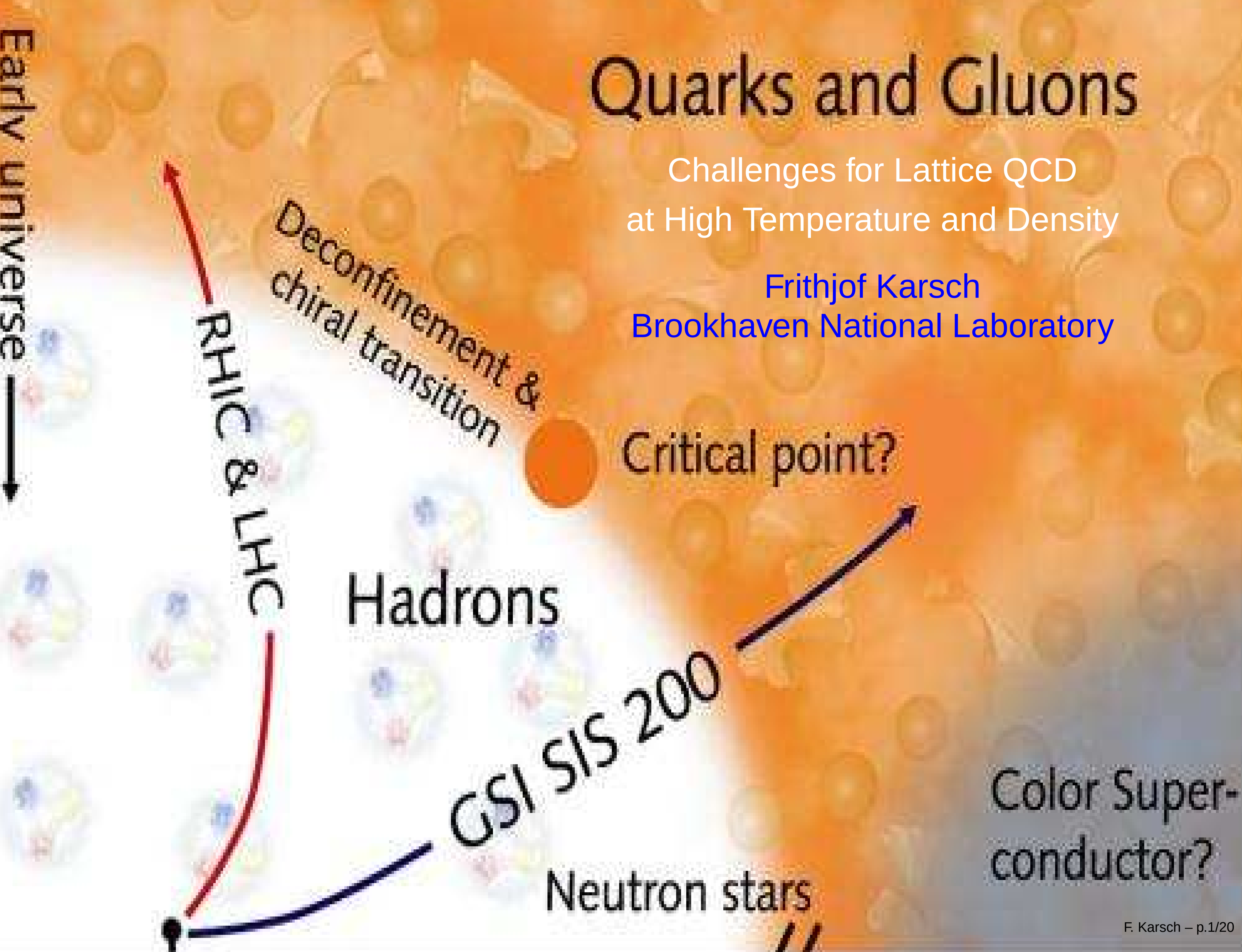


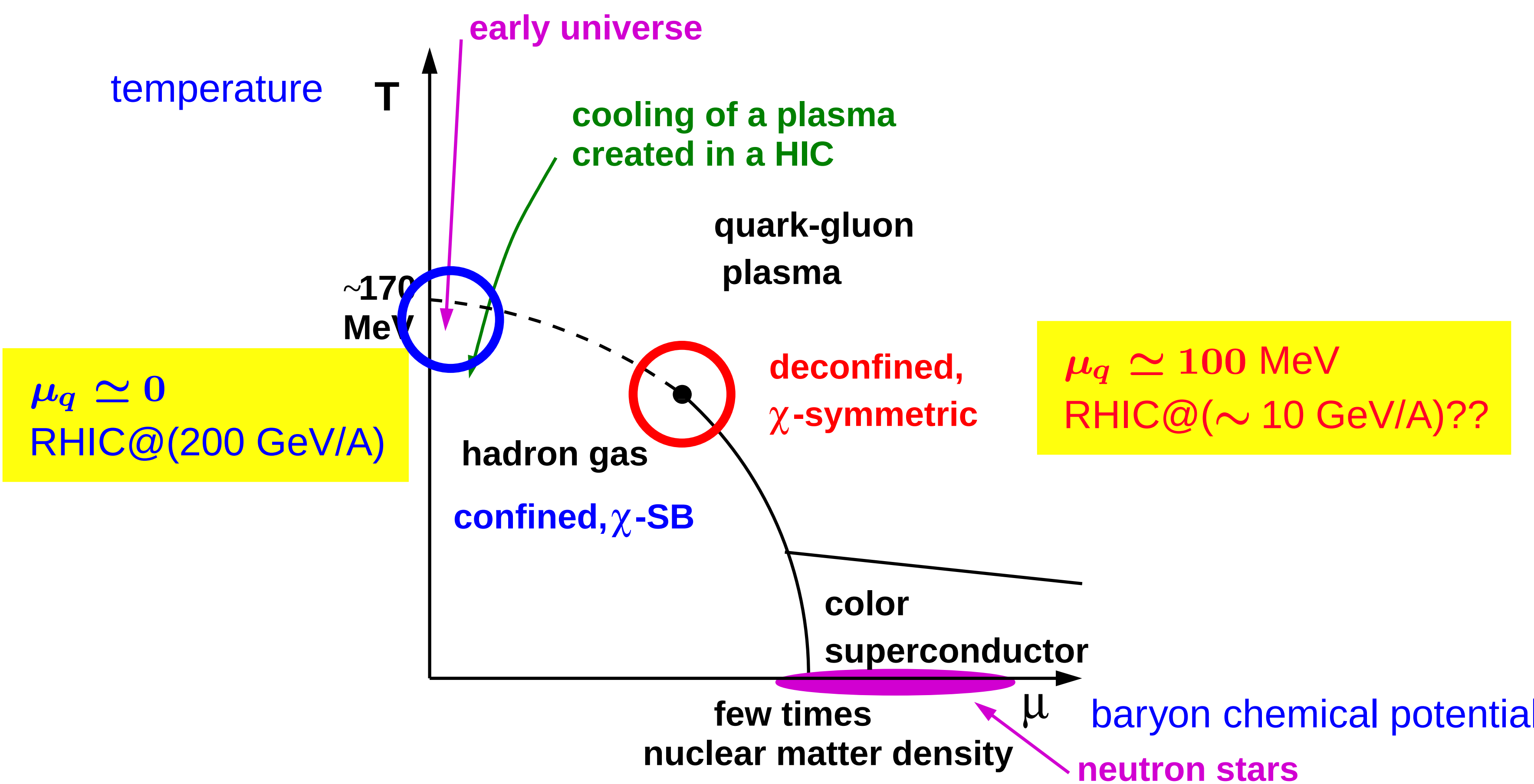
Challenges for Lattice QCD at High Temperature and Density

Frithjof Karsch
Brookhaven National Laboratory

Phase diagram of strongly interacting matter

RHIC I/II & LHC ⇔ LGT at vanishing chemical potential

RHIC at low energy ⇔ LGT at non zero chemical potential



QCD Thermodynamics 2007-2012: What do we want to know?

