Transverse Momentum Dependent Distributions

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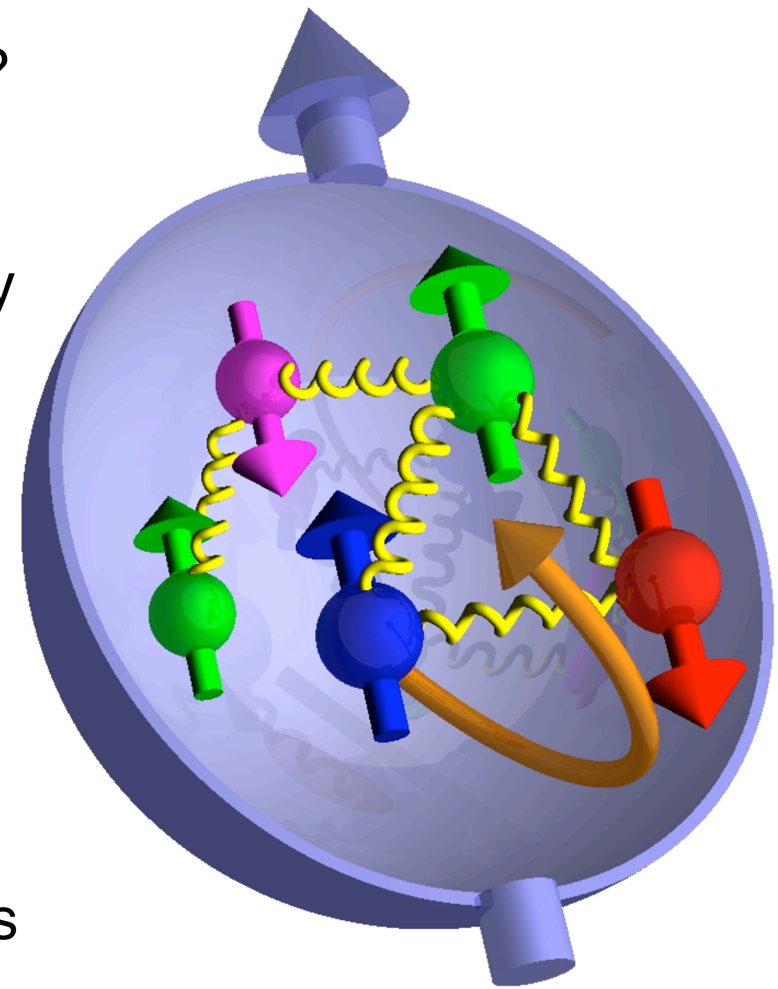
& Josh Rubin,
the Animagician!

- The rich world of **proton substructure**:
  What *are* **TMDs** & why are they interesting?

- **Single-spin asymmetries**:
  How to *measure* the TMDs ... and what they tell us about quark **orbital motion** & **spin**?

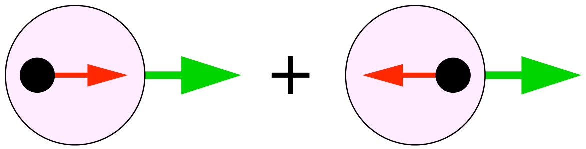
- **Highlights** of the past 5 years:
  what we've learned from **theory** & experiments with **transverse spin**

  - The **Collins Effect**: spin-orbit effects in fragmentation
  - The **Sivers Effect**: spin-orbit correlations within the proton

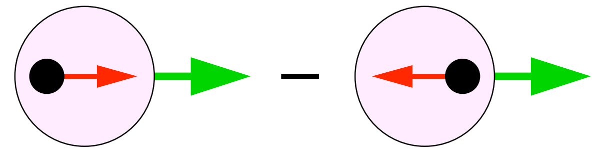


A particular puzzle: Where does the proton spin come from?

$$q(x) = q^{\uparrow}(x) + q^{\downarrow}(x)$$



$$\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$



only three possibilities

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g}$$

❶ Quark polarization

$$\Delta\Sigma \equiv \int dx\, (\Delta u(x) + \Delta d(x) + \Delta s(x) + \Delta\bar{u}(x) + \Delta\bar{d}(x) + \Delta\bar{s}(x)) \approx 20\%\ \text{only}$$

❷ Gluon polarization

$$\Delta G \equiv \int dx\, \Delta g(x) \quad ?$$

❸ Orbital angular momentum

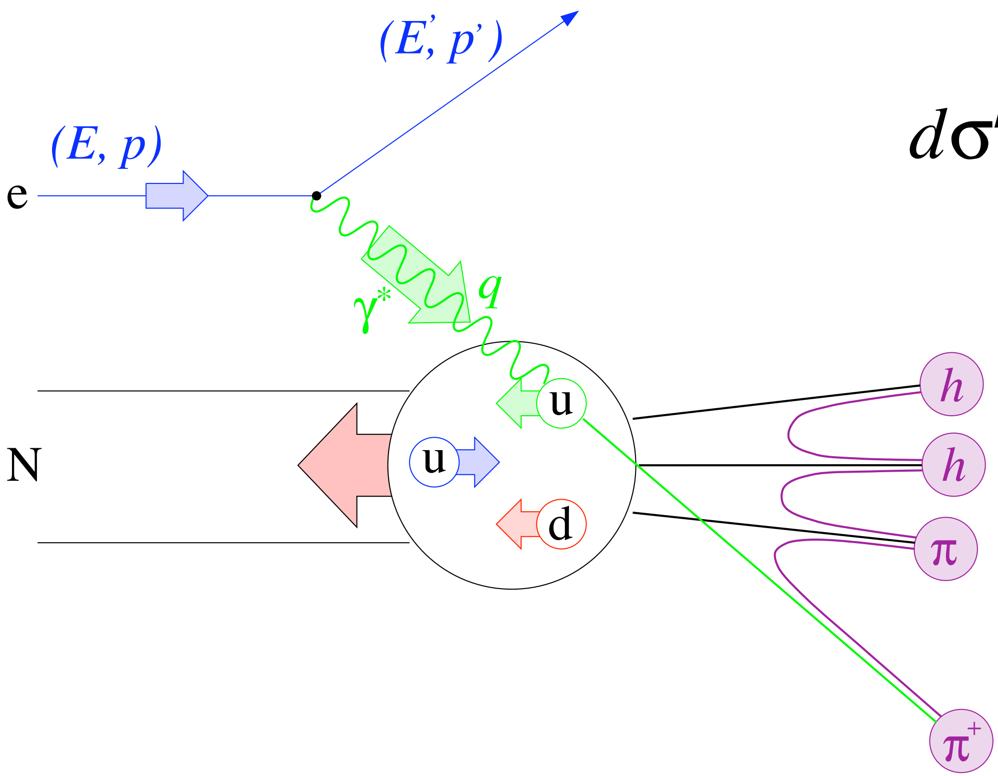
$$L_z \equiv L_q + L_g \quad ?$$

In friendly, **non-relativistic** bound states like atoms & nuclei (& constituent quark model), particles are in ***eigenstates of L***

Not so for bound, ***relativistic Dirac particles*** ...
Noble "l" is ***not a good quantum number***

Semi-Inclusive Deep-Inelastic Scattering (SIDIS)

*In SIDIS, a **hadron h** is detected in **coincidence** with the scattered lepton:*



Factorization of the cross-section:

$$d\sigma^h \sim \sum_q e_q^2 \, q(x) \cdot \hat{\sigma} \cdot D^{q\rightarrow h}(z)$$

- **The perturbative part**

  Cross-section for elementary photon-quark ***subprocess***

  Large energies ➡ asymptotic freedom
  ➡ can calculate!

- **The Distribution Function**

  momentum ***distribution of quarks q*** within their proton bound state

  ➡ **lattice QCD** progressing steadily

- **The Fragmentation Function**

  momentum ***distribution of hadrons h*** formed from quark *q*

  ➡ not even lattice can help ...

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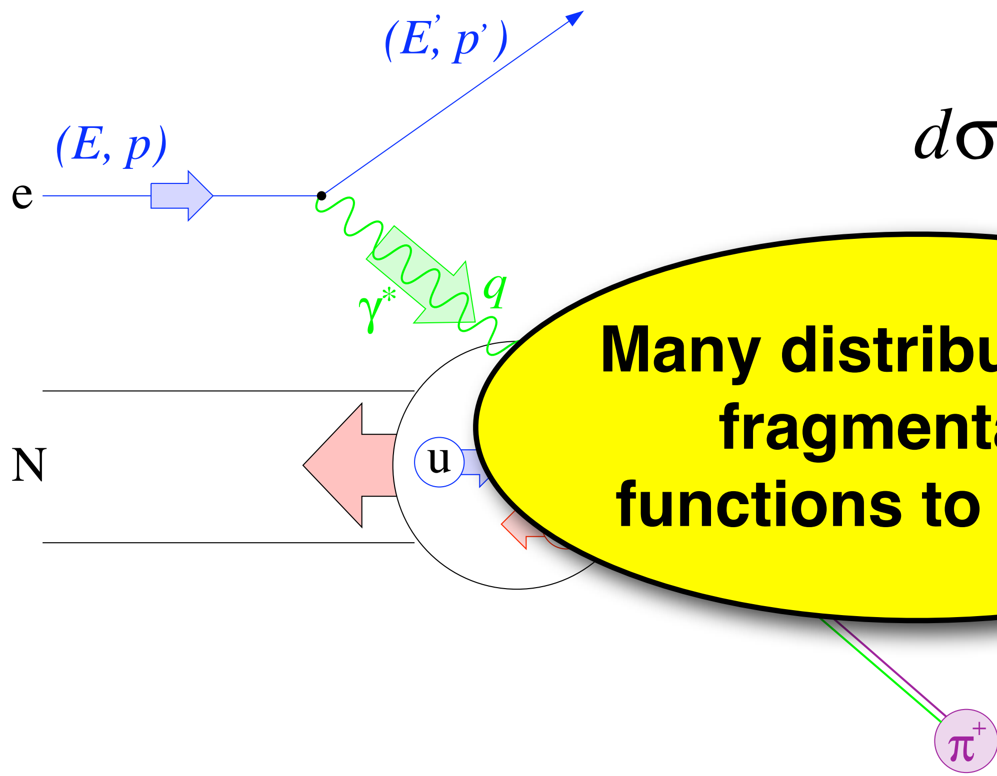

Factorization of the cross-section:

$$d\sigma^h \sim \sum_q e_q^2 \, q(x) \cdot \hat{\sigma} \cdot D^{q\rightarrow h}(z)$$

Many distribution and fragmentation functions to explore!

…e perturbative part

…section for elementary …on-quark ***subprocess***

Large energies ➡ asymptotic freedom
➡ can calculate!

