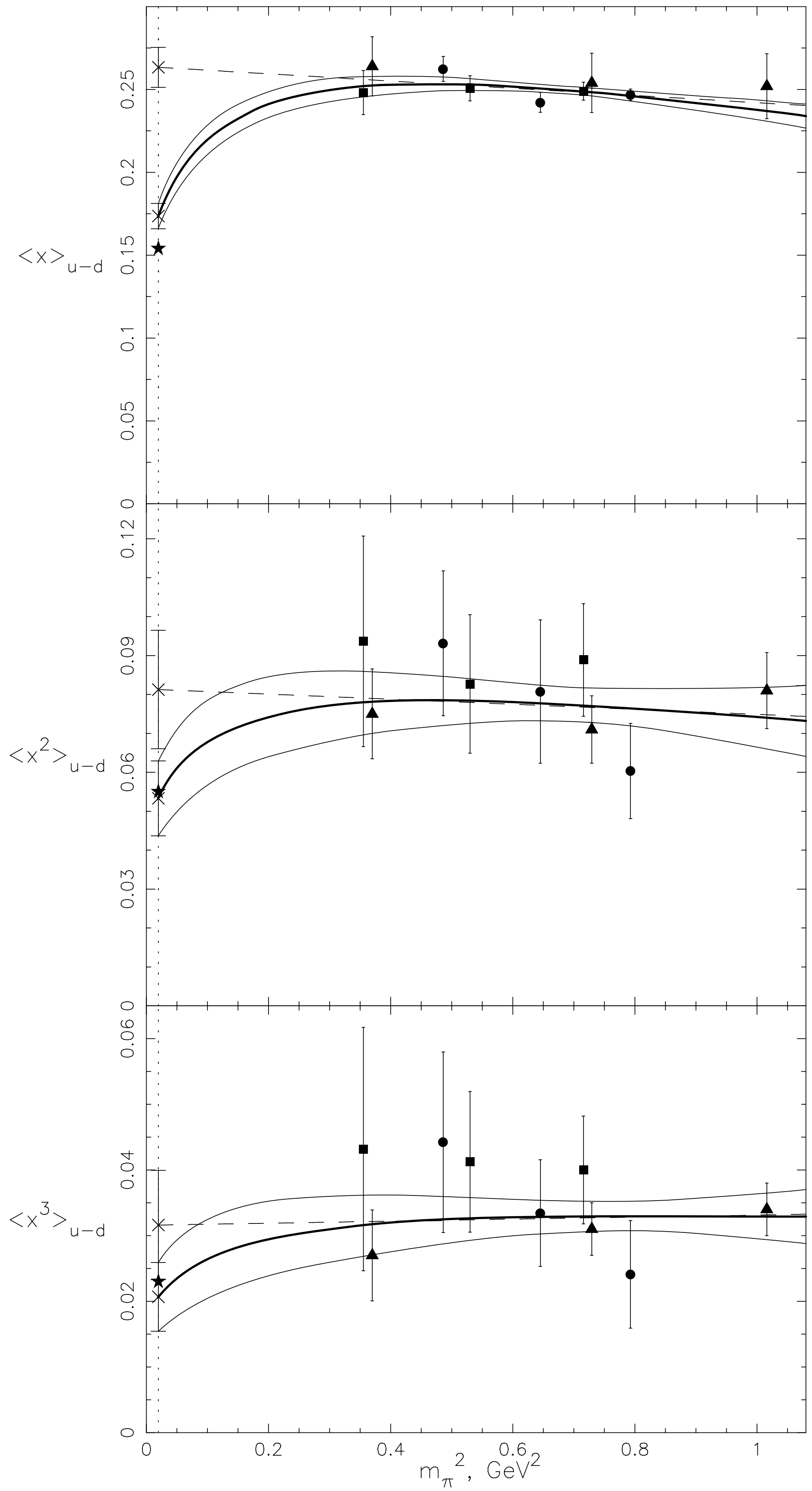
# Form Factors (future)

- Understand continuum and chiral extrapolations
- Low $Q^2$ : Comparison with experiment provides a benchmark
- Predict the High $Q^2$ behavior
- Computation of flavor singlet form factors (proton and neutron)
  - This requires the next generation of computational resources
  - Exploratory studies can be done now
  - New algorithmic developments could provide further aid
- From factors of resonances (Decouplet baryons, Ropper etc.)
- Transition form factors
  - N- Delta already done in quenched and heavy dynamical quarks

[Alexandrou et.al]

# Unpolarized moments of Structure Functions

(year 2002)




LHPC-SESAM:

diamonds - quenched,

squares - dynamical

QCDSF:

quenched - triangles

[hep-lat/0201021]

$$\langle x\rangle_{u-d} \sim a_1\left[1-\frac{(3g_A^2+1)m_\pi^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_\pi^2}{m_\pi^2+\mu^2}\right)\right]+b_1 m_\pi^2.$$

Where $\mu = 550 MeV$

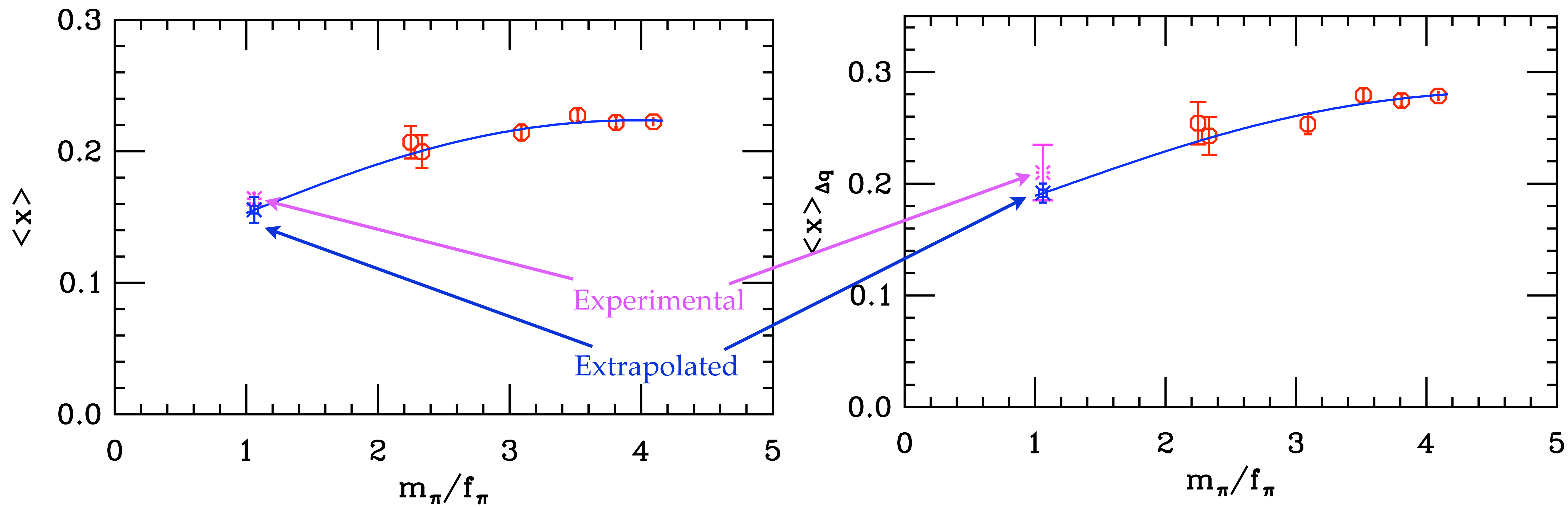
The log coefficient is valid for full QCD

[Detmold et.al. Phys.Rev.D87 2001]