I. T_c, ϵ_c, EoS:

- basic input to hydrodynamic modeling of heavy ion collisions; confront models with lattice calculations (resonance gas, quasi-particle gas, high-T pert. theory, HTL-resummation, AdS/CFT ...) test universal aspects of deconfinement and chiral symmetry restoration in $2,\ (2+1)$-flavor QCD

II. search for the critical point at $\mu > 0$:

- chiral critical point: does it exist?; location of the chiral critical point; direct evidence for $1^{\rm st}$ order regime; location of the transition line for $\mu > 0$; density fluctuations; $T_c(\mu) \equiv T_{\rm freeze}$?

III. In-medium properties of hadrons

- quarkonium spectroscopy and heavy quark diffusion
- light quark bound states and thermal dilepton/photon rates

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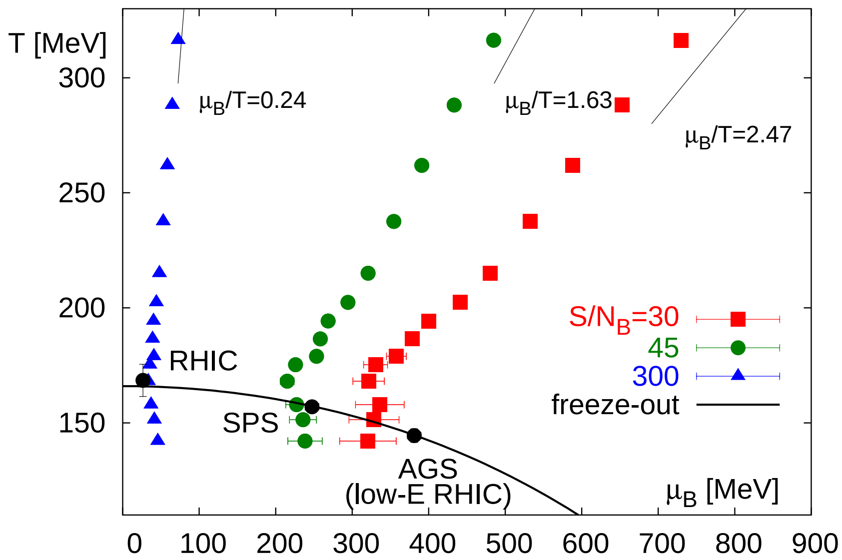
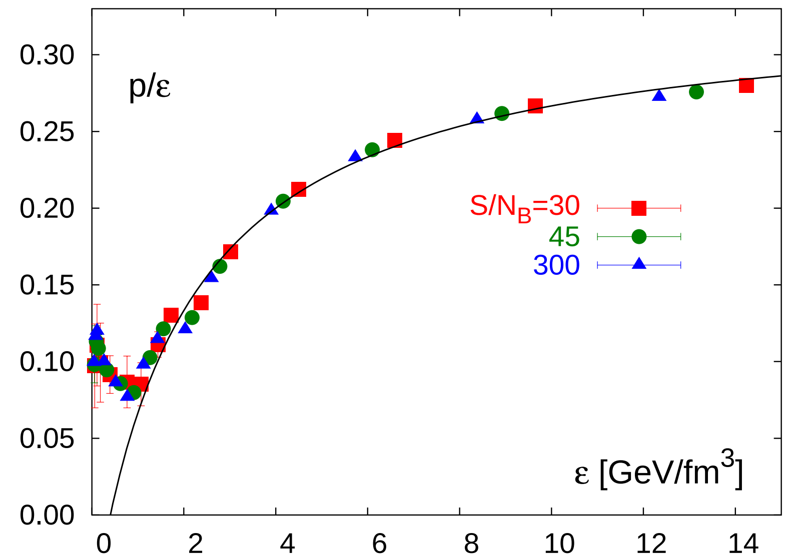
  test universal aspects of deconfinement and chiral symmetry restoration in $2,\ (2+1)$-flavor QCD

Isentropic Equation of State: p/ϵ

S. Ejiri, F. Karsch, E. Laermann and C. Schmidt, Phys. Rev. D73 (2006) 054506





- p/ϵ vs. ϵ shows almost no dependence on S/N_B
- softest point: $p/\epsilon \simeq 0.075$
- phenomenological EoS for $T_0 \lesssim T \lesssim 2T_0$

$$\frac{p}{\epsilon} = \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\ \epsilon\ \mathrm{fm}^3/\mathrm{GeV}}\right)$$

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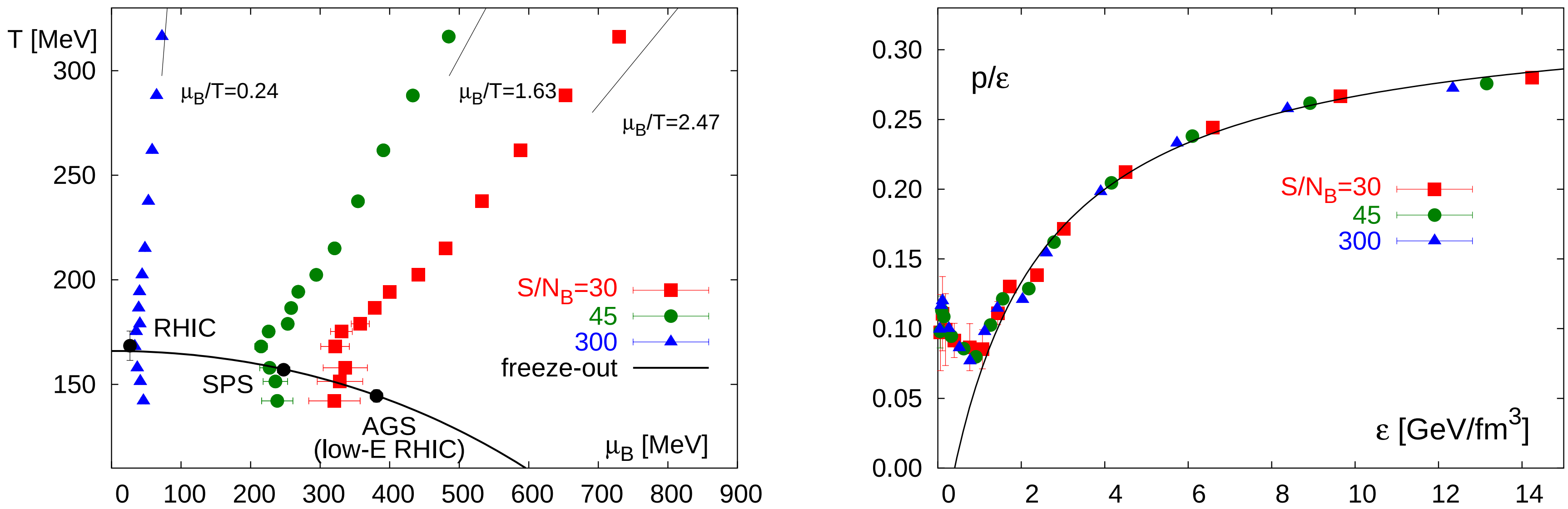