The Distribution Function

momentum ***distribution of quarks q*** within their proton bound state

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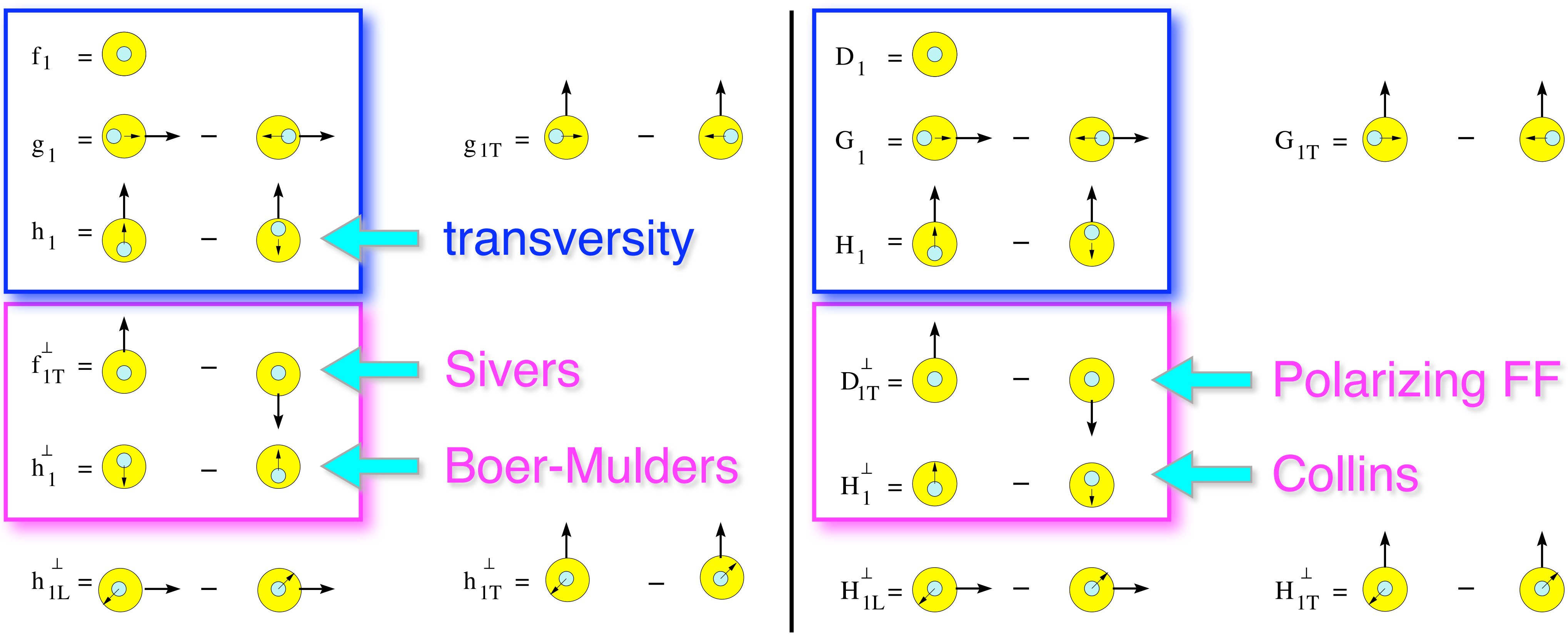
Functions surviving on integration over Transverse Momentum

TMD's: the **others** are sensitive to ***intrinsic k_T*** in the nucleon & in the fragmentation process

Mulders & Tangerman, NPB 461 (1996) 197

Distribution Functions | Fragmentation Functions



Functions **odd** under **naive time reversal** ⇒ generate **SSA's**

Sensitive to ***spin-orbit*** correlations of quarks and gluons ⇒ ***orbital angular momentum***

SURA QCD Workshop
Dec 2006

Key Issues in Proton Structure

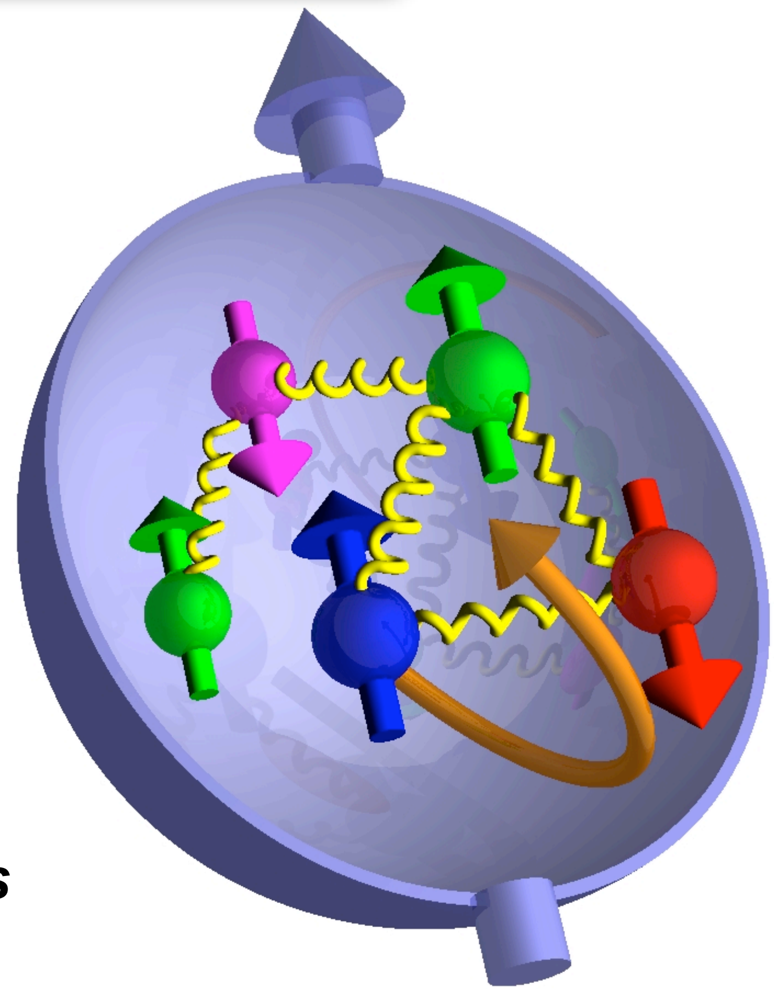
Overarching Goal:
Explore and Understand QCD

Consists of Gaining Insight

- map the ***basic features*** of the proton
- discover the best ***degrees of freedom*** to fully describe the proton
- explore how ***hadrons emerge*** from the QCD vacuum

and of Precision Tests of QCD

- can QCD provide ***precise calculations*** of hadron structure from 1st principles?



TMD's

Key Questions in Hadron Structure

→ ***Talk by Rolf Ent***

1. What is the role of *gluons* in nucleons and nuclei?

2. What is the internal *spin and flavor landscape* of hadrons?

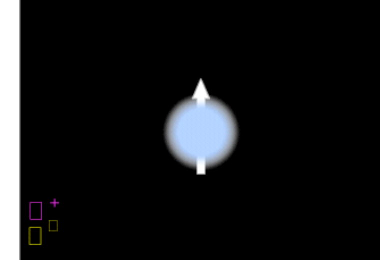
- What are the **spin-orbit correlations** of quarks and gluons **within the proton**?

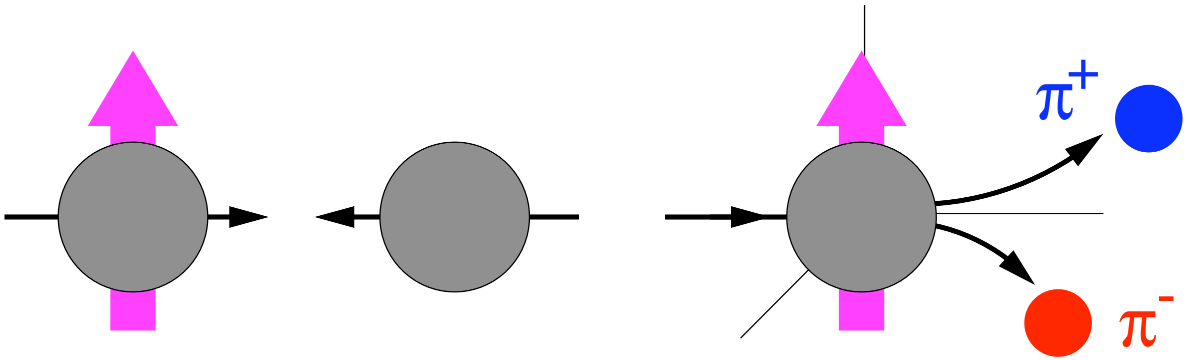
3. How do *hadronic final-states* form in QCD?

- What role do **spin** and **angular momentum** play in **fragmentation**?

Single-Spin Asymmetries

Fermilab E704: $p^{\uparrow} p \rightarrow \pi X$ at 200 GeV

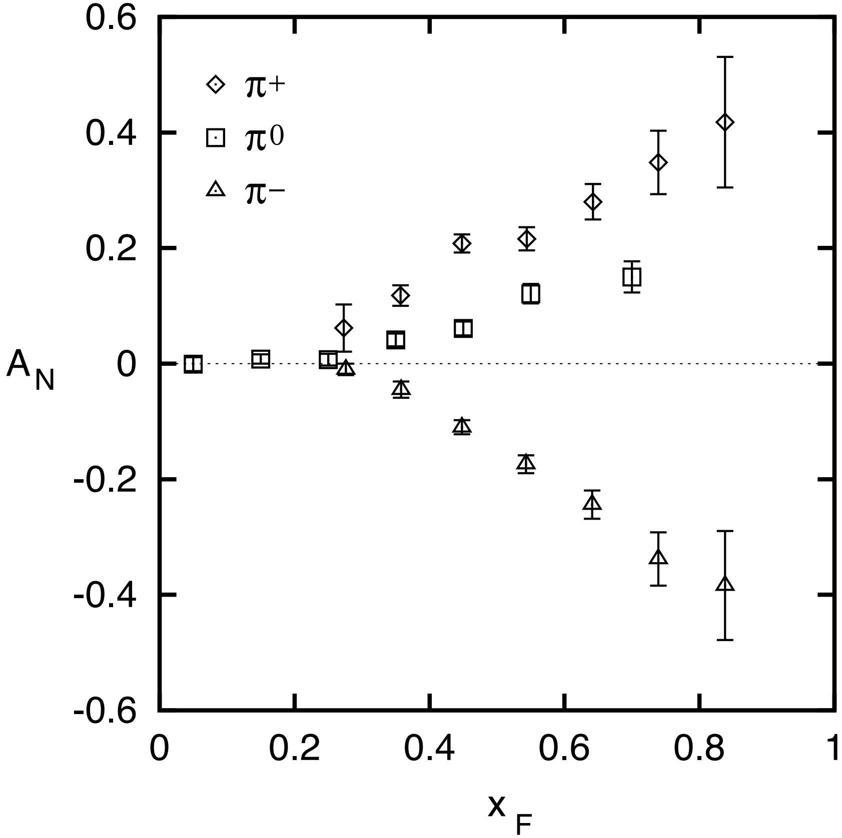




Analyzing Power

$$A_N = \frac{1}{P_{\text{beam}}} \frac{N^{\pi}_{\text{left}} - N^{\pi}_{\text{right}}}{N^{\pi}_{\text{left}} + N^{\pi}_{\text{right}}}$$

Huge single-spin asymmetry !

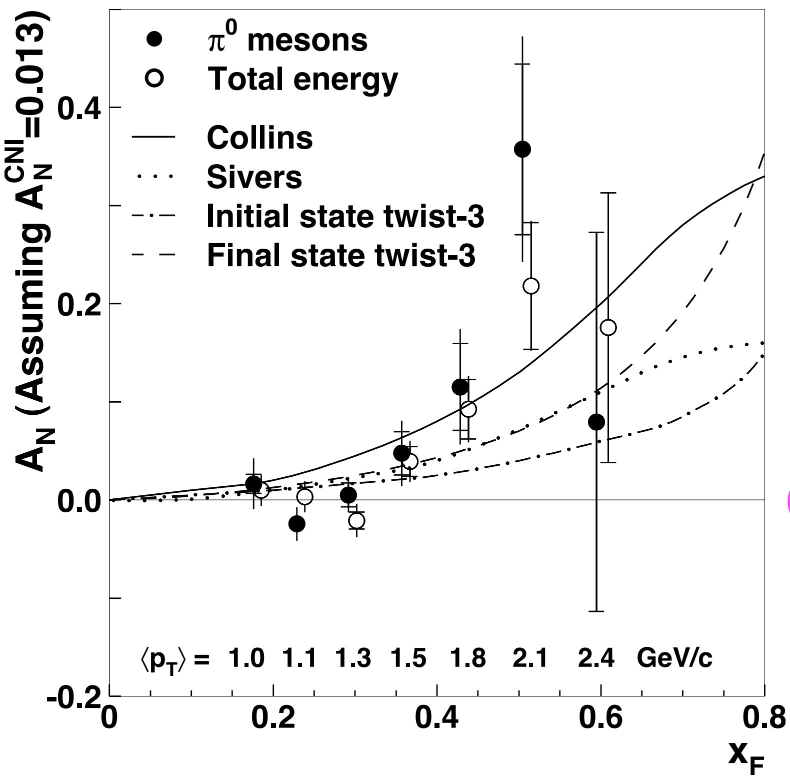


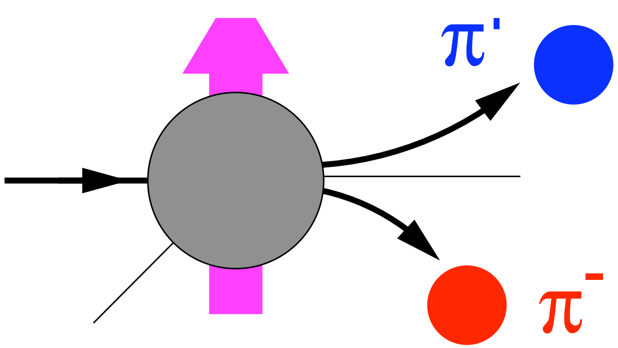
- Opposite sign for $\pi^+ = u\bar{d}$ than for $\pi^- = d\bar{u}$
- Effect larger for forward production
- Observable: $\vec{S}_{\text{beam}} \cdot (\vec{p}_{\text{beam}} \times \vec{p}_{\pi})$
  odd under naive Time-Reversal

Surprising observation! ..... Why?