# First Moments of Structure functions (present) [LHPC]

Domain Wall Valence, Staggered Sea (MILC)

μ = 2 GeV



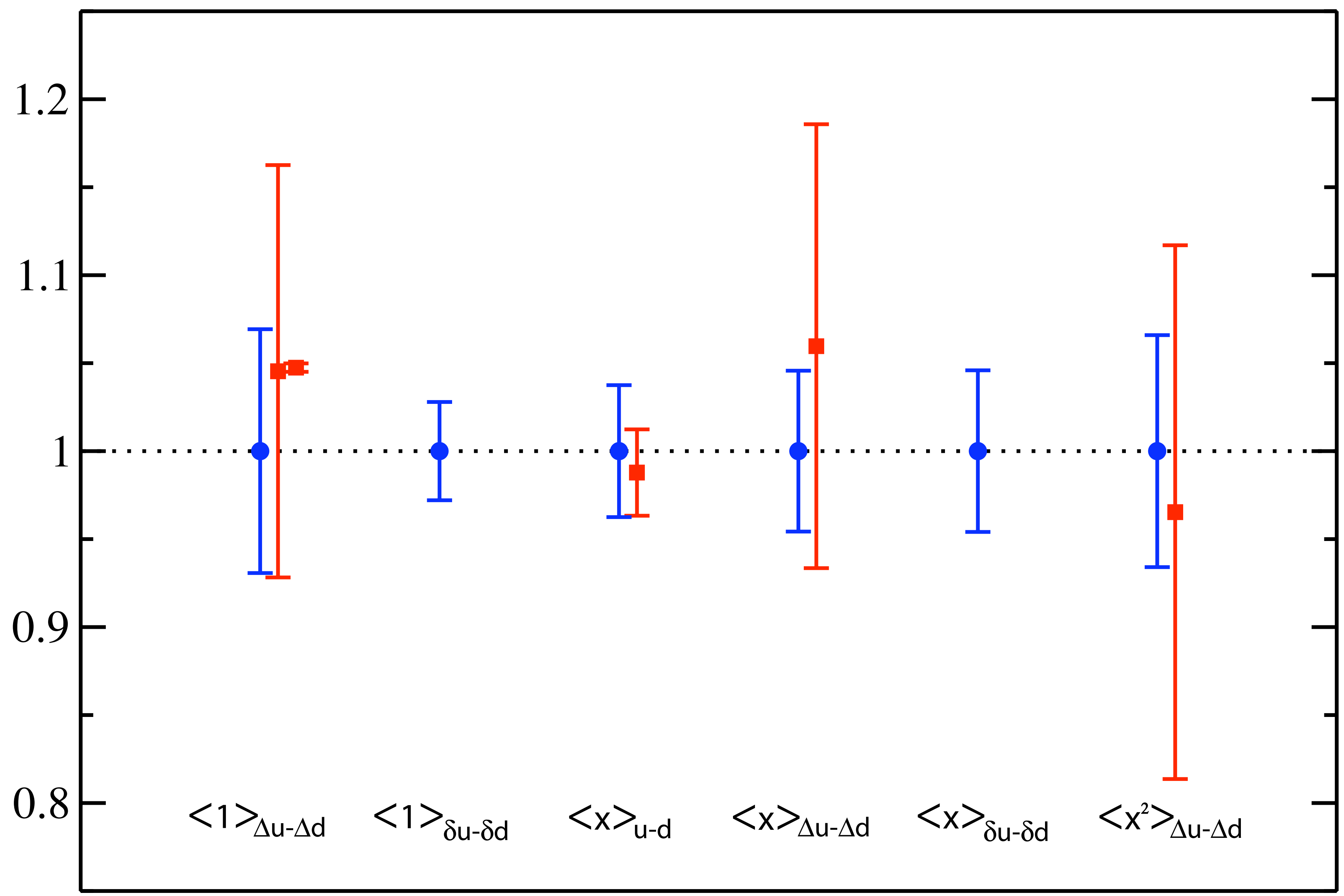

[Renner Lattice '06]

- Hint of curvature just below 400MeV
- Isovector moments only
- Fit to leading ChiPT keeping the mass dependence of $g_A$
- Purturbative renormalization [Bistrovic '05 (thesis)]

$$\langle x \rangle_{u-d} = a\left[1 - \frac{3g_A^2+1}{8\pi^2}\left(\frac{m_\pi^2}{f_\pi^2}\right)\ln\left(\frac{m_\pi^2}{f_\pi^2}\right)\right] + c\frac{m_\pi^2}{f_\pi^2}$$

$$\langle x \rangle_{\Delta u-\Delta d} = a'\left[1 - \frac{2g_A^2+1}{8\pi^2}\left(\frac{m_\pi^2}{f_\pi^2}\right)\ln\left(\frac{m_\pi^2}{f_\pi^2}\right)\right] + c'\frac{m_\pi^2}{f_\pi^2}$$

# Moments of Structure functions
# Experiment vs LQCD




[Renner Lattice '06]

Need to include the Delta in the fits [Arndt and Savage, '02]

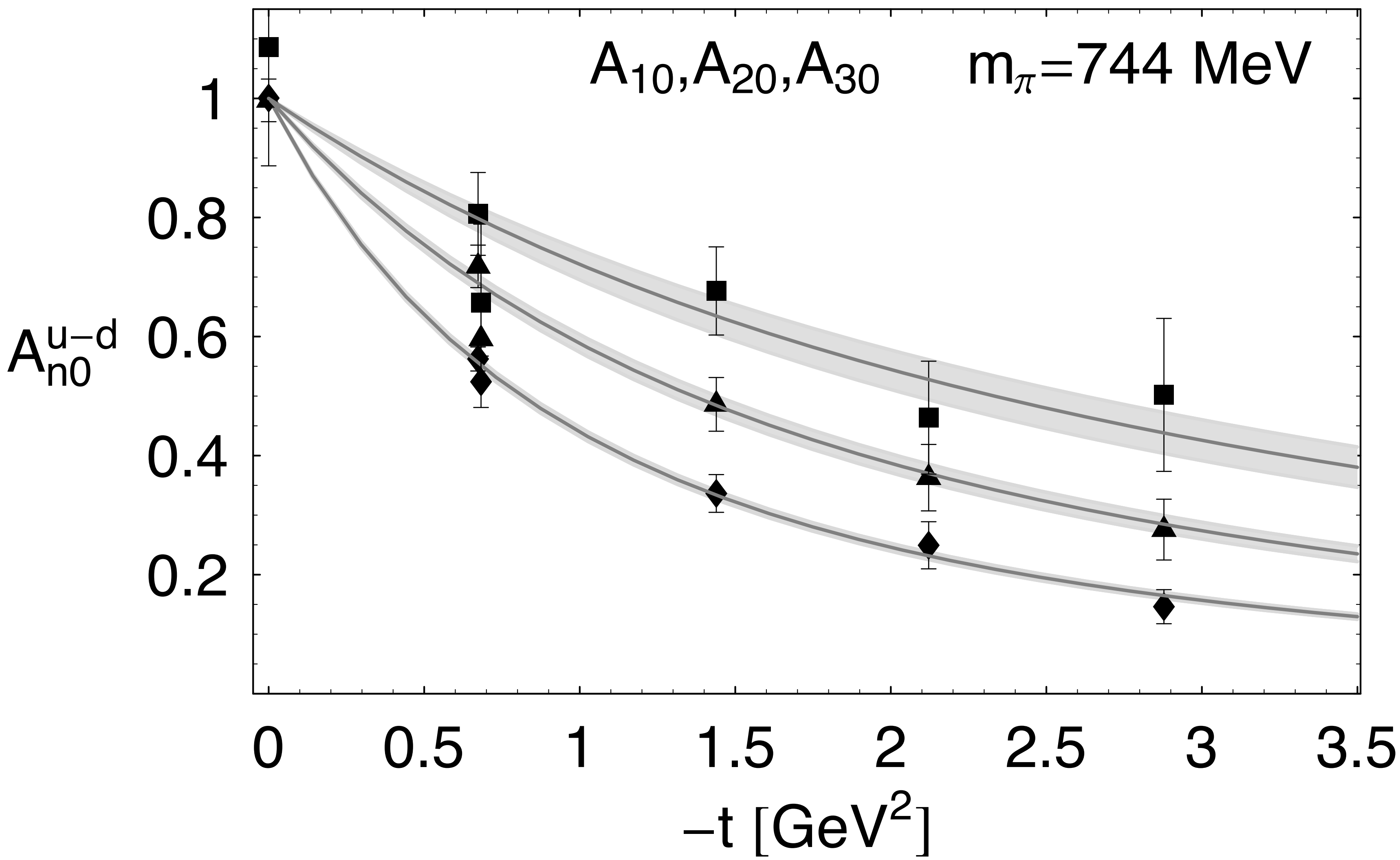
# Moments of Structure functions (future)