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- softest point: $p/\epsilon \simeq 0.075$
- phenomenological EoS for $T_0 \lesssim T \lesssim 2T_0$

so far analyzed only for $m_\pi \simeq 770$ MeV

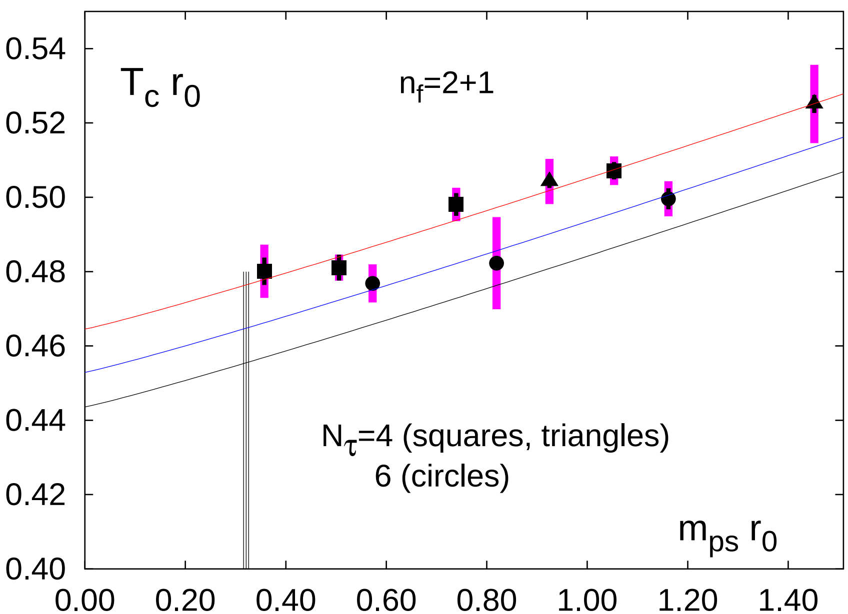
$$\frac{p}{\epsilon} = \frac{1}{3}\left(1 - \frac{1.2}{1 + 0.5\ \epsilon\ \mathrm{fm}^3/\mathrm{GeV}}\right)$$

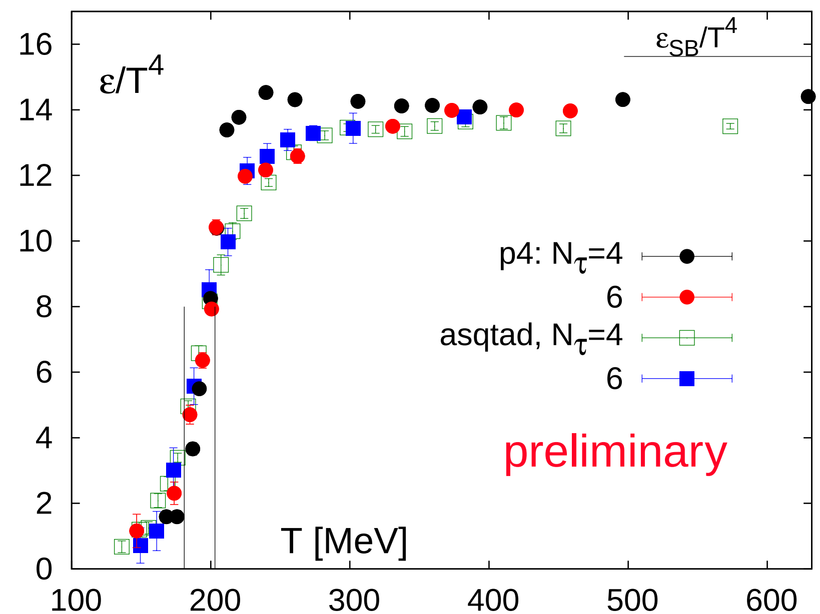
awaits confirmation in (2+1)-flavor QCD with light quarks

Transition Temperature; QCD Equation of State

N. H. Christ et al. (RBC-Bielefeld collaboration), Phys. Rev. D74, 054507 (2006)
F. Karsch, for RBC-Bielefeld collaboration, presented at Quark Matter 2006
MILC, hep-lat/0611031

- equation of state with almost physical light and strange quark masses
- extrapolation to chiral and continuum limit





$\Rightarrow \quad r_0 T_c = 0.456(7)^{+3}_{-1}$

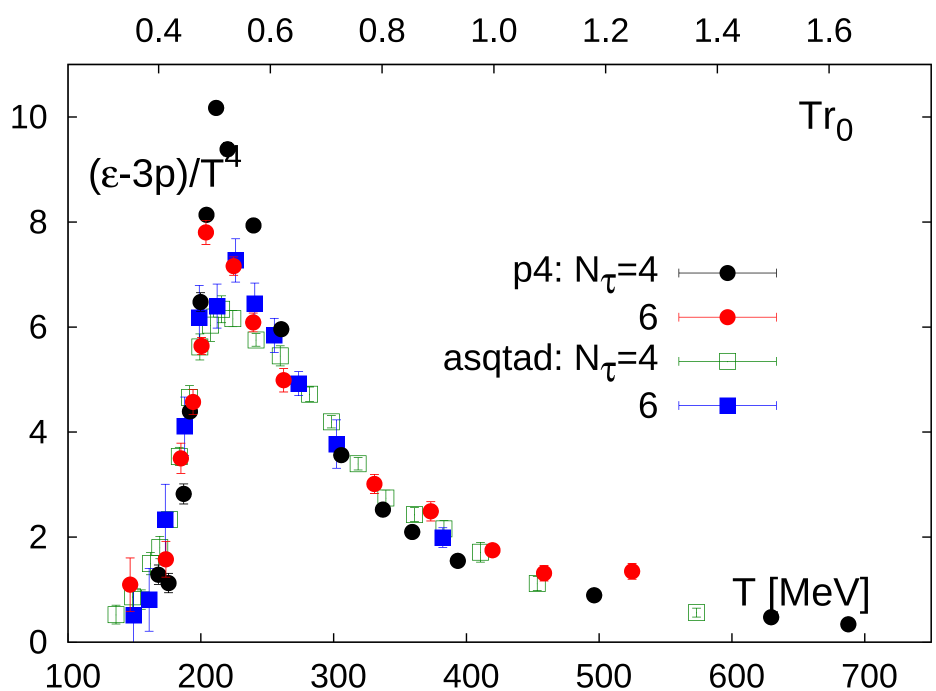
$\Rightarrow \quad T_c = 192(7)(4)$ MeV

weak cut-off dependence at T_c

$\epsilon_c/T_c^4 \simeq 6 \Rightarrow 1$ GeV/fm^3

$(\epsilon - 3p)/T^4$ on LCP

- requires good control over $T > 0$ observables (action differences, chiral condensates) as well as an
- accurate determination of $T = 0$ scales AND β-functions