SSA's at high-energies

Now confirmed at STAR at much higher energies!





T-odd observables

SSA observables $\sim \vec{\mathbf{J}} \cdot (\vec{\mathbf{p_1}} \times \vec{\mathbf{p_2}})$
$\Rightarrow$ ***odd*** under naive ***time-reversal***

Since QCD amplitudes are T-even, must arise from **interference** between **spin-flip** and non-flip amplitudes with **different phases**

Can't come from perturbative subprocess xsec:

- q helicity flip suppressed by $m_q/\sqrt{s}$
- need α_s-suppressed loop-diagram to generate necessary phase

At hard (enough) scales, SSA's must arise from soft physics: T-odd distribution / fragmentation functions

SSA's at high-energies

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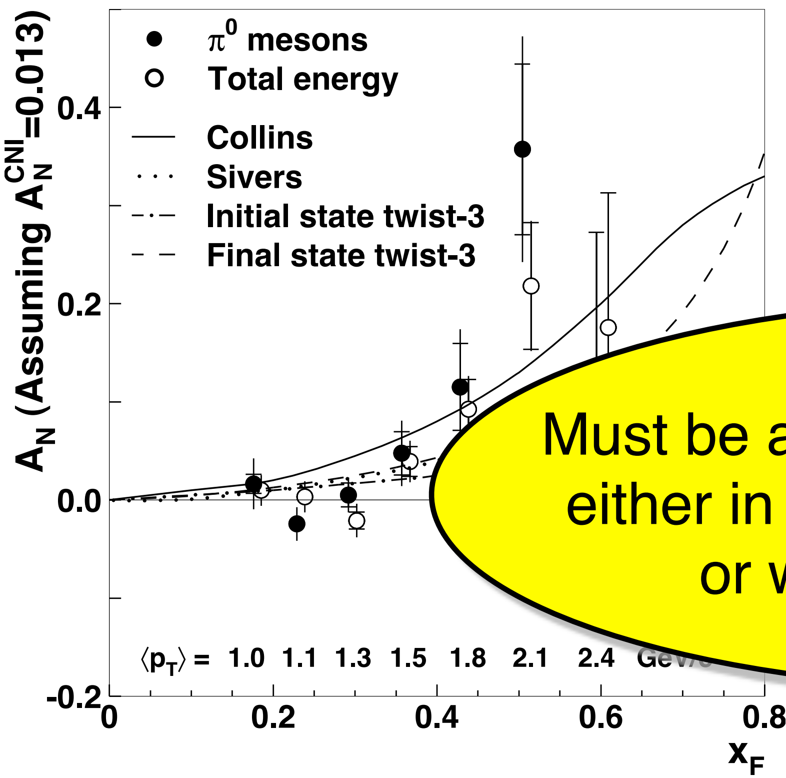


T-odd observables

SSA observables $\sim \vec{\mathbf{J}} \cdot (\vec{\mathbf{p_1}} \times \vec{\mathbf{p_2}})$

$\Rightarrow$ ***odd*** under naive ***time-reversal***

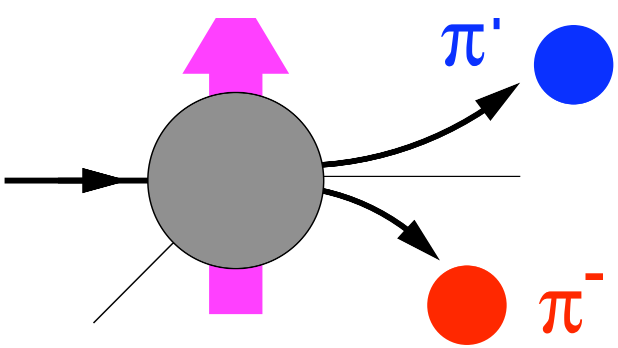
Since QCD amplitudes are T-even, must arise **from interference** between **spin-flip** and ... **different phases**

Must be a new, **spin-orbit structure** either in the fragmentation process or within the proton itself

...ubprocess xsec:

...ed by $m_q/\sqrt{s}$

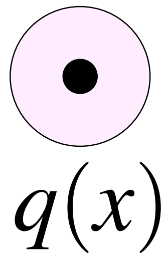
- need α_s-suppressed loop-diagram to generate necessary phase

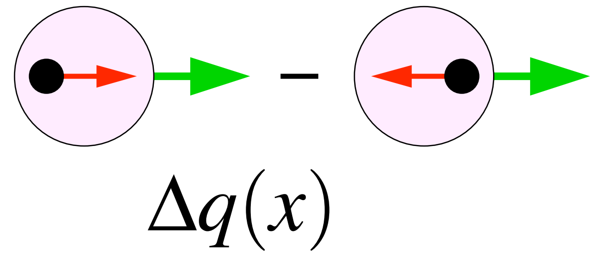


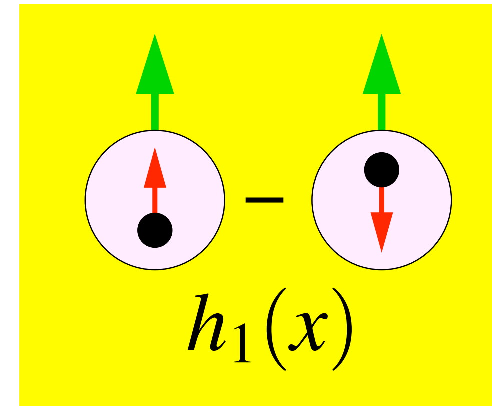
At hard (enough) scales, SSA's must arise from soft physics: T-odd distribution / fragmentation functions

E704 Possible Mechanism #1: The "Collins Effect"

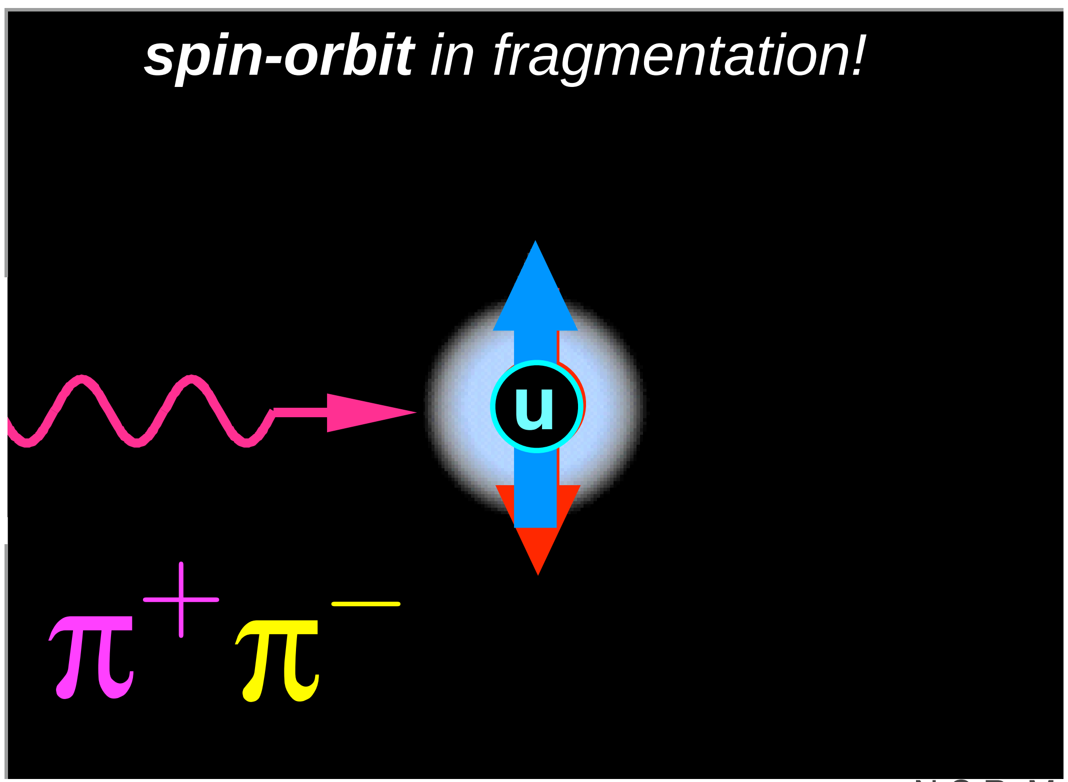
Need an ordinary distribution function ... **transversity**



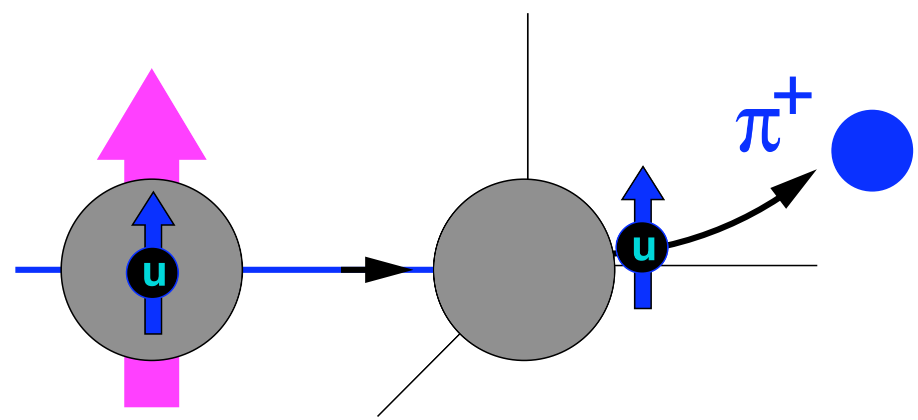




... with a new, **T-odd "Collins" fragmentation function** $H_1^\perp(z, p_T)$



E704 effect:



$$h_1(x) \otimes H_1^\perp(z, p_T)$$

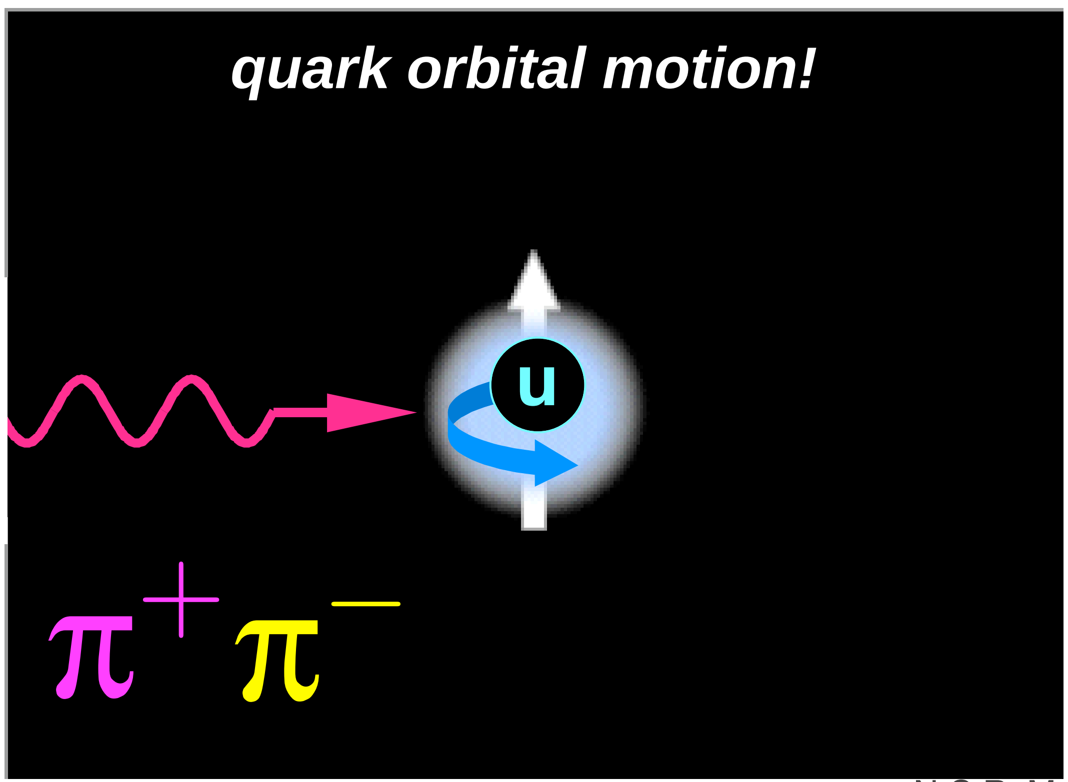
E704 Possible Mechanism #2: The "Sivers Effect"