- Study systematics from chiral extrapolations
- Continuum Limit
  - Currently one lattice spacing (a=0.125fm)
  - Plan to do in the next few years two more (a=0.09fm and a=0.06fm)

Cost: ~ 5-6 Tflop-years

- Compute flavor singlet moments.
  - Next generation of resources (Cost: ~ 10 Tflop-years)
  - Exploratory studies near future
- New ideas
  - Go beyond the first few moments [**Detmold and Lin '05**]

# Moments of Generalized Parton Distributions [LHPC 2003]




- Heavy dynamical quarks
- Slope at small t decreases as we go to higher moments
- Higher moments dominated by higher x

# Transverse Structure

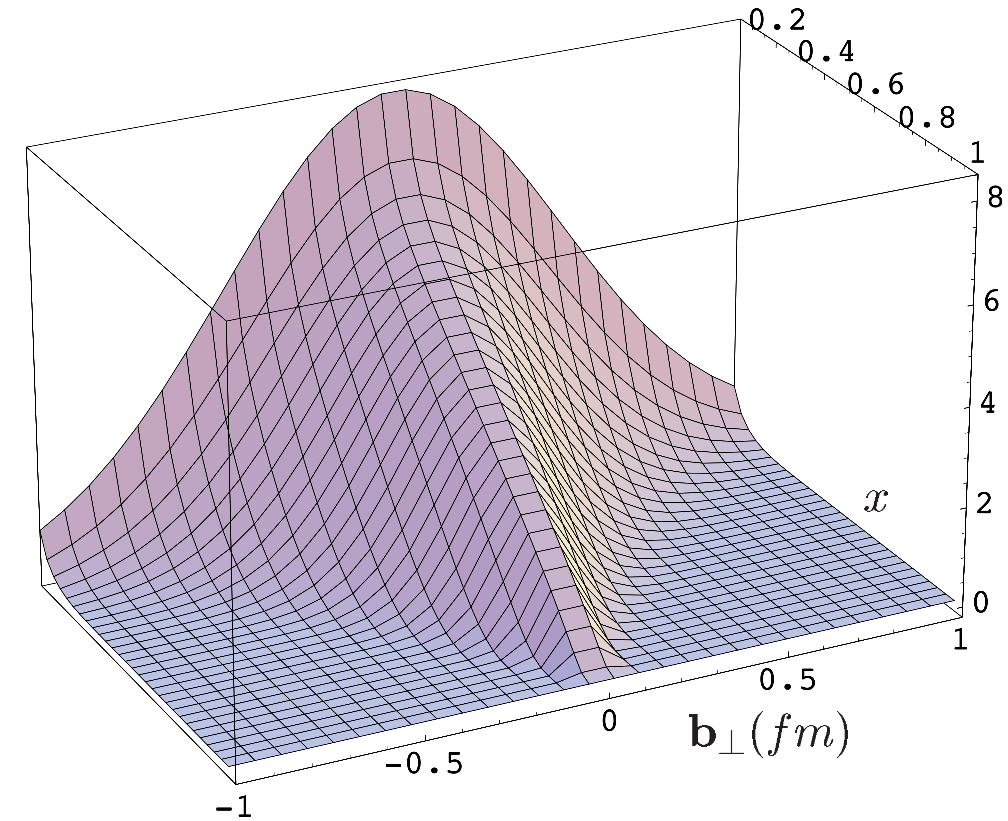
ξ=0 limit

$H(x, 0, t)$

- The generalized form factors have simple interpretation

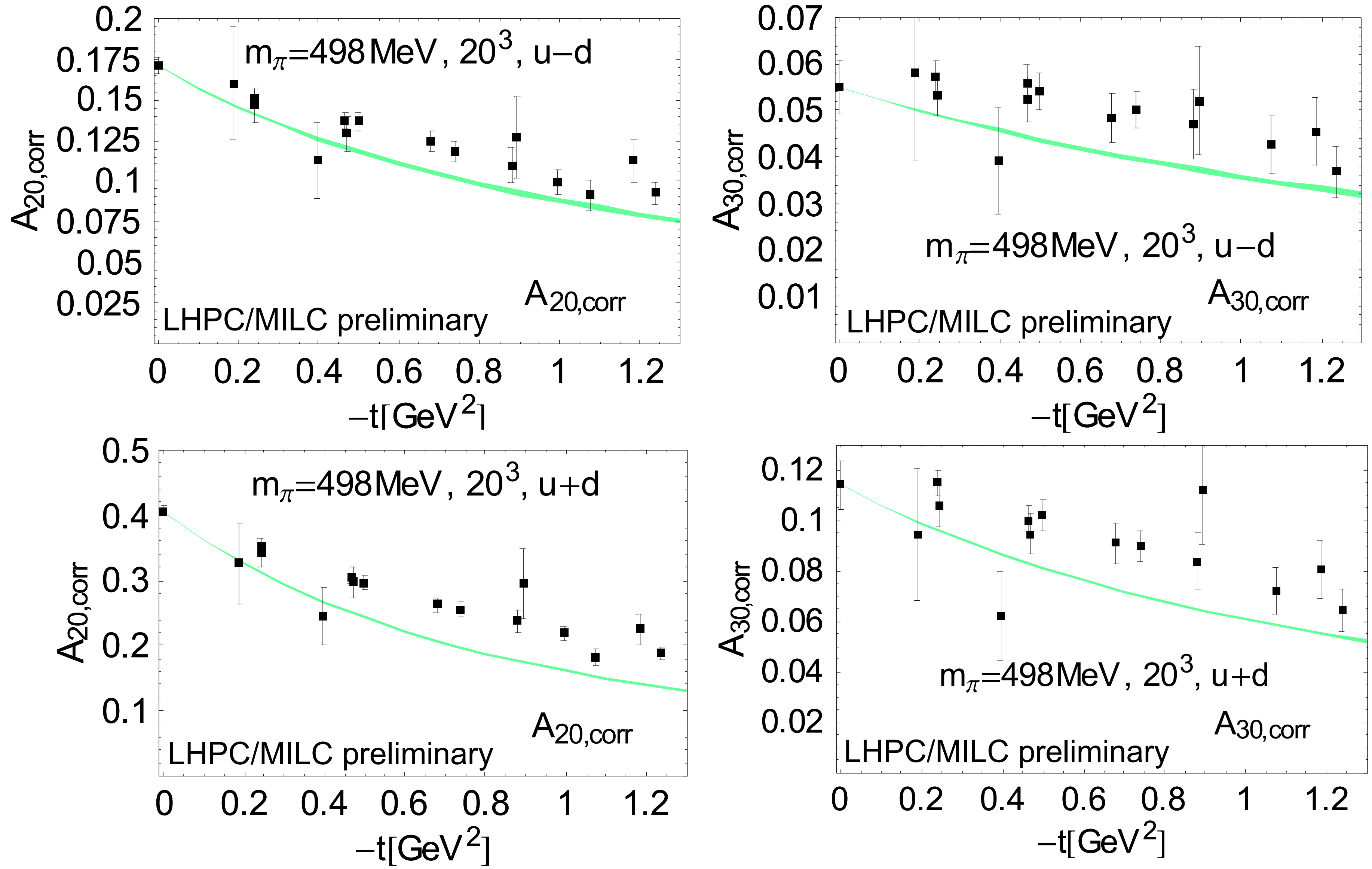
$$\int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\Delta_T \cdot b} \longrightarrow \rho(x, b)$$