RBC-Bielefeld, (QM'06, preliminary)

$T < T_c$	make contact to hadron gas phenomenology
$T < 2T_c$	analyze large deviation from conformal limit
$T > 2T_c$	make contact to (resummed) perturbation theory

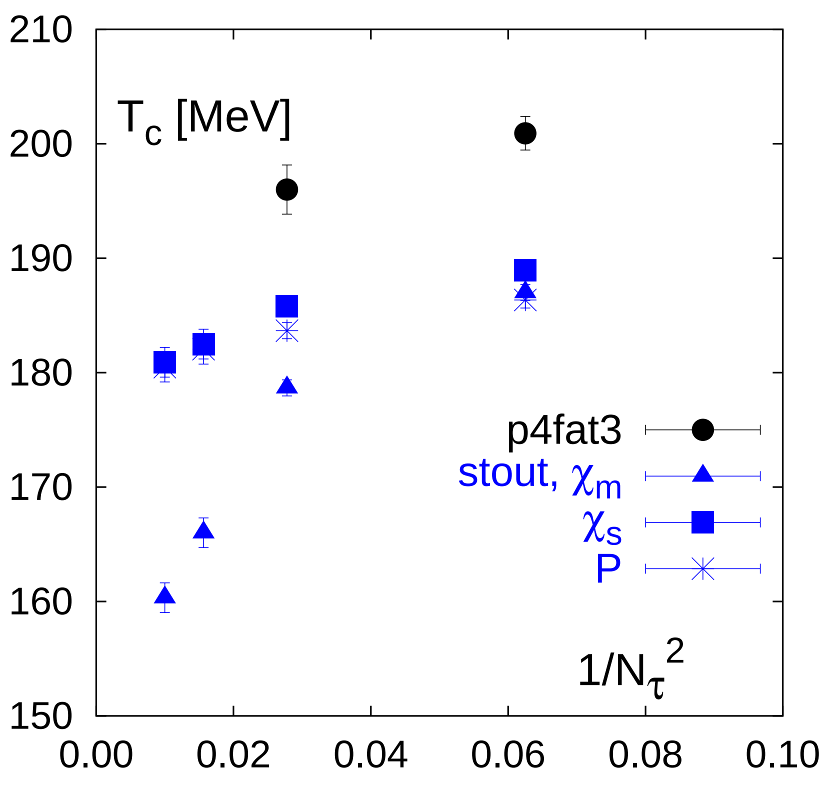
p4 vs. asqtad: overall good agreement

Note:
T-scale is not dependent on T_c determination

asqtad data:
C. Bernard et al., hep-lat/0611031

extrapolations to physical point

- RBC-Bielefeld (p4fat3 (p4)) vs. Wuppertal (stout (stand. staggered))
- results for $N_\tau = 4, 6$ differ by 15% but show similar cut-off dependence
- stout results for different observables no longer consistent with each other for $N_\tau = 8$, 10



overall scale set with $r_0 = 0.469$ fm

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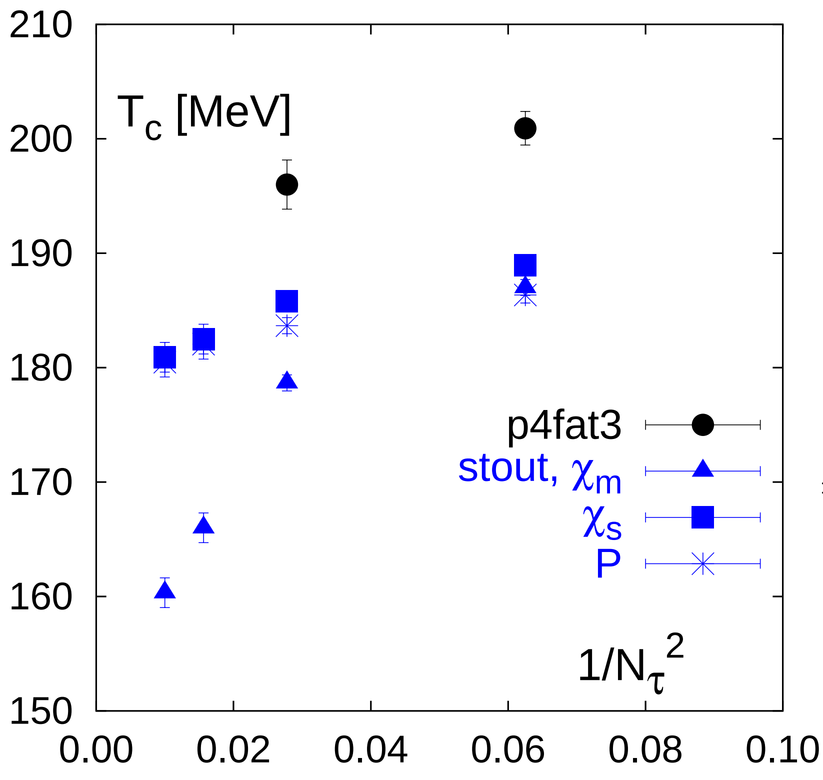
Drastic changes closer to the continuum limit?



- widely different crossover temperatures should also be reflected in the EoS
- current EoS at $T \simeq 150$ MeV: $\epsilon \simeq 65$ MeV/fm^3 $\simeq \epsilon_0$ ($\epsilon_0 \equiv \epsilon$ in uncompressed nuclear matter)

$\Rightarrow$ need to confirm results on EoS closer to the continuum limit

overall scale set with

$r_0 = 0.469$ fm

New projects on BlueGene/L

– 360TFLops at LLNL; 100TFlops at BNL –

Modeling the QCD equation of state closer to the continuum limit
joint project: BNL-RIKEN-Columbia, LANL, LLNL and MILC
collaboration on the Livermore BlueGene/L

- T_c, EoS on $N_\tau = 8$ lattices with light dynamical quarks:
  (2+1)-flavor QCD, close to physical m_π/m_K ratio;
  exploring the continuum limit: $N_\tau = 4,\ 6,\ 8$
  analyzing the thermodynamic limit: $V \simeq 500\ \mathrm{fm}^3$

  EoS on $32^3 \times 8$ lattices; CPU-time: $\sim$ (20-40) TFlops-years

- FUTURE: test universal properties, details of χ-symmetry restoration
  $\Rightarrow$ bulk thermodynamics with chiral fermions
  requires $\mathcal{O}(50)$ more computing resources
  $\Rightarrow$ PETAFLOPS computing

QCD Thermodynamics 2007-2012:
What do we want to know?

II. search for the critical point at $\mu > 0$:

- chiral critical point: does it exist?; location of the chiral critical point; direct evidence for 1^{st} order regime;
  location of the transition line for $\mu > 0$; density fluctuations; $T_c(\mu) \equiv T_{freeze}$?

Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\; \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\mathcal{D}\; det\; M(\mu)\; \mathrm{e}^{-S_E(V,T)}$$

$\Uparrow$ complex fermion determinant;

long standing problem

$\Rightarrow$ three (partial) solutions for large T, small μ

- exact evaluation of $detM$: works well on small lattices; requires reweighting
  Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$: works well for small μ; requires reweighting
  C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
- imaginary chemical potential: works well for small μ; requires analytic continuation
  Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290

Extending the phase diagram to non-vanishing chemical potential

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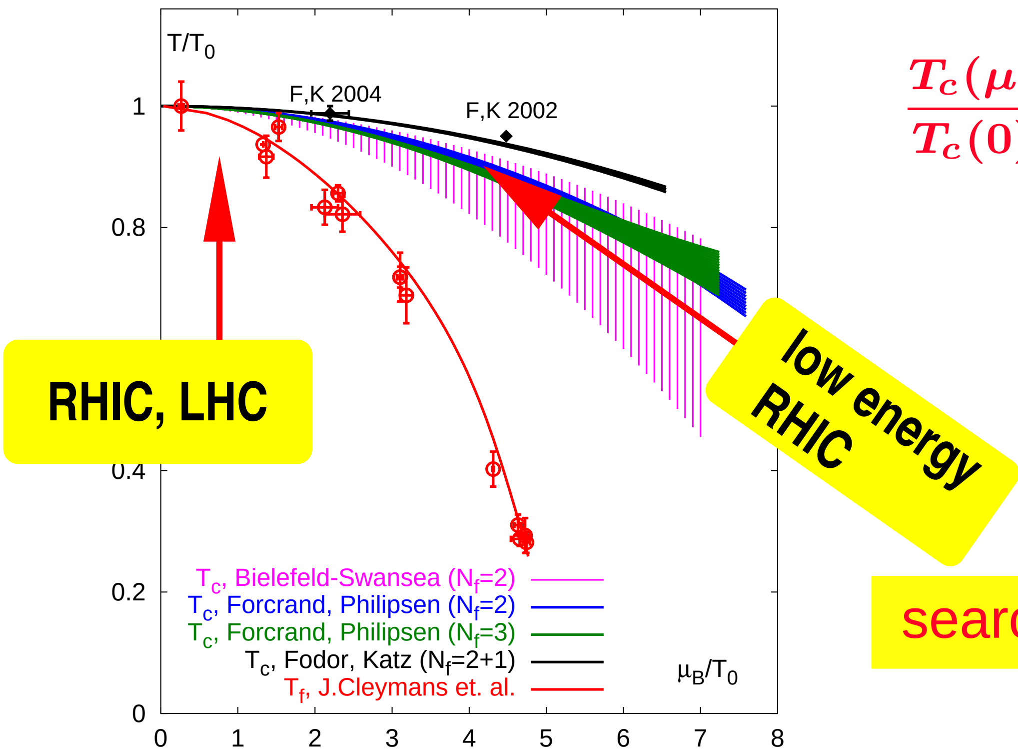


$\frac{T_c(\mu)}{T_c(0)}$: $1 - 0.0056(4)(\mu_B/T)^2$

deForcrand, Philipsen (imag. μ)

$1 - 0.0078(38)(\mu_B/T)^2$

Bielefeld-Swansea

($\mathcal{O}(\mu^2)$ reweighting)

search for critical point

Fluctuations of the baryon number density ($\mu \geq 0$)

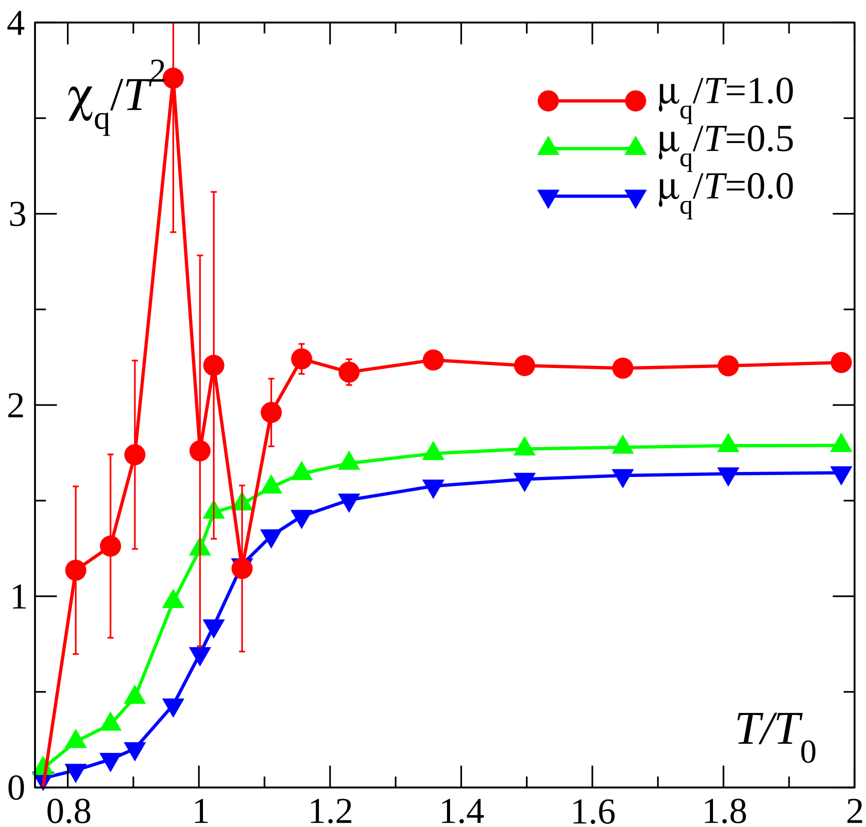
baryon number density fluctuations:

(Bielefeld-Swansea, PRD68 (2003) 014507)

$\mu \geq 0$, $n_f = 2$

$$\frac{\chi_q}{T^3} = \left(\frac{d^2}{d(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$

$$= \frac{9\,T}{V} \left(\langle N_B^2 \rangle - \langle N_B \rangle^2 \right)$$



susceptibilities

to be studied in event-by-event fluctuations

recent papers:

V. Koch, E.M. Majumder, J. Randrup, nucl-th/0505052

S. Ejiri, FK, K. Redlich, hep-ph/05090521

R.V. Gavai, S. Gupta, hep-lat/0510044

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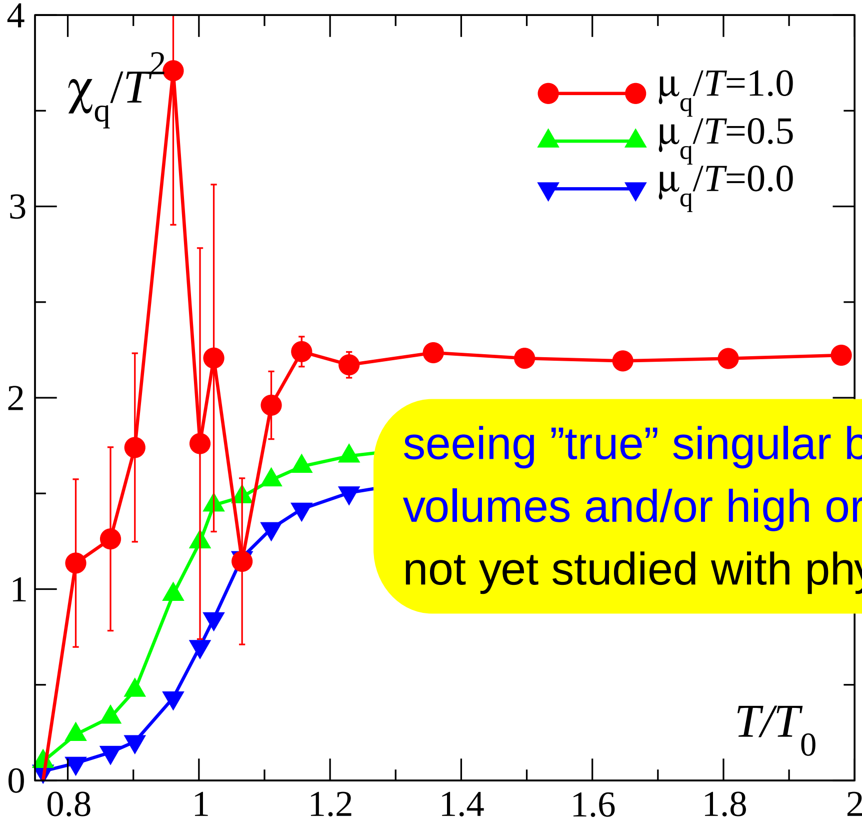

susceptibilities

to be studied in event-by-event fluctuations

seeing "true" singular behaviour requires large volumes and/or high order Taylor expansions
not yet studied with physical quark masses !!!