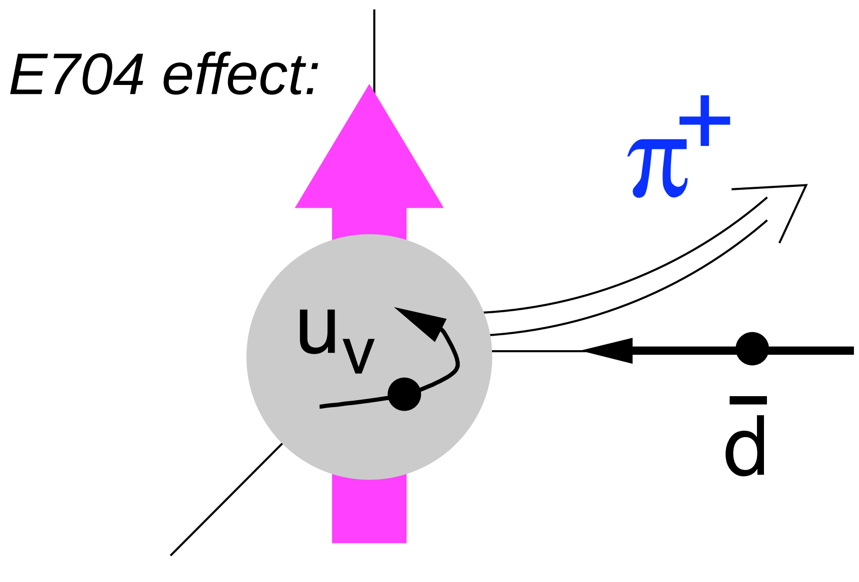
Need the ordinary fragmentation function $D_1(z)$

... with a new, **T-odd "Sivers" distribution function** $f_{1T}^{\perp}(x, k_T)$

Phenomenological model of **Meng & Chou**:

Forward π+ produced from **orbiting valence-u quark** by recombination at <u>*front surface*</u> of beam protons

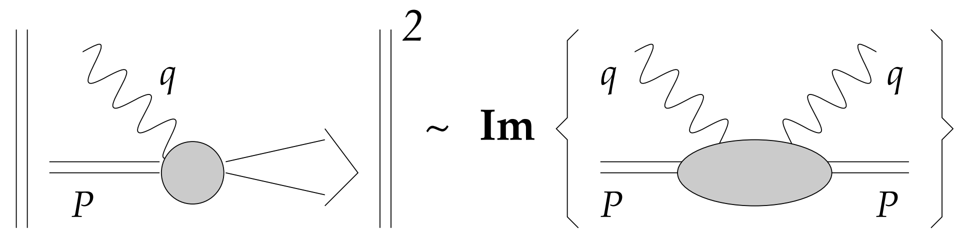




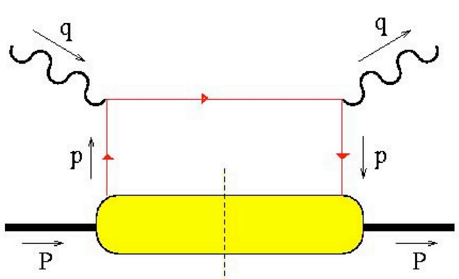
$$f_{1T}^{\perp}(x, k_T) \otimes D_1(z)$$

The Leading-Twist Sivers Function: Can it Exist in DIS?

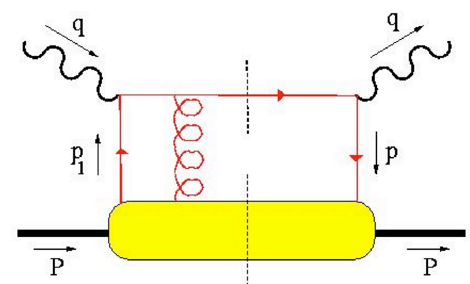
A T-odd function like $f_{1T}^{\perp}$ ***must*** arise from ***interference*** ... but a distribution function is just a forward scattering amplitude, how can it contain an interference?



Brodsky, Hwang, & Schmidt 2002



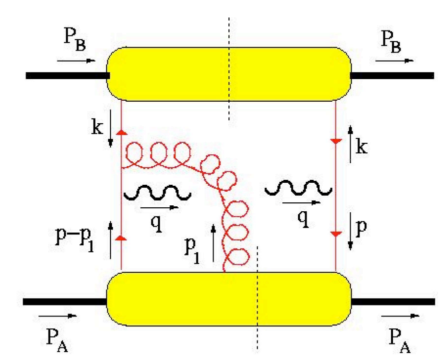
can interfere with



and produce a T-odd effect! (also need $L_z \neq 0$)

It looks like higher-twist ... but no, these are soft gluons = "gauge links" required for color gauge invariance

Such soft-gluon reinteractions with the soft wavefunction are ***final (or initial) state interactions*** ... and may be ***process dependent !*** ⇒ new ***universality issues***

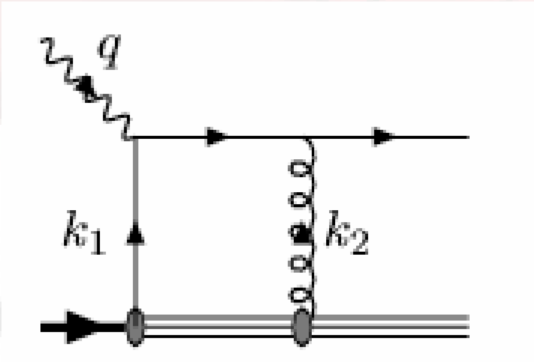


e.g. Drell-Yan

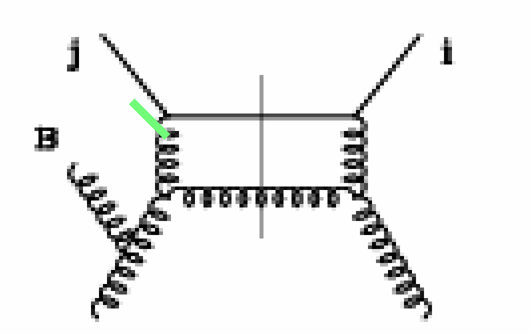
SSA Descriptions: **TMDs** ... or **Twist-3**?

***TMD** parton distribution and fragmentation functions*	***Twist-3** correlations (collinear factorization)*

- TMD: the quark orbital angular momentum leads to hadron helicity flip
- The factorizable final state interactions --- the gauge link provides the phase



- Twist-three: the gluon carries spin, flipping hadron helicity
- The phase comes from the poles in the hard scattering amplitudes



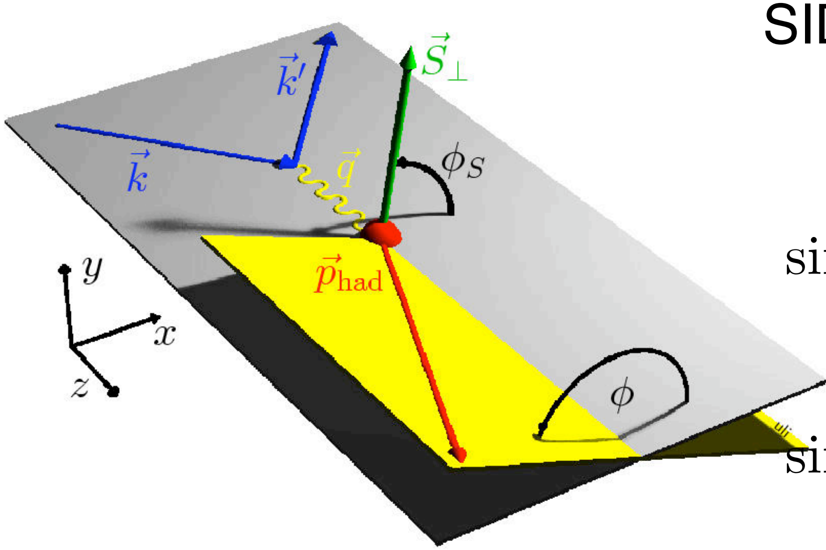
Unifying the Two Descriptions
($P_\perp$ dependence of SSAs)

- At **low $P_\perp$**, the non-perturbative TMD Sivers function will be responsible for the SSA
- When **$P_\perp \gg Q$**, purely twist-3 contributions
- For intermediate $P_\perp$, $\Lambda_{QCD} \ll P_\perp \ll Q$, we should see the transition between these two
- *An important issue: at $P_\perp \approx Q$, these two should **merge**, showing consistence of the theory*

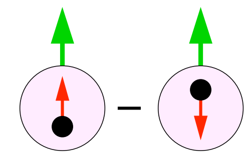
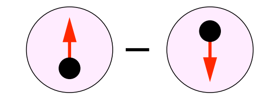
(Ji, Qiu, Vogelsang, Yuan, PLB638,178; PRD73,094017; PRL97, 082002, 2006)

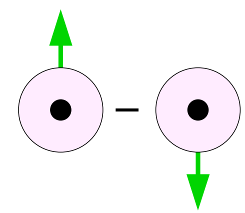
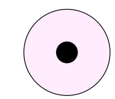
Separating Collins & Sivers:
New Experimental Observables

Lepto-production: SIDIS with Transverse Target

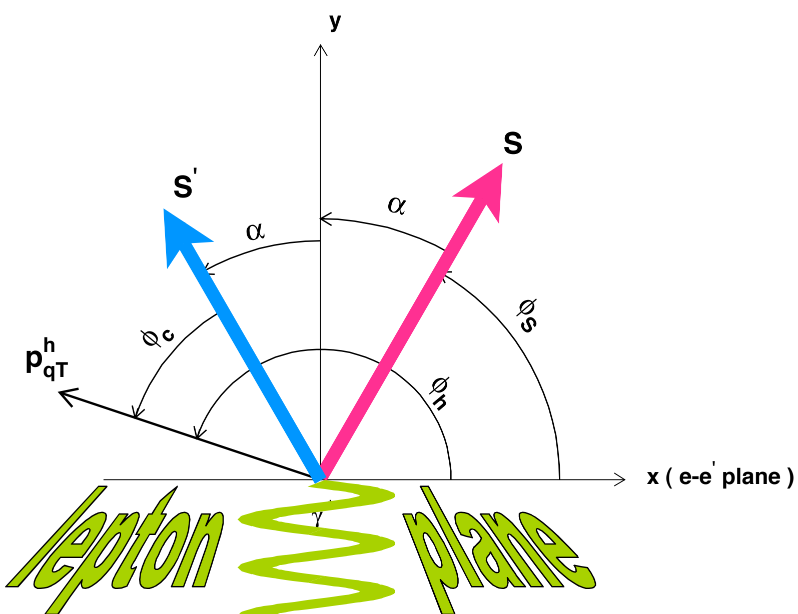


SIDIS xsec with *transverse target* polarization has ***two*** similar terms:

$\sin(\phi_h^l + \phi_S^l) \Rightarrow h_1 =$  $\otimes H_1^\perp =$ 

$\sin(\phi_h^l - \phi_S^l) \Rightarrow f_{1T}^\perp =$  $\otimes D_1 =$ 

seperate Sivers and Collins mechanisms

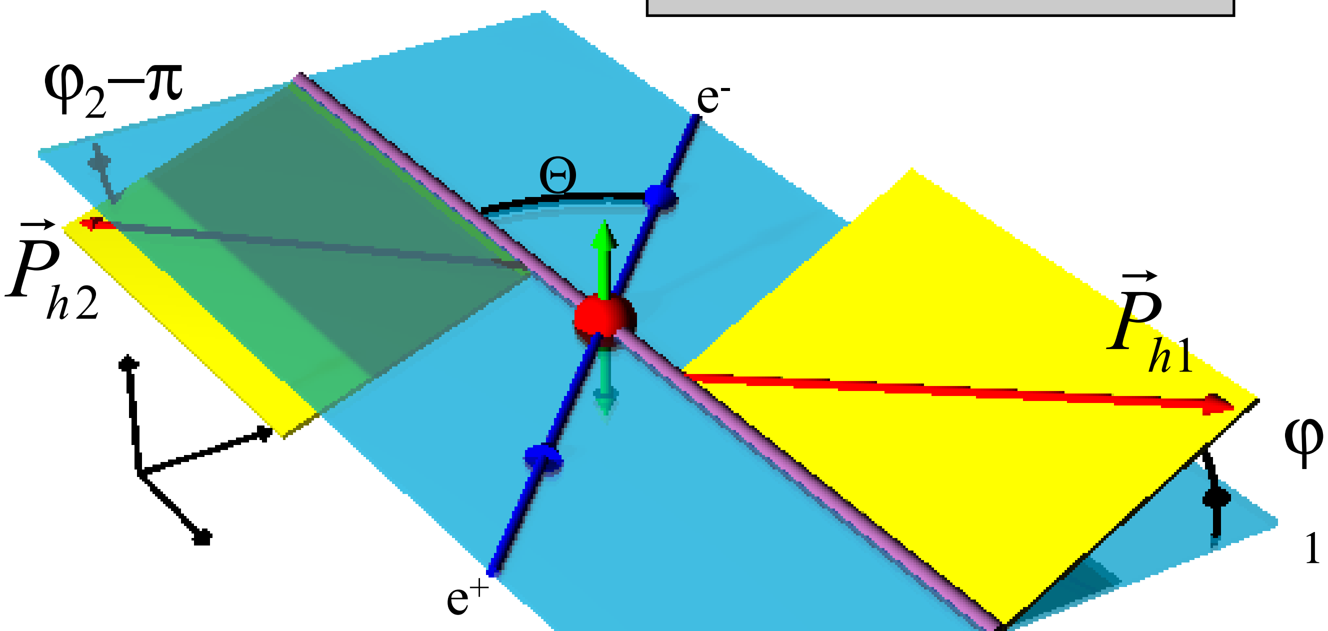


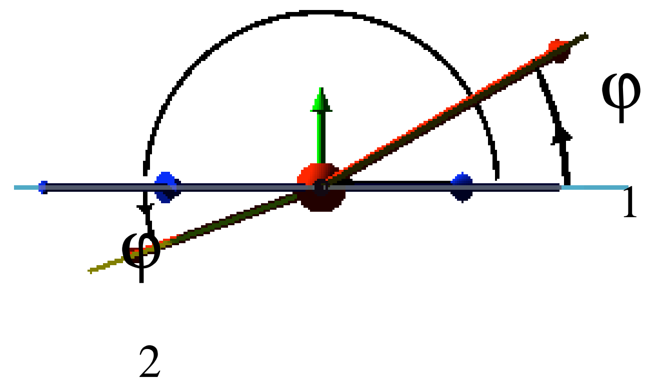
- $(\phi_h^l - \phi_S^l)$ = angle of hadron relative to **initial** quark spin
- $(\phi_h^l + \phi_S^l) = \pi + (\phi_h^l - \phi_S^{l'})$ = hadron relative to **final** quark spin

Collins fragmentation: Angles and Cross section $\cos(\phi_1+\phi_2)$ method

BELLE

e⁺e⁻ CMS frame:





2-hadron inclusive transverse momentum dependent cross section:

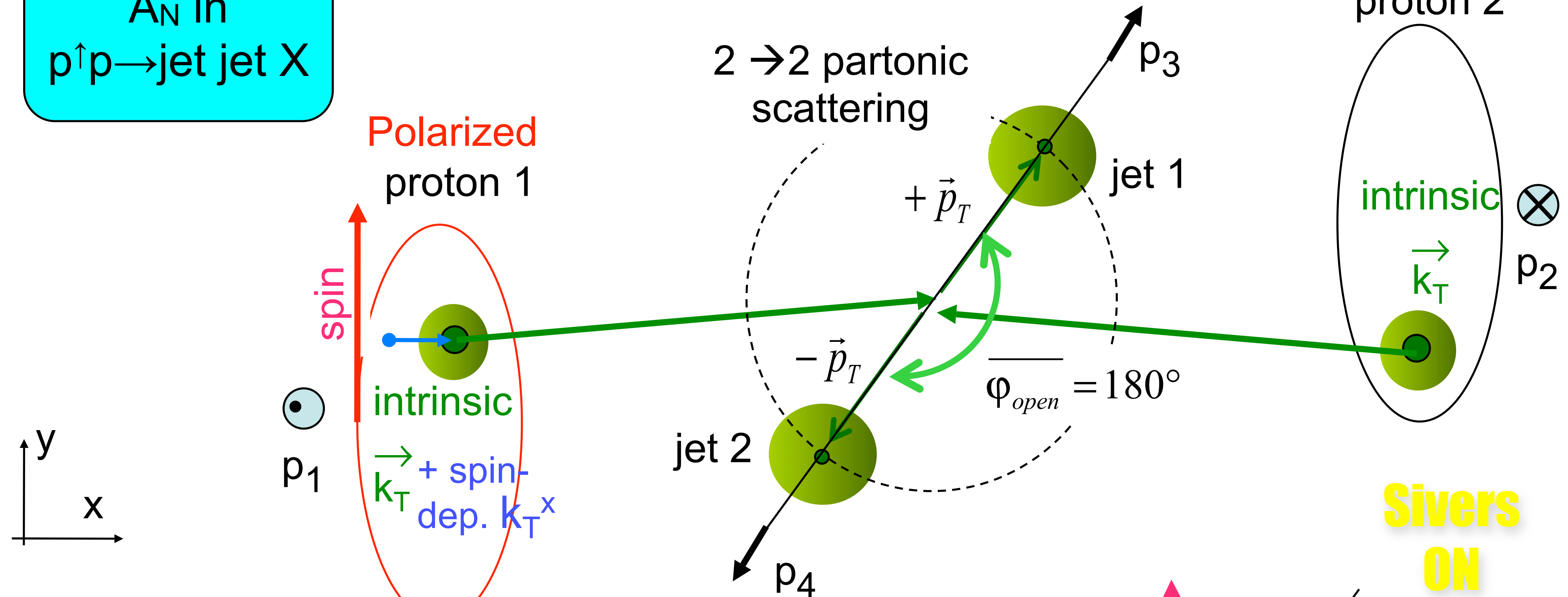
$$\frac{d\sigma(e^+e^- \rightarrow h_1h_2X)}{d\Omega dz_1dz_2d^2\boldsymbol{q}_T} = \cdots B(y)\cos(\varphi_1+\varphi_2)H_1^{\perp[1]}(z_1)\overline{H}_1^{\perp[1]}(z_2)$$

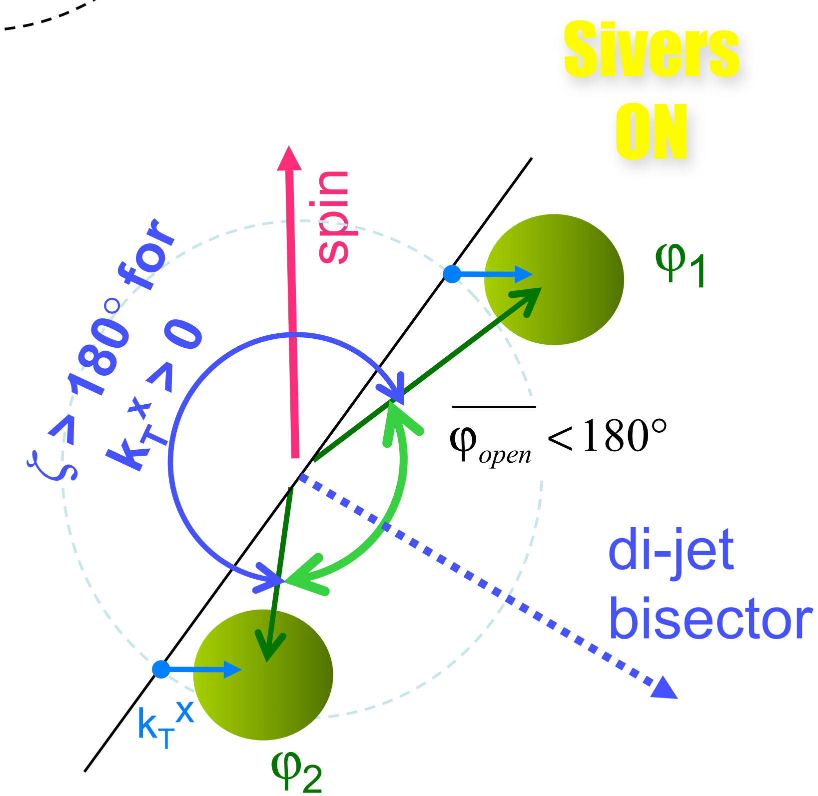
$$B(y) = y(1-y) \overset{cm}{=} \frac{1}{4}\sin^2\Theta$$

Net anti-alignment of transverse quark spins

Sivers Mechanism of SSA

A_N in
$p^{\uparrow}p \rightarrow$ jet jet X





Spin dependent k_T^x offset

- deflects both jets in the same direction
- reduces average di-jet opening angle
- **can be measured** from **correlation** between **di-jet bisector vs. spin direction**
- from accumulated **ζ-spectra (spin)**

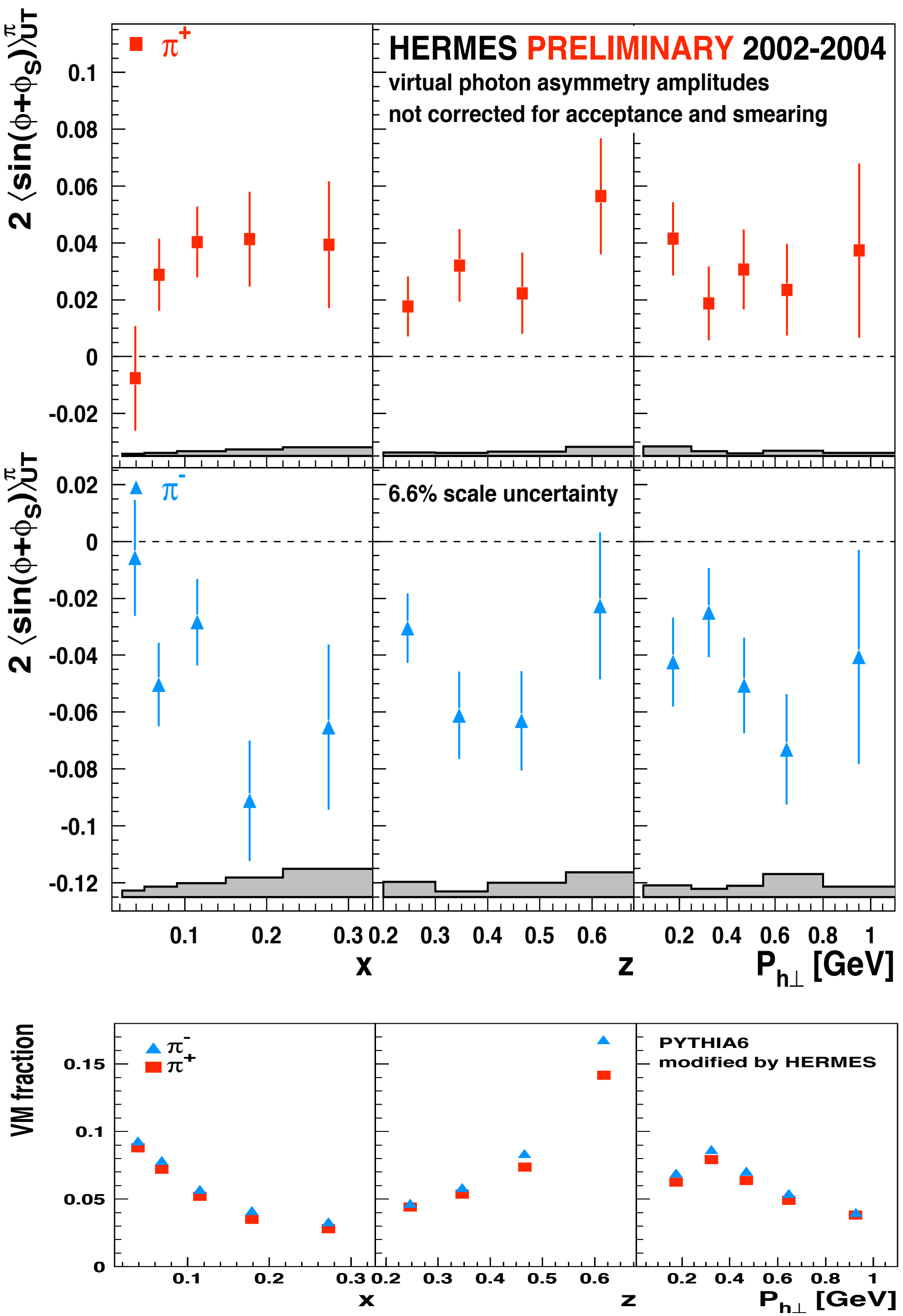
Collins Effect Results :
SIDIS and e^+e^- Annihilation