- x ⟶ 1 ρ is a delta function




[Burkardt hep-ph/0207047]

# Moments of Generalized Parton Distributions (present)




- Lighter pion masses are attainable
- Green band is a phenomenological parameterization using Form factor data, CTEQ patron distributions, and Regge Anzatz as input [Deihl et.al.]

# The Proton Spin

- What is the contribution of the quark spin to the spin of the proton?
  - EMC (1988) $Q^2 = 10\ GeV^2$ $\Delta\Sigma = 0.00(24)$
  - SMC (1998) $Q^2 = 5\ GeV^2$ $\Delta\Sigma = 0.13(17)$
- Quarks contribute nothing to the spin of the proton!
- Spin crisis

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

# The Proton Spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_z \quad \text{or} \quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_z^q + J_z^g$$

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

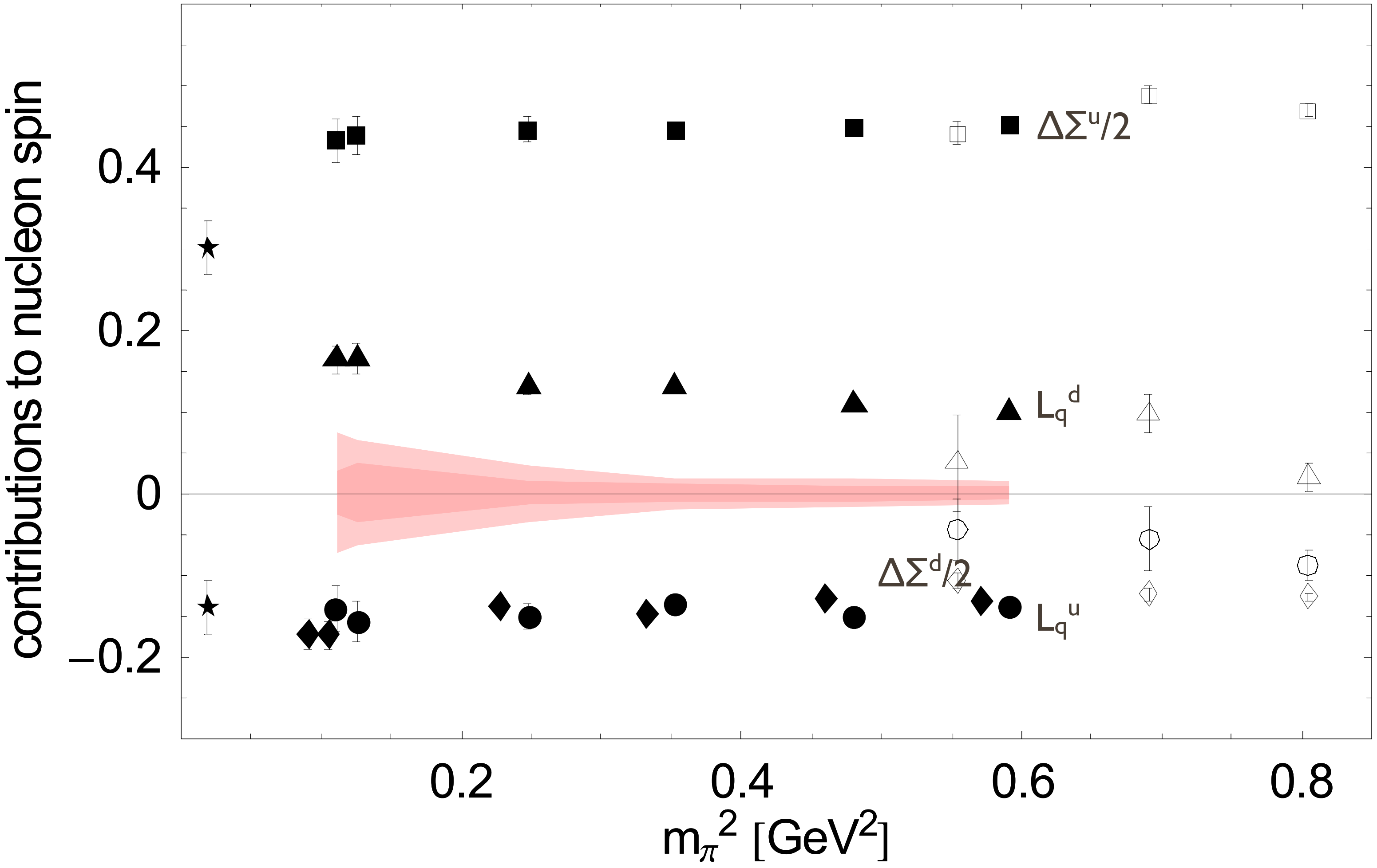
- The quark angular momentum can be computed on the lattice
- $$J_z^q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$$ [Ji,'98]
- A and B are the generalized form factors of

$$\mathcal{O}_{\mu\nu} = \bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q = T_{\mu\nu}$$

[Mathur et. al. '00] [LHPC: Hagler et. al. '03] [QCDSF: Hagler et. al. '03]

# Proton SPIN

μ = 2 GeV




- Quark orbital angular momentum almost zero due to cancelation of the up and down contributions
- Disconnected diagrams are missing

# GPDs (future)

- Study systematics from chiral extrapolations
- Continuum Limit
  - Currently one lattice spacing (a=0.125fm)
  - Plan to do in the next few years two more (a=0.09fm and a=0.06fm)
    Cost: ~ 5-6 Tflop-years
- Compute flavor singlet moments.
  - Next generation of resources (Cost: ~ 10 Tflop-years)
  - Exploratory studies near future
- New ideas
  - Go beyond the first few moments [**Detmold and Lin '05**]

# Hadronic Interactions

- Effective field theory description of few nucleon systems
- Use lattice QCD to extract the low energy constants needed
  - Decay constants: $f_\pi$, $f_K$
  - Axial couplings: $g_A$, $g_{N\Delta}$, $g_{\Sigma\Sigma}$, $g_{\Xi\Xi}$, $g_{\Sigma\Lambda}$, ...
  - Scattering lengths: NPLQCD
- Lattice Nuclear physics [Lee et al., Borasoy et al.]
- Lattice offers flexibility!
  - Study quark mass dependence
  - Compute experimentally inaccessible quantities (Hyperons)

# Hadronic Interactions

- Scattering processes from Lattice QCD are not straight forward
- Miani-Testa no-go theorem ('90) [and C. Michael '89]
- Infinite Volume:

  Euclidean $\not\longrightarrow$ Minkowski

- Finite volume: discrete spectrum
  - Avoids Miani-Testa no-go theorem [M. Luscher]

# Luscher Formula

Energy level shift in finite volume:

$$\Delta E_n \equiv E_n - 2m = 2\sqrt{p_n^2 + m^2} - 2m$$

$p_n$ solutions of:

$$p \cot\delta(p) = \frac{1}{\pi L}\mathbf{S}\left(\frac{p^2L^2}{4\pi^2}\right)$$

$$\mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} - 4\pi\Lambda$$

$$p_n \cot\delta(p_n) = \frac{1}{a} + \cdots$$

$$\frac{1}{a} = \frac{1}{\pi L} S\left(\frac{p_0^2L^2}{4\pi^2}\right) + \cdots$$

Expansion at p~0 :

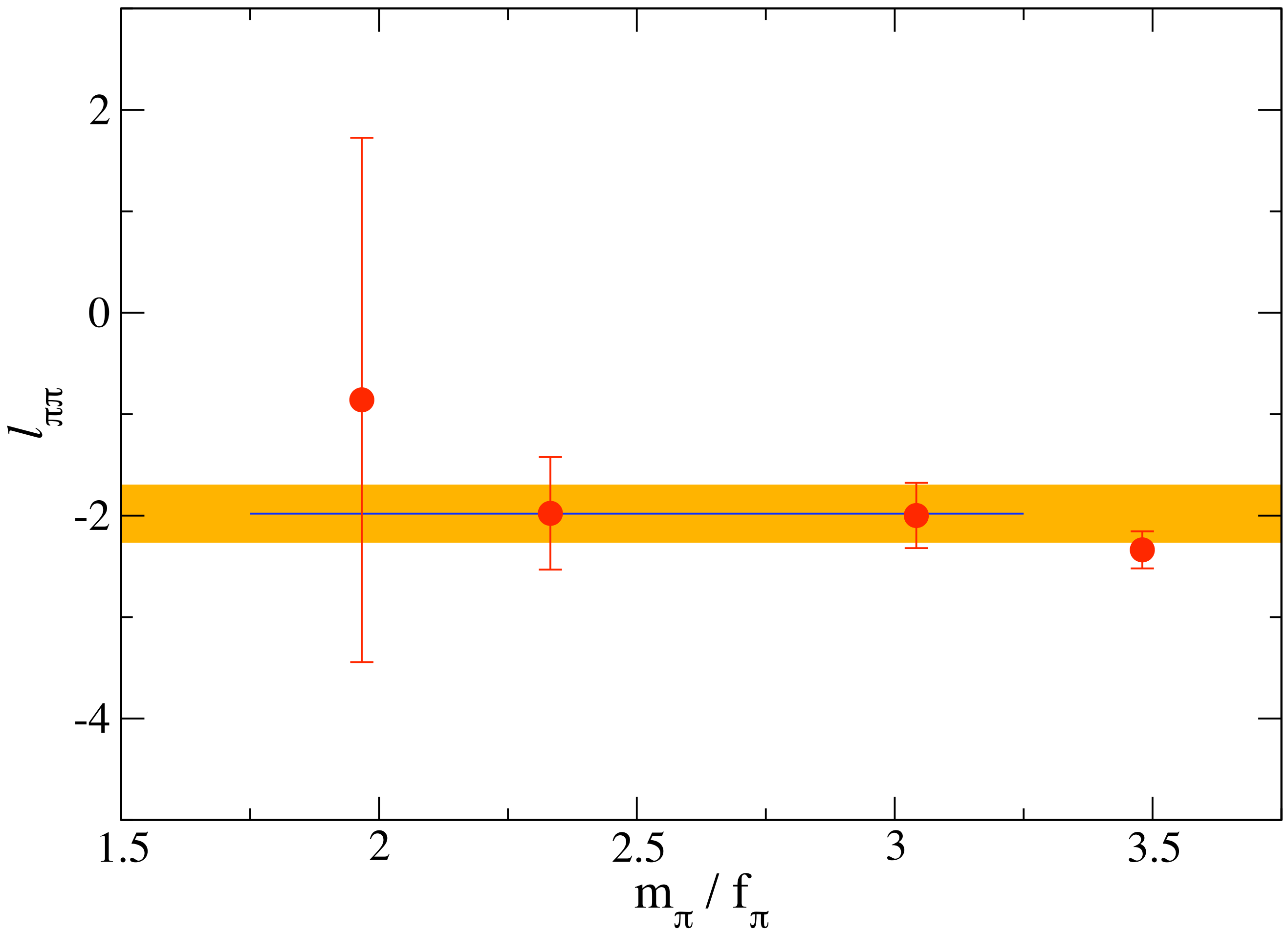
$$\Delta E_0 = -\frac{4\pi a}{mL^3}\left[1 + c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2\right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