V. Koch, E.M. Majumder, J. Randrup, nucl-th/0505052
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III. In-medium properties of hadrons

- quarkonium spectroscopy and heavy quark diffusion
- light quark bound states and thermal dilepton/photon rates

Analyzing the (quasi-particle) structure of HG and QGP phases

Response and correlation functions:

$T \le T_c$: **chiral symmetry restoration**

- hadronic resonance gas;

  MEM analysis of thermal masses and widths, $\pi, \ \rho,$ ...
- (baryon) density fluctuations, strangeness fluctuations, ...

$T > T_c$: **deconfinement**

- free energies, potentials and screening masses, running coupling at short and large distances,...
- MEM analysis of heavy and light quark bound states, quark and gluon propagators, dilepton and photon rates,
- transport coefficients: heavy quark diffusion,...

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requires light dynamical quarks
$\Rightarrow$ PETAFLOPs era

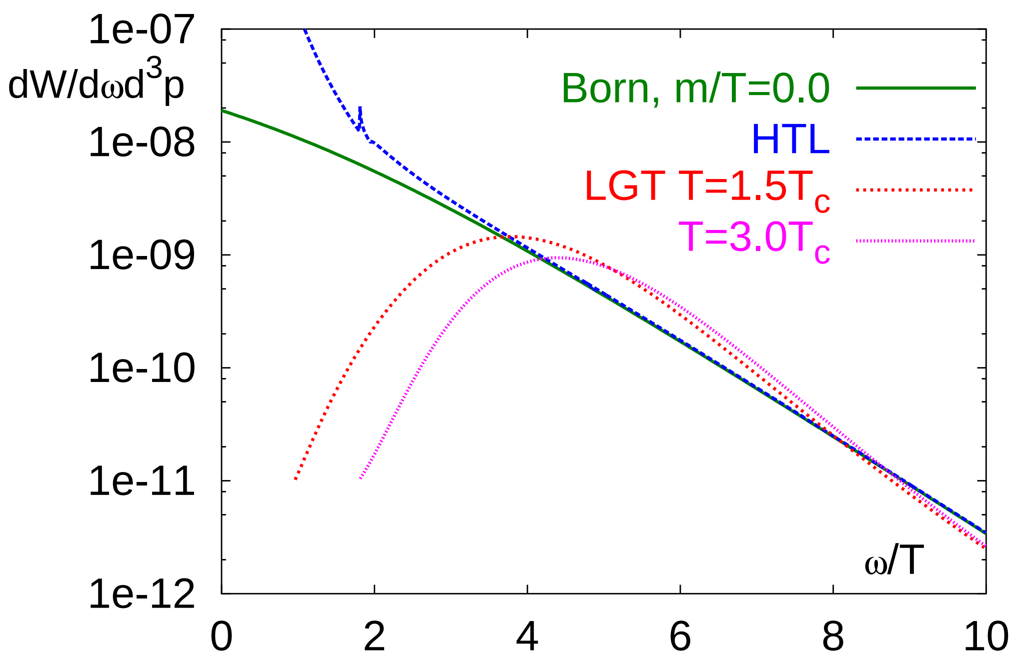
thermodynamics with
chiral fermions (DWF, overlap, ...)

$T > T_c$: deconfinement

- free energies, potentials and screening masses, running coupling at short and large distances,...
- MEM analysis of heavy and light quark bound states, quark and gluon propagators, dilepton and photon rates,
- transport coefficients: heavy quark diffusion,...

meaningful already in quenched QCD
$\Rightarrow$ TERAFLOPs era

Dilepton rate:
HTL and lattice calculations



thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\sigma_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

HTL and lattice disagree for

$\omega/T \lesssim (3-4)$

- infra-red sensitivity of HTL-calculations ⇔ "massless gluon" cut in HTL-propagator
- infra-red sensitivity of lattice calculations ⇔ thermodynamic limit, $V \rightarrow \infty$
- $VT^3 = (N_\sigma/N_\tau)^3 < \infty$ ⇒ momentum cut-off: $p/T > 2\pi N_\tau/N_\sigma$

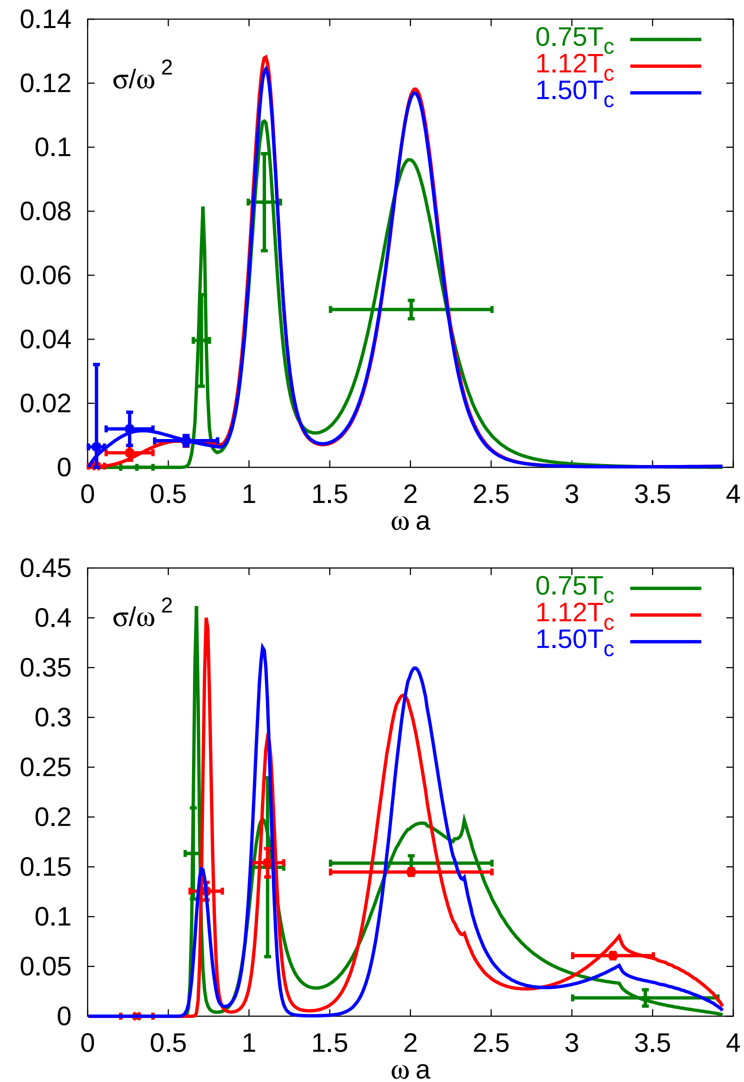
⇒ need large lattices to analyze infra-red regime