hermes

COMPASS

BELLE

Collins Moments for π^+ π^- from 2002–2004 $H^\uparrow$ Data



It exists!

- First evidence for **non-zero Collins function** ...
  and **transversity**!

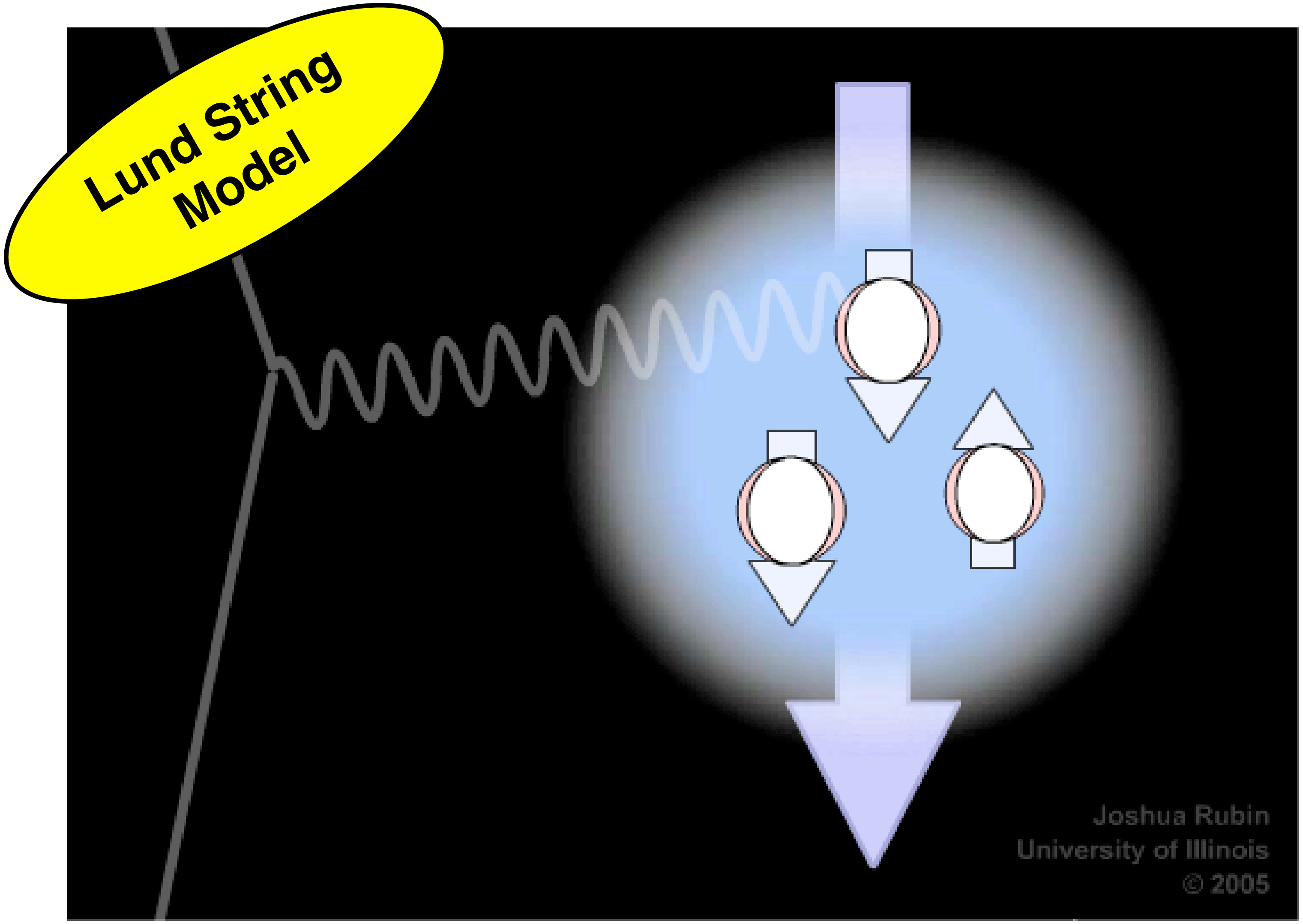
- Positive for π^+...
  Negative and *larger* for π^-...

- Systematic error bands include acceptance and smearing effects, and contributions from unpolarized $<\cos(2\phi)>$ and $<\cos(\phi)>$ moments

Understanding the Collins Effect

The Collins function exists! ➡ **spin-orbit** correlations in π formation

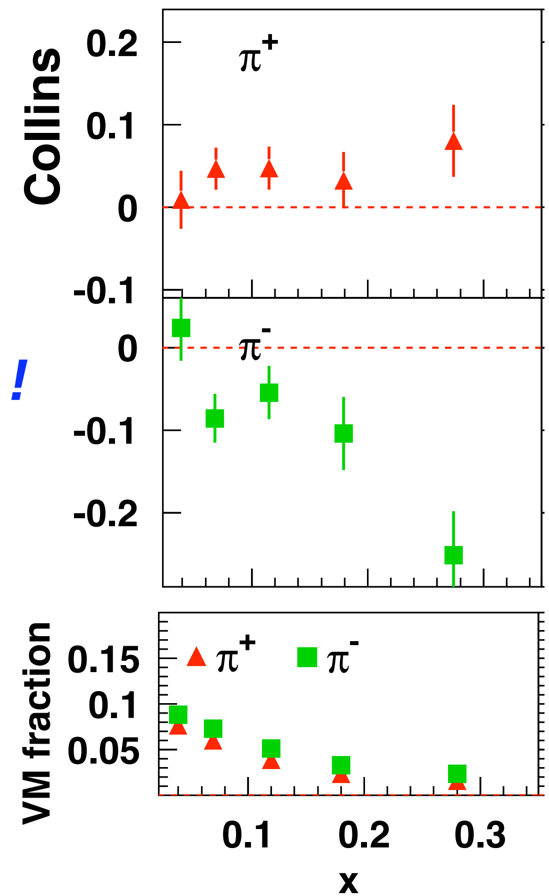
Is the Artru mechanism responsible?



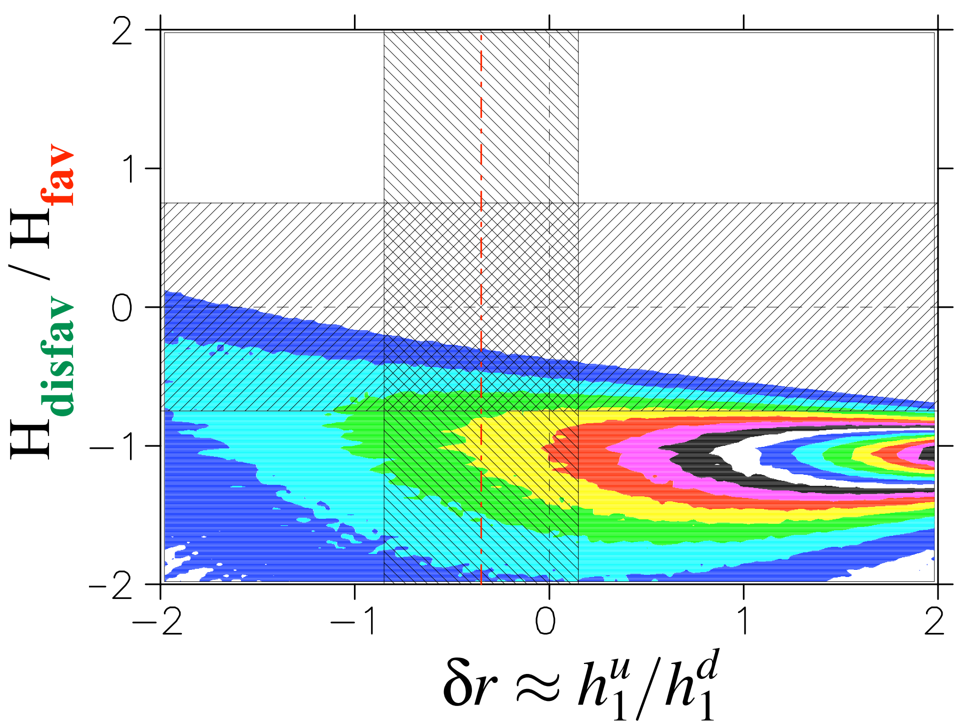
Why are the Collins π⁻ asymmetries so large?

DIS on proton target always dominated by ***u-quark scattering***

- $A_{Col}^{\pi^+} \sim h_1^u \, H_{1,\mathbf{favored}}^{\perp}$ ... expect: **positive**
- $A_{Col}^{\pi^-} \sim h_1^u \, H_{1,\mathbf{disfavored}}^{\perp}$ ... expect: **~ zero**



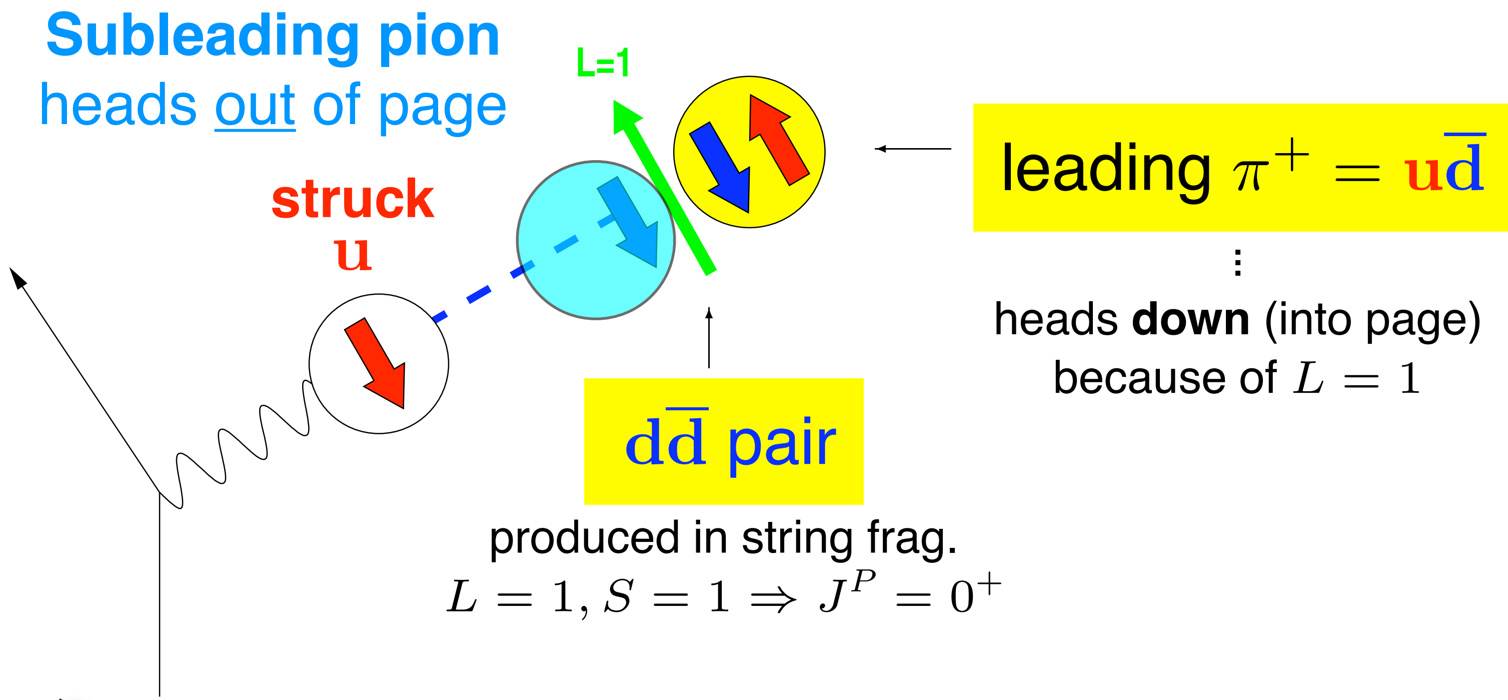
Data indicate ***disfavored*** CollinsFF is ***large*** & ***negative !***