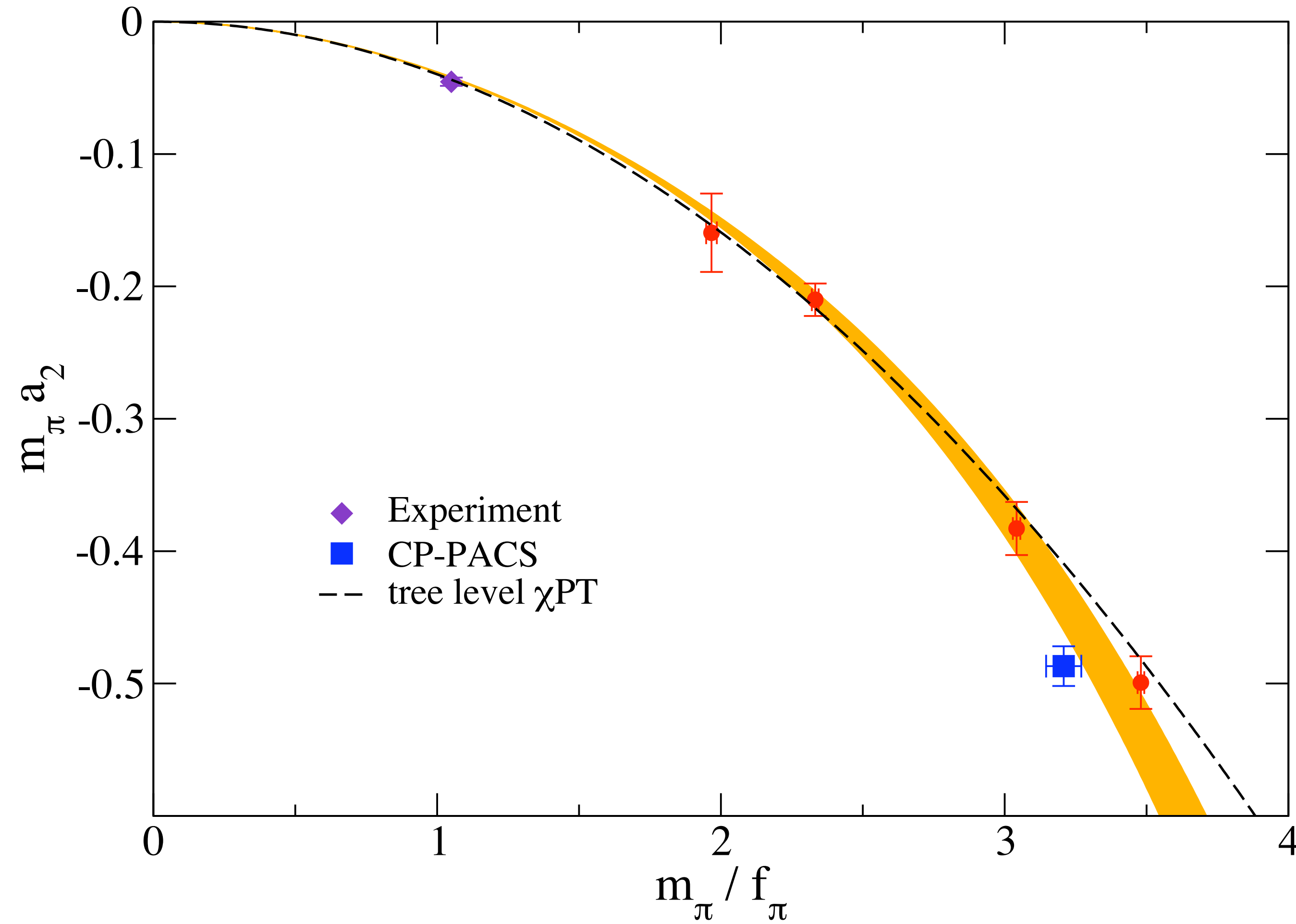
$a$ is the scattering length

$c_1$ and $c_2$ are universal constants

# I=2 Pion Scattering

NPLQCD



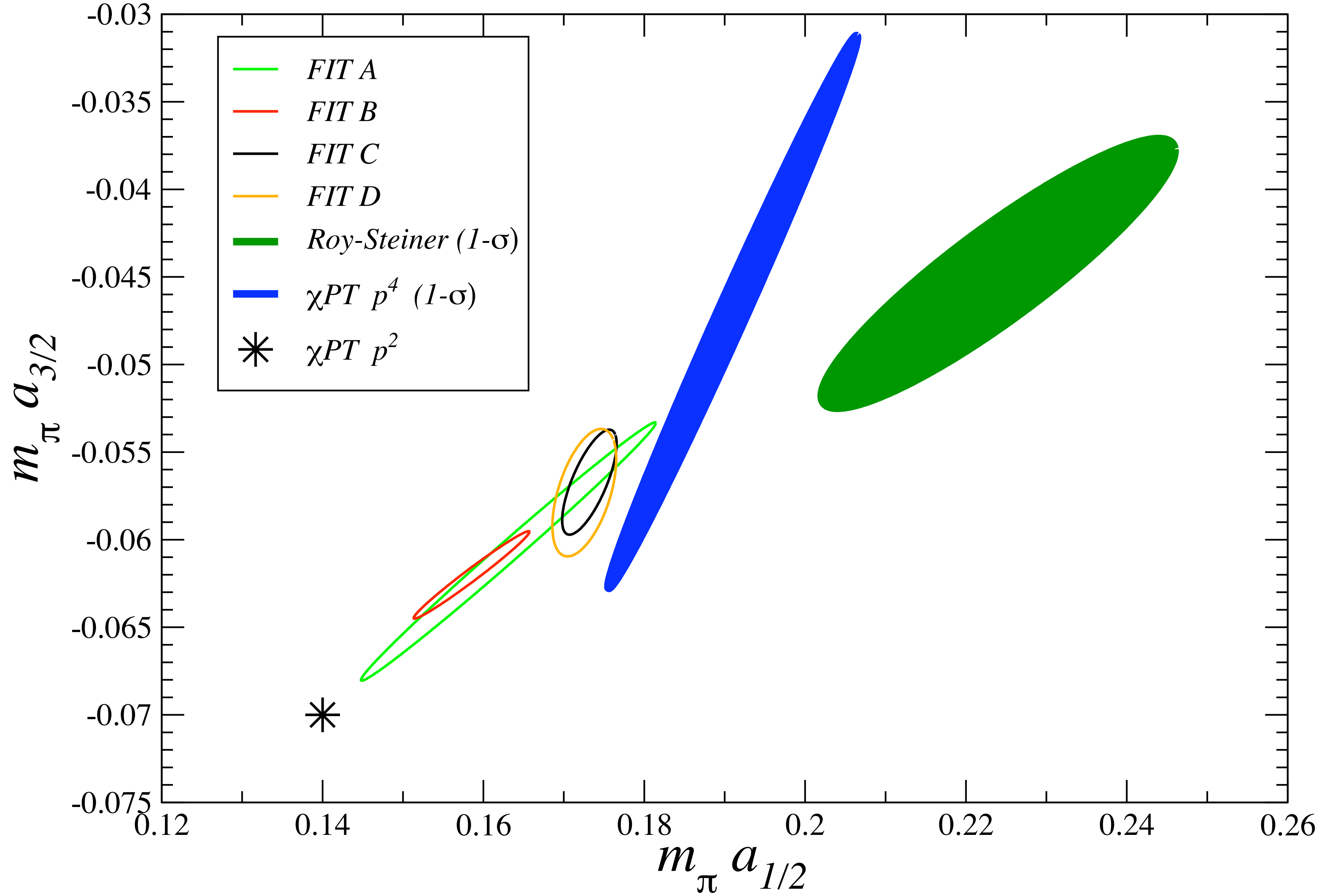

$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[1 + \frac{3m_\pi^2}{16\pi^2 f_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + l_{\pi\pi}(\mu)\right)\right]$$

[Gasser-Leutwyler ‘84]
[Colangelo et al. ‘01]

- $m_\pi$ $a_2$ = -0.0422(3)(18)
- Experiment: $m_\pi$ $a_2$ = -0.0454(31)
- SχPT has insignificant effect to the result [Chen et al. ‘05]