⇒ in future also thermal photon rates

need $N_\tau \sim \mathcal{O}(30)$ AND $N_\sigma \sim 6\ N_\tau$

Heavy quark spectral functions and correlation functions

reconstructed spectral functions
using the Maximum Entropy Method



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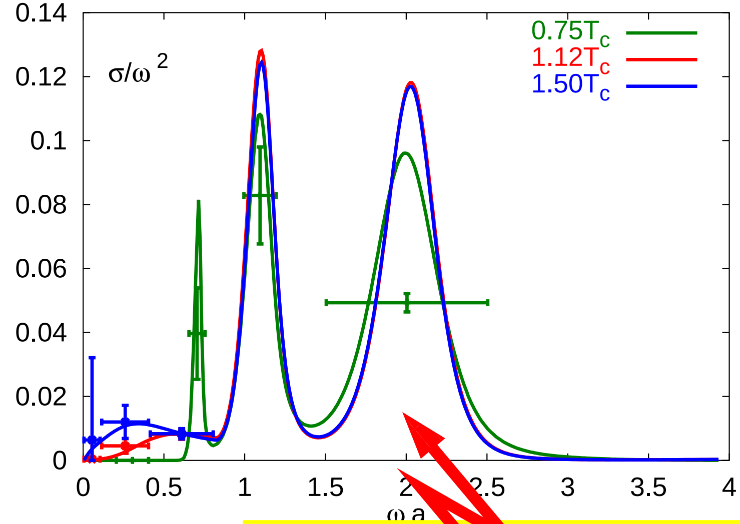
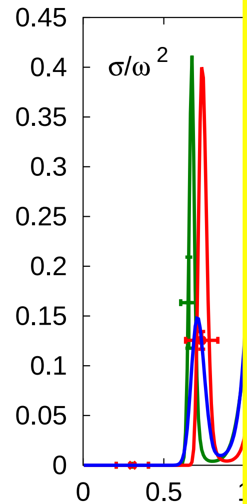




need to get better control over
ultra-violet cut-off effects
(Wilson-doublers)

use better fermion actions
- overlap fermions
- domain wall fermions
- (truncated) perfect actions...

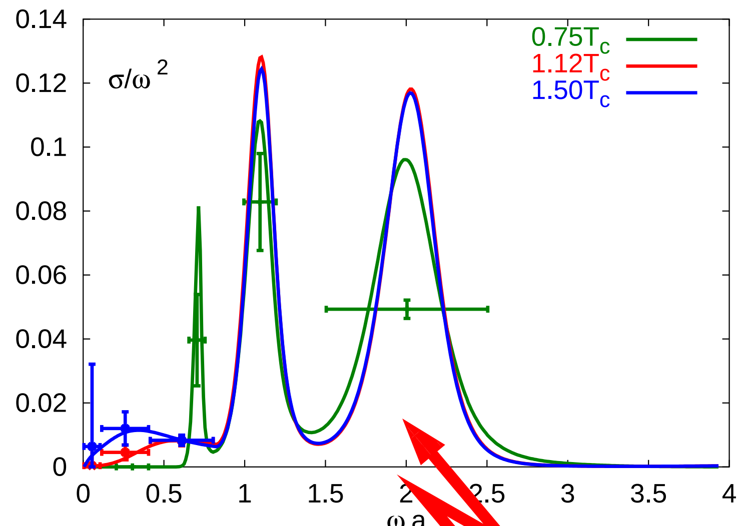
Heavy quark spectral functions and correlation functions

Want to get access to transport properties

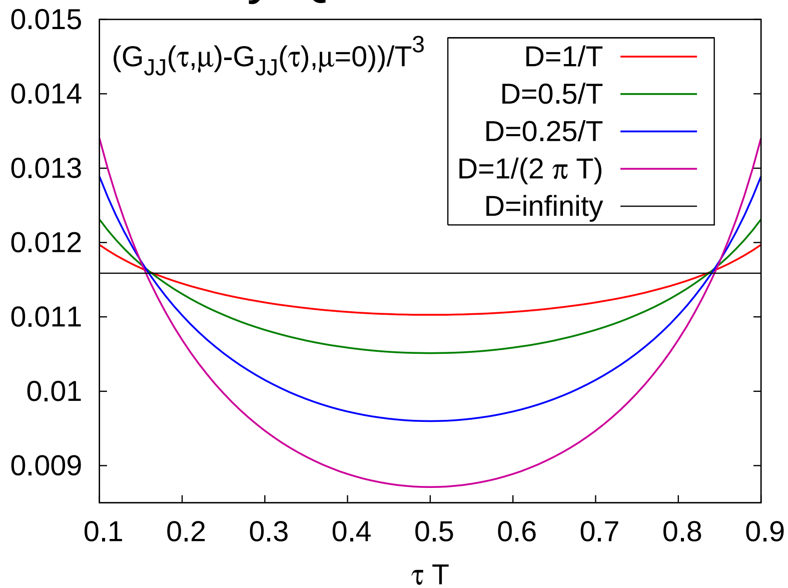
low energy ⇔ large distances

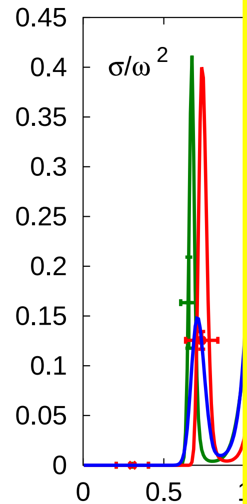
need to get better control over low energy regime

reconstructed spectral functions using the Maximum Entropy Method



Heavy Quark Diffusion





need to get better control over ultra-violet cut-off effects (Wilson-doublers)

use better fermion actions

- overlap fermions
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- (truncated) perfect actions...

P. Petreczky and D. Teaney, PRD73, 014508 (2006)

Heavy quark spectral functions and correlation functions

- so far finite-T quarkonium spectroscopy has been analyzed in quenched QCD; except for a few exploratory studies with dynamical staggered fermions

FUTURE:

spectroscopy with dynamical chiral fermions at finite temperature

- need large lattices $\sim \quad \mathbf{120^3 \times 40}$
- need light, dynamical quarks with good chiral properties (DWF, overlap,..)