Map out solution space ...
find $H_{\mathbf{disfav}} \approx -H_{\mathbf{fav}}$

Interpretation of Collins Results

Lund model + $^{3}P_{0}$ ***hypothesis*** once more:



- leading π^{+} = **favored** transition, heads into page
- subleading particle (prob π^{-}) = **disfavored** transition, heads out of page

Perhaps $H_{\mathrm{dis}} \approx -H_{\mathrm{fav}}$ ***is not only reasonable, but likely ?***

Collins Global Fit: HERMES & BELLE

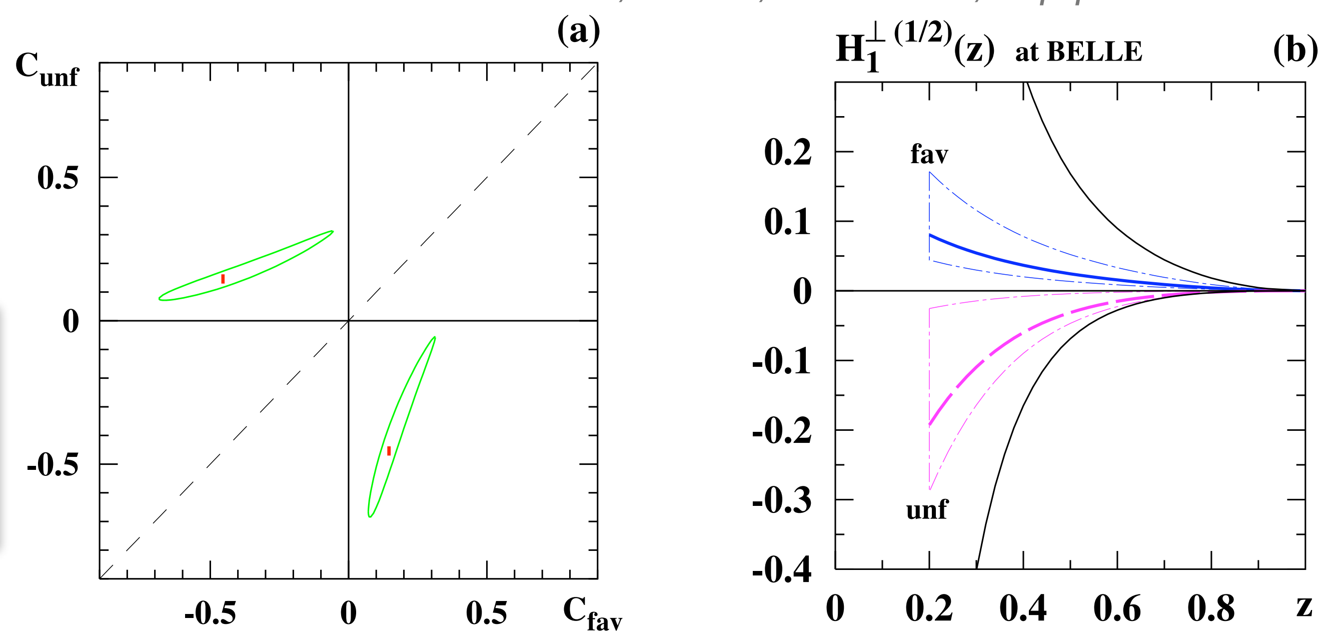
Efremov, Goeke, Schweitzer, hep-ph/0603054

Fit **BELLE** *z*-dependent results to

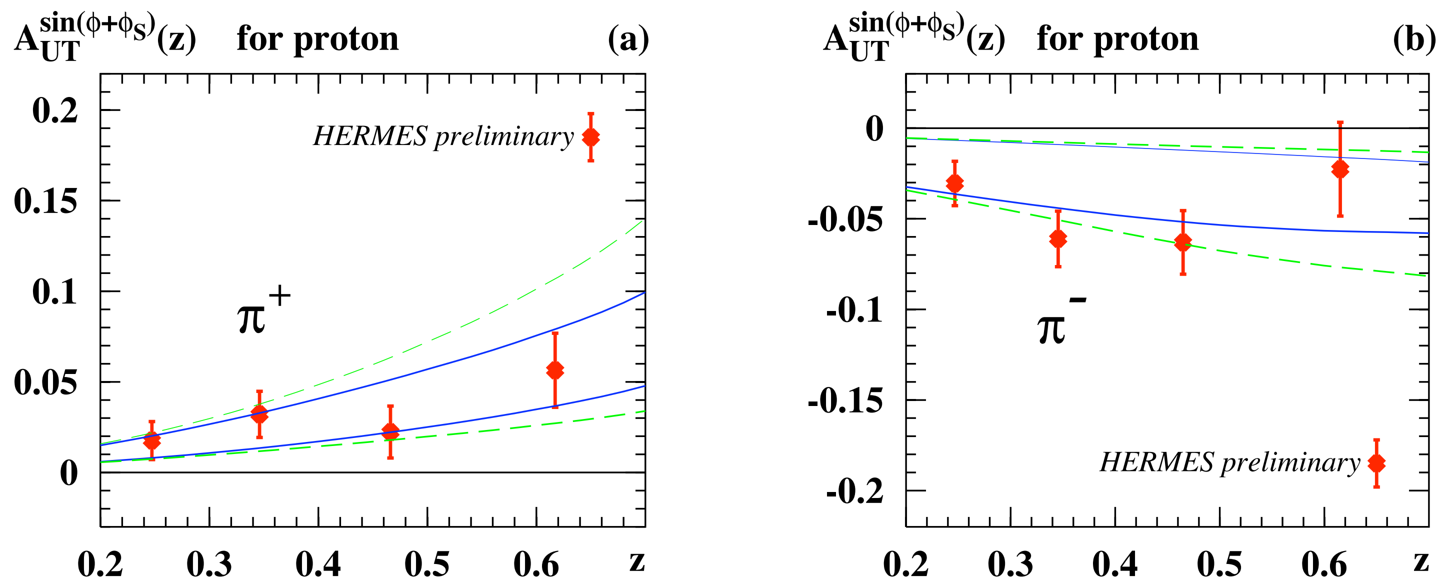
$$H_1^{\perp(1/2)a}(z) = C_a\, z\, D_1^a(z)$$

$C_{fav} = 0.15,\ C_{unf} = -0.45$

and so $H_1^{fav} \approx -H_1^{unf}$



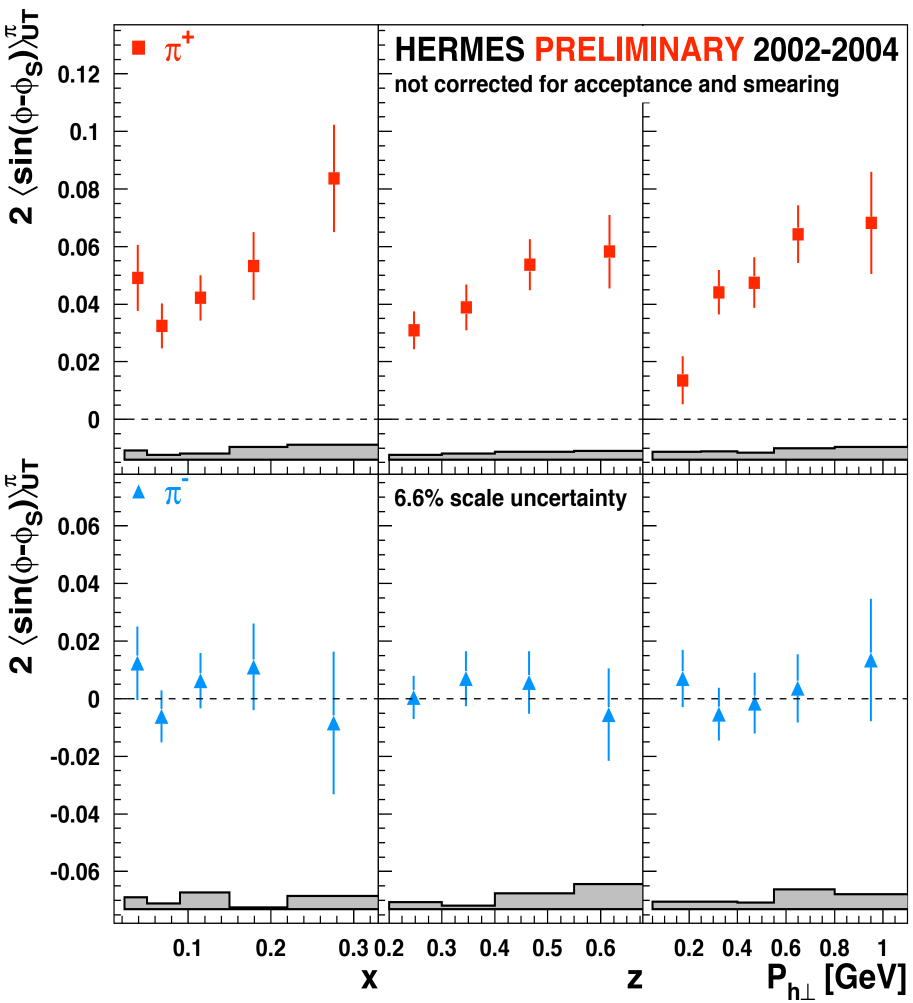
Resulting Collins FF also fit **HERMES** data well