# Kaon Pion Scattering Lengths

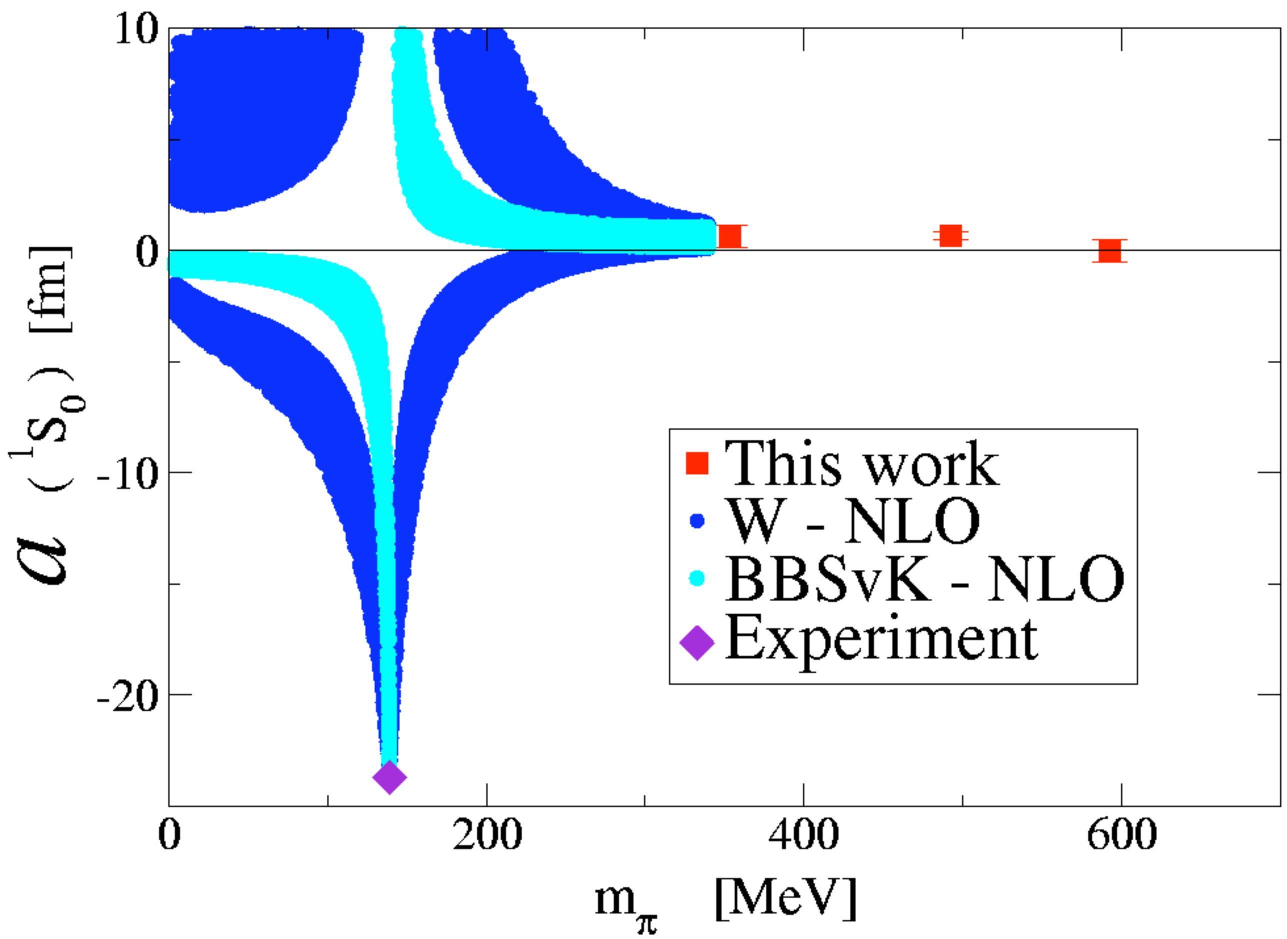
NPLQCD

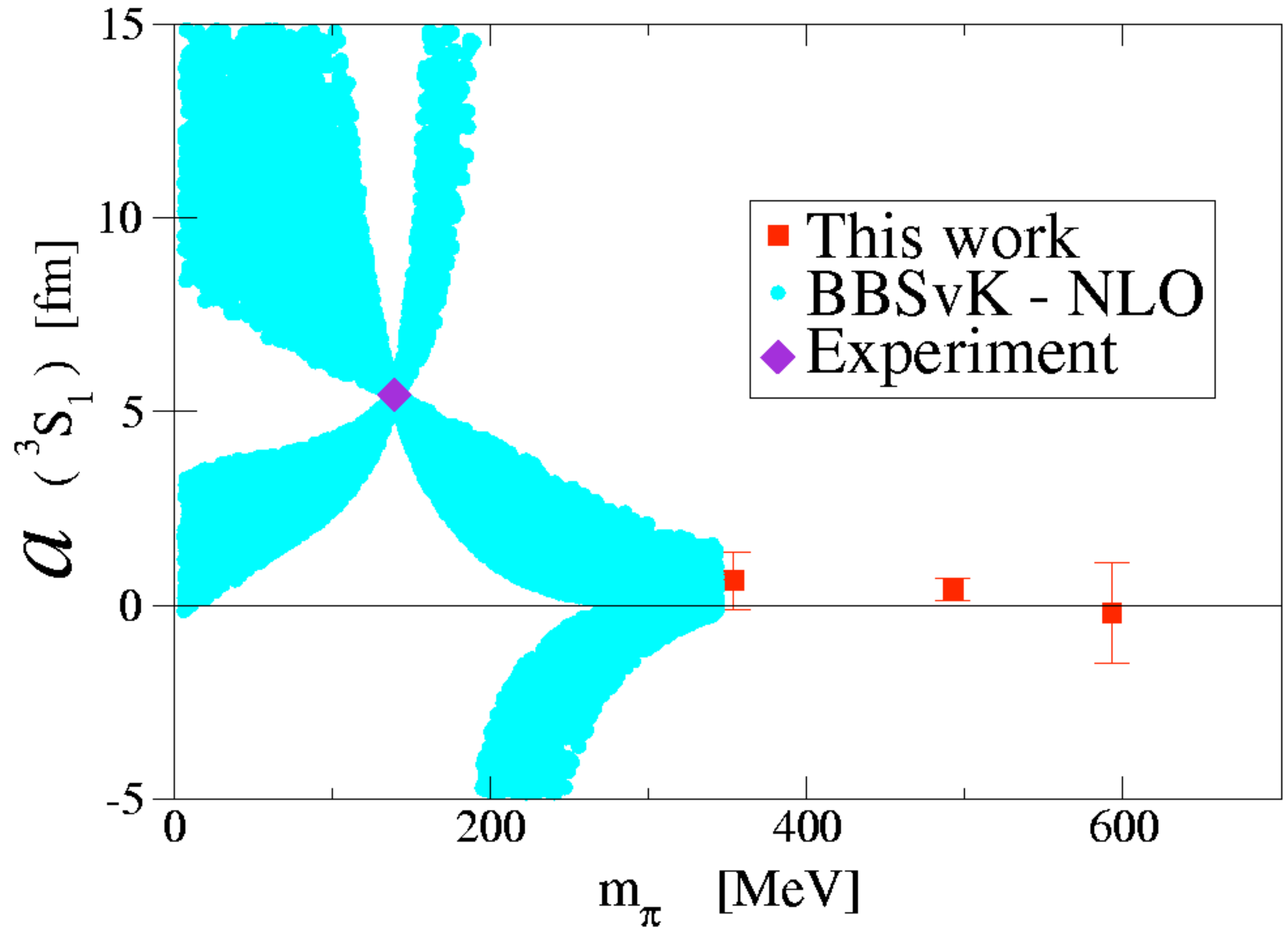



- Upcoming experiments on Kaon - pion molecules (DIRAC collab.)
- Continuum extrapolation still needed