$\Rightarrow$ thermal properties of ρ-meson
$\Rightarrow$ low energy part of spectral functions
$T > 0$ spectroscopy on PETAFLOPS computers

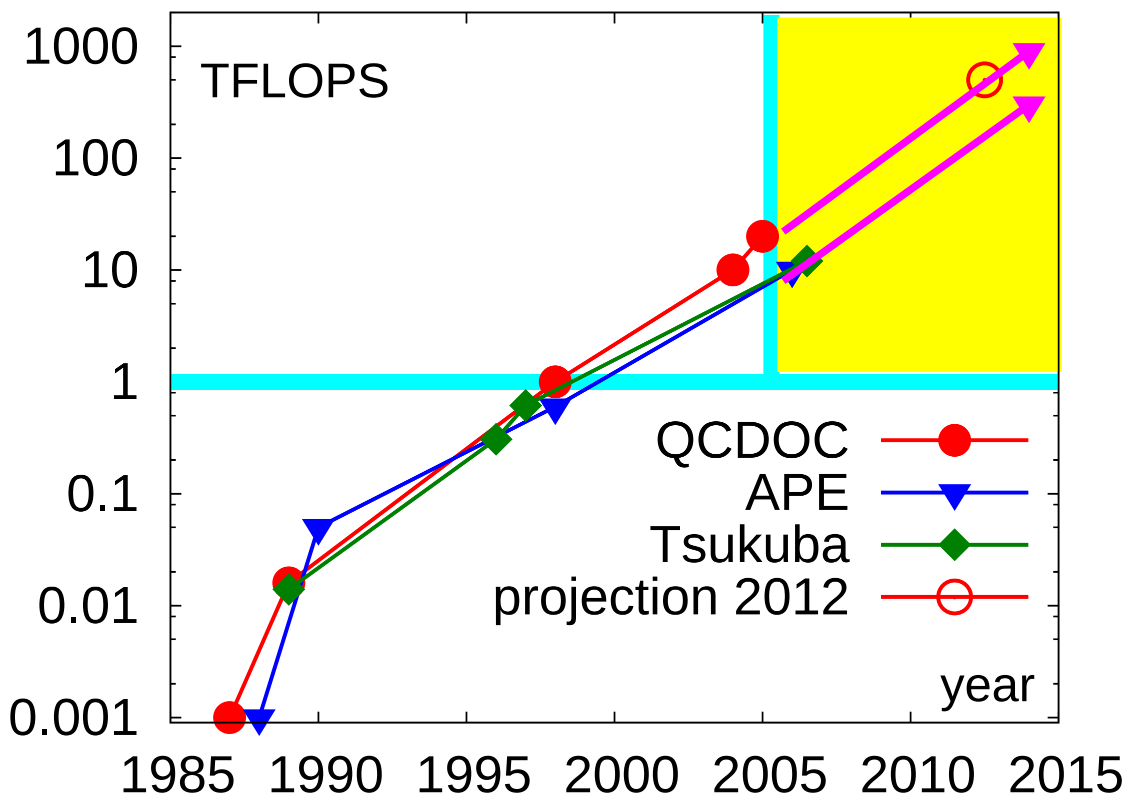
Progress in lattice calculations depends on...

- stable funding for a QCD thermodynamics group that guarantees continuity of know-how...
- access to dedicated computer hardware

development of special purpose computer hardware

⇓

towards PETAFLOPS computing



Outlook: projects on future machines...

...towards thermodynamics on Petaflops computers
(extension of (exploratory) studies on current Teraflops computers)

- Thermodynamics of QCD with chiral fermion formulations
- In-medium properties of light quark bound states:
  QCD with light, dynamical quarks on fine lattices become possible;
  mass shifts and modification of widths below T_c
- finite density QCD:
  aim at definite answer on the existence and location of a critical point;
  try to reach lower temperatures around $T \sim 0.5\, T_c$
- transport properties:
  calculation of "gluonic correlator" (energy momentum tensor) should become possible; spectral functions in the $\omega \longrightarrow 0$ limit may become accessible
  (dilepton rates; heavy quark diffusion coefficient,..)

LGT needs open access to large scale computing

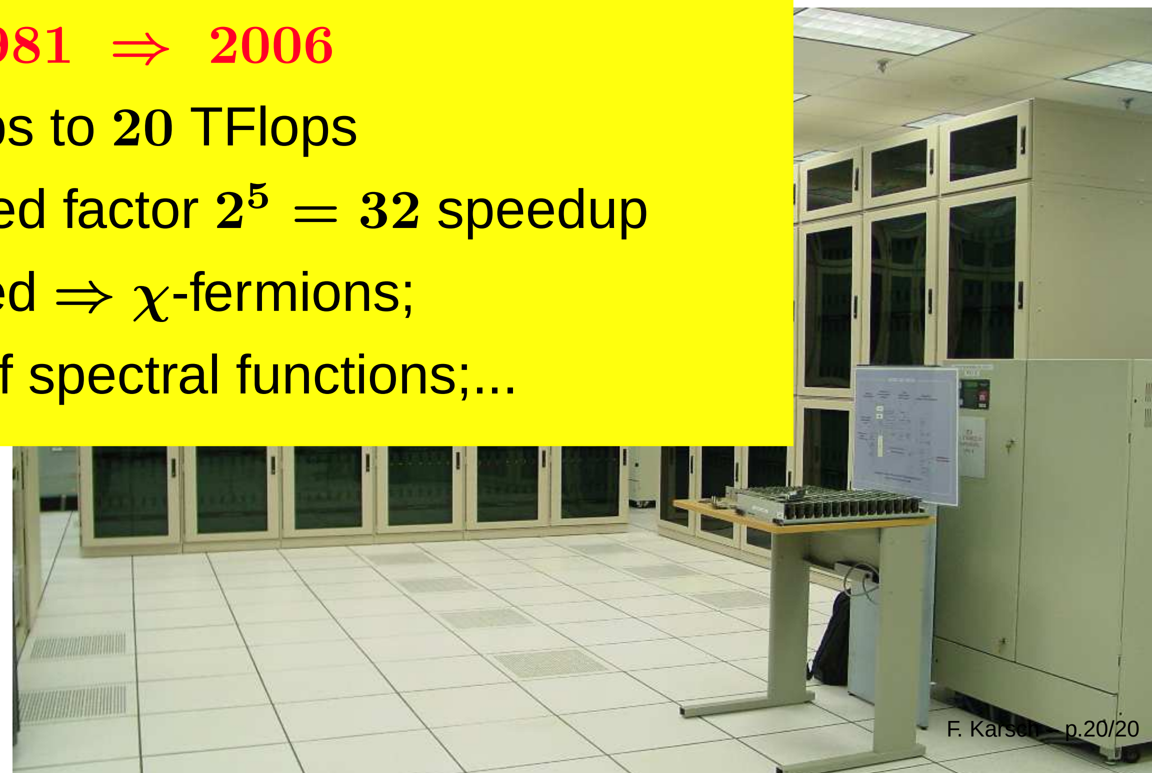
360TFlops BlueGene/L
at Livermore
EoS and T_c for $N_\tau = 8$
~ 40 TFlop-years

20TFlops QCDOC
at BNL
$\mu_q \geq 0, N_\tau = 4,\ 6$
~ 5 TFlop-years

LGT needs open access to large scale computing

360TFlops BlueGene/L
at Livermore
EoS and T_c for $N_\tau = 8$
$\sim \mathbf{40}$ TFlop-years

finite-T LGT: $\mathbf{1981 \ \Rightarrow \ 2006}$
from $\mathbf{800}$ KFlops to $\mathbf{20}$ TFlops
2007-2012: need factor $\mathbf{2^5 = 32}$ speedup
buys: staggered $\Rightarrow \chi$-fermions;
details of spectral functions;...



20TFlops QCDOC
at BNL
$\mu_q \geq \mathbf{0}$, $N_\tau = \mathbf{4,\ 6}$
$\sim \mathbf{5}$ TFlop-years