# Nucleon-Nucleon

NPLQCD: **Phys.Rev.Lett.97 2006**



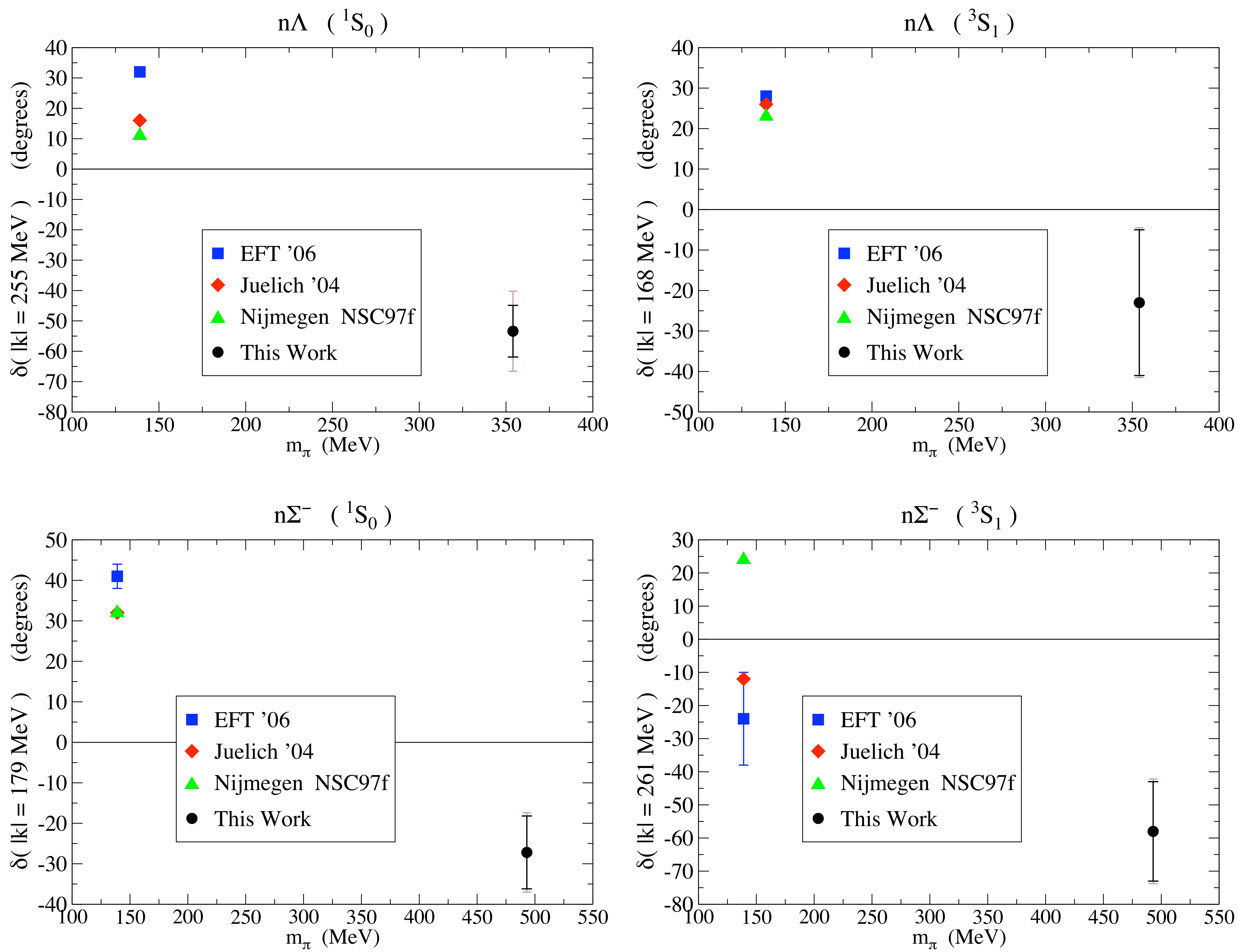




BBSvK: Beane Bedaque Savage van Kolck '02
W: Weinberg '90;Weingberg '91; Ordonez et.al '95

Fukugita et al. '95: Quenched heavy pions

# Nucleon-Hyperon

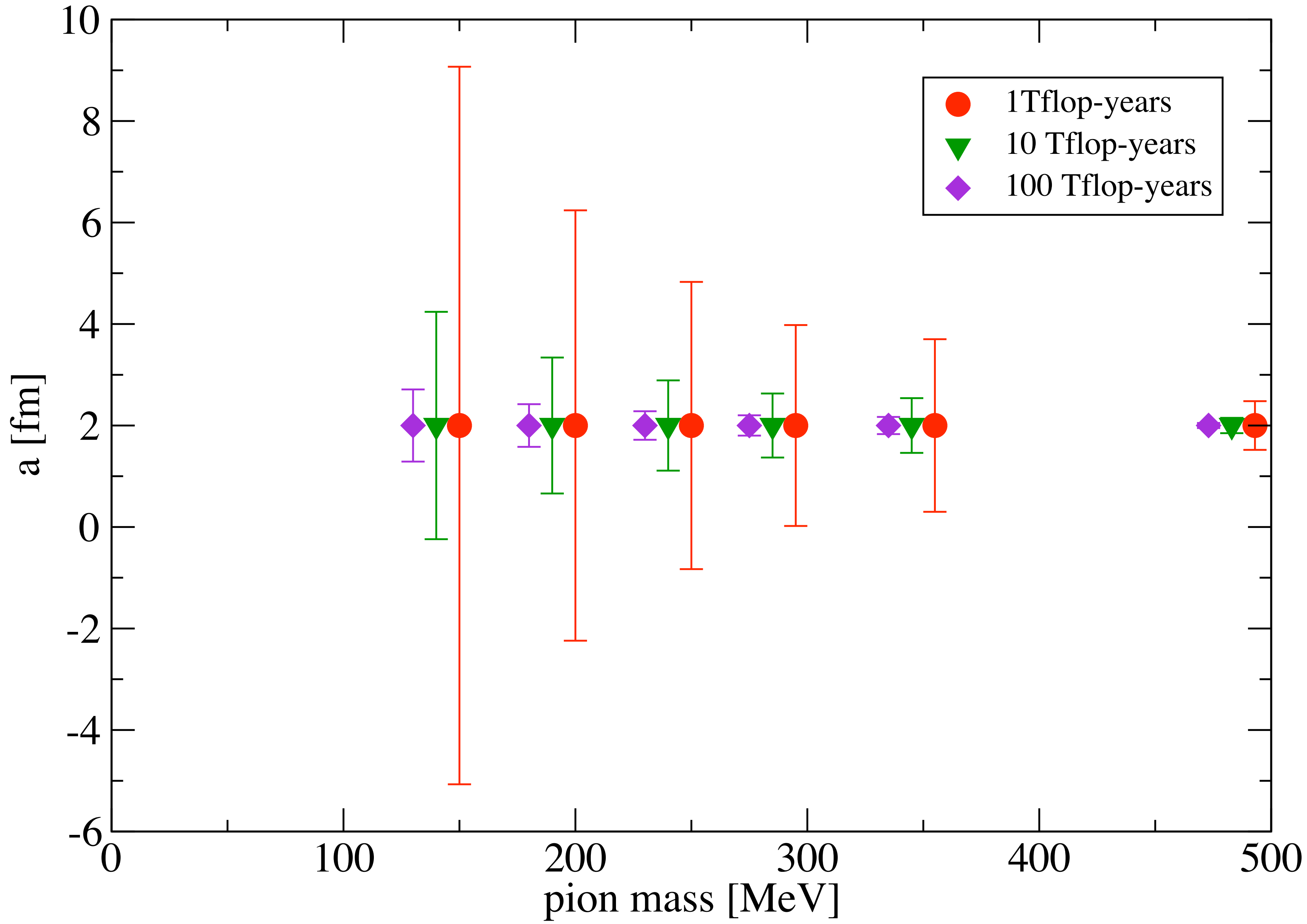
NPLQCD: **hep-lat/0612026**



# Hadronic interactions (Future)

- These calculations are the beginning of the beginning!
- Need lighter pion masses, multiple volume sizes, and lattice spacings
  - Determine if we see scattering states
- Meson baryon channels: (K-n, K-Σ ...) ----- Neutron stars
- Hyperon-Hyperon and Hyperon-Nucleon channels [NPLQCD: **hep-lat/0612026**]
  - Hyper-nuclear physics and Neutron stars
- Need to make lattices designed for this project
- Higher statistics: (JLAB spectrum program -- INCITE recent award)

# Hadron Interactions: Projected errors




- Errors on scattering nucleon-nucleon scattering length as function of computational resources
- Only cost for correlation function calculation presented

# Conclusions

- Lattice QCD is a mature field
  - Direct comparison with experiment is now possible for several observables
  - Predictions are now emerging
- Extraction of GPDs: Synergy between experiment, Lattice and phenomenology
- Petaflop computing is around the corner
  - Will allow us to perform precision calculations for spectrum, form factors and GPDs
  - Will takes us long ways in understanding the nuclear force

# Polarizabilities

- Compute the response of the hadron in external electric or magnetic field
- Use the external field method
- Electric polarizabilities of neutral hadrons [Christensen, Wilcox, Lee '04 ]
- Magnetic polarizabilities of hadrons [Lee, Zhou, Wilcox, Christensen '05 ]
- Spin polarizabilities and electric polarizabilities of charged particles proposed to be measured [Detmold, Tiburzi, Walker-Loud '06]