... with $h_1(x)$ from XQSM

Sivers Effect Results:
SIDIS and dijet production

Sivers Moments for π^+ π^- from 2002–2004 $H^{\uparrow}$ Data

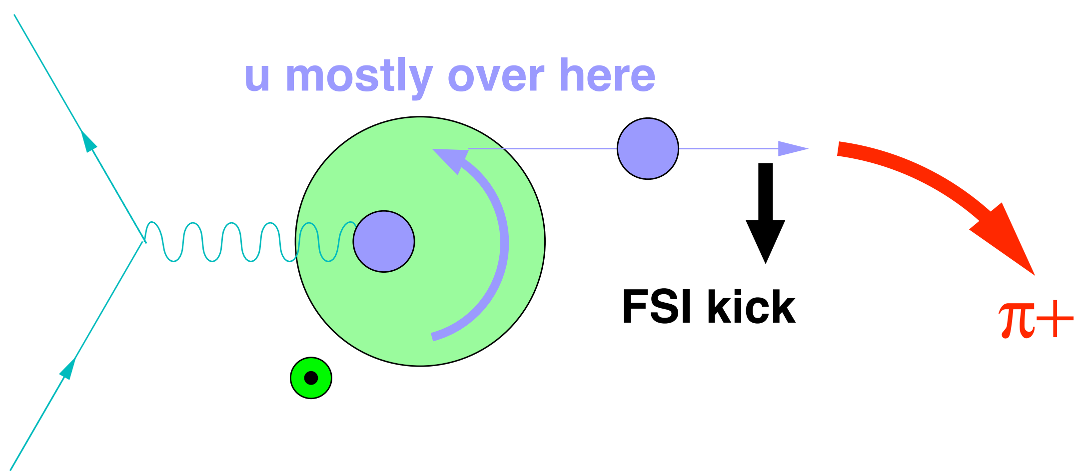


$\Rightarrow$ presence of non-zero **quark orbital angular momentum**

M. Burkardt: Chromodynamic lensing

Electromagnetic coupling $\sim (J_0 + J_3)$
stronger for ***oncoming*** *quarks*

... and most models predict $L_u > 0$

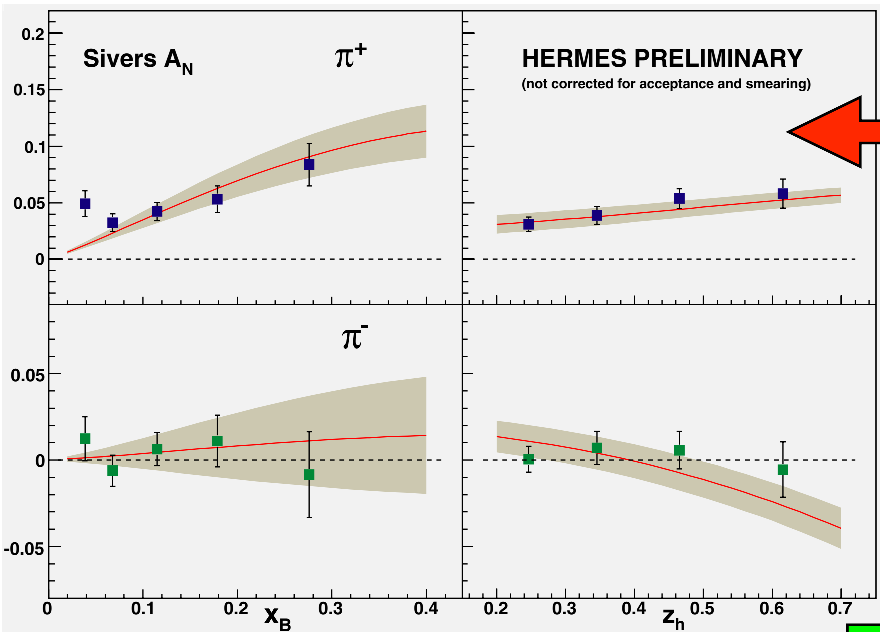


It exists !

Model gives correct sign!

Sivers Global Fit: HERMES & COMPASS

Vogelsang & Yuan, PRD 72 (2005) 054028



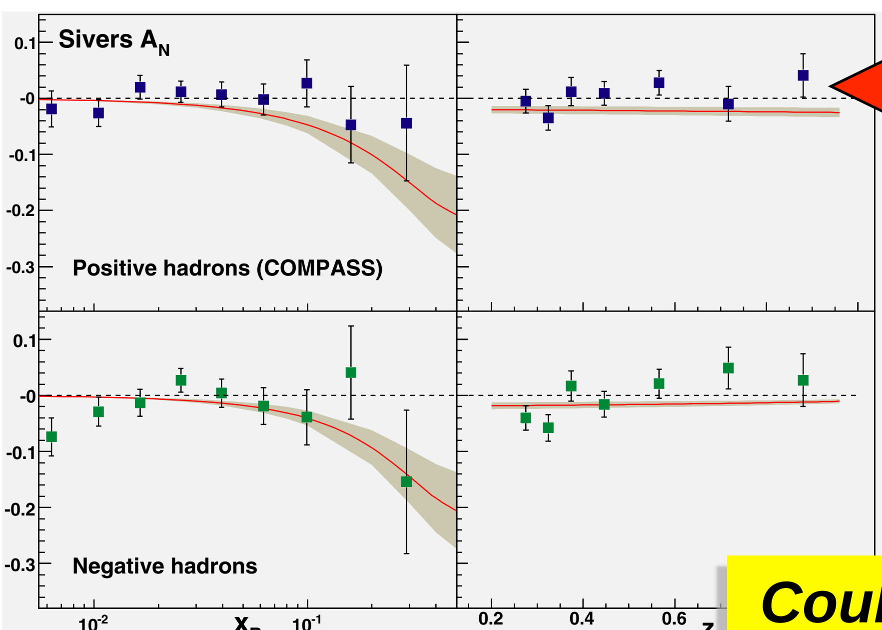
For convenience: $q_T(x) \equiv f_{1T}^{\perp,q}(x)$

Fit HERMES A_{UT} to Sivers func^n of form:

$$\frac{u_T^{(1/2)}(x)}{u(x)} = S_u\, x(1-x), \quad \frac{d_T^{(1/2)}(x)}{u(x)} = S_d\, x(1-x)$$

- assume **no antiquark** Sivers func: $\bar{q}_T(x) = 0$
- unpol PDFs = GRV-LO, unpol FFs = Kretzer

$$\boxed{S_u = -0.81 \pm 0.07, \quad S_d = 1.86 \pm 0.28}$$



Fits COMPASS deuterium data well

But a surprise! $|S_d| \gg |S_u|$!

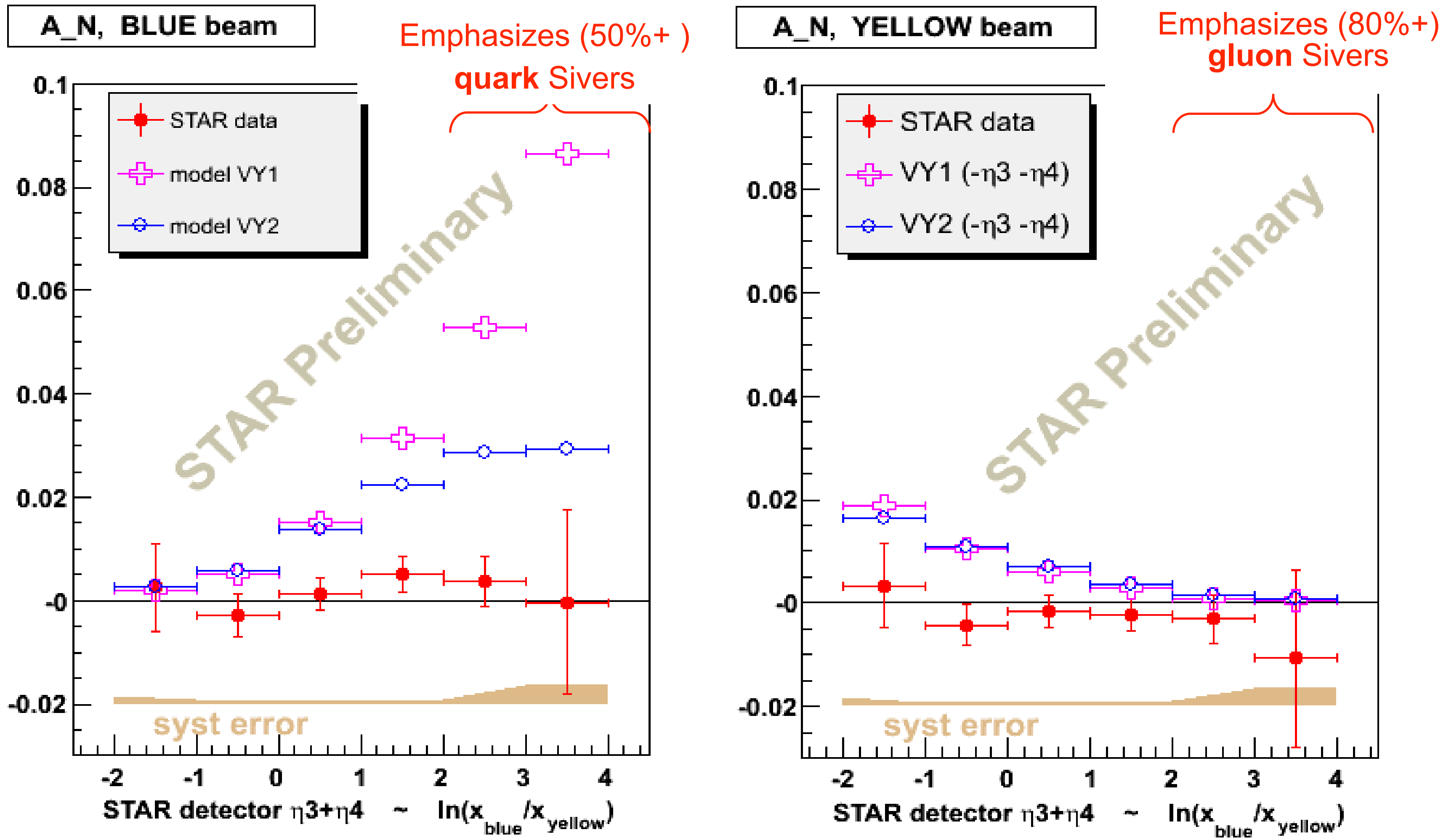
e.g., large-N_C expectation: $u_T(x) \approx -d_T(x)$

Hmm ... S_u *actually* reflects $u_T + \bar{d}_T/4$

... S_d *actually* reflects $d_T + 4\bar{u}_T$

Could Sivers (and L) be large for antiquarks?

Measured Sivers A_N for Di-jets vs. Theory



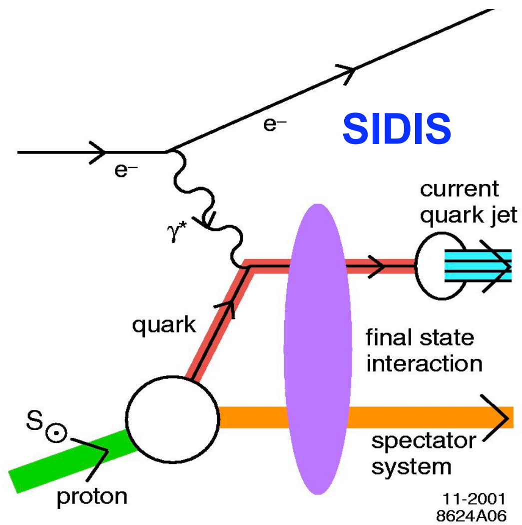
- Model w/o hadronization, integrated over STAR η, 5<p_T<10 GeV/c, includes only quark Sivers -- predicts $A_N \sim A_N^{HERMES}$ where q Sivers dominates
- Sign of predictions reversed to adhere to Madison A_N sign convention

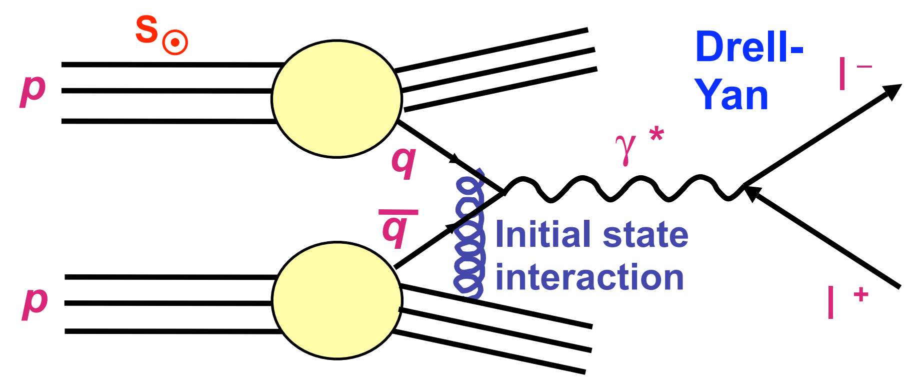
STAR measured A_N all consistent with zero ⇒ both quark and gluon Sivers effects much smaller in $\vec{p}p$ → di-jets than in HERMES SIDIS !!

Theory - exp't discrepancy raises questions!

Are observed di-jet Sivers SSA much smaller than predictions because:

- ***ISI & FSI both important in $\vec{p}p \rightarrow$ jets and tend to cancel?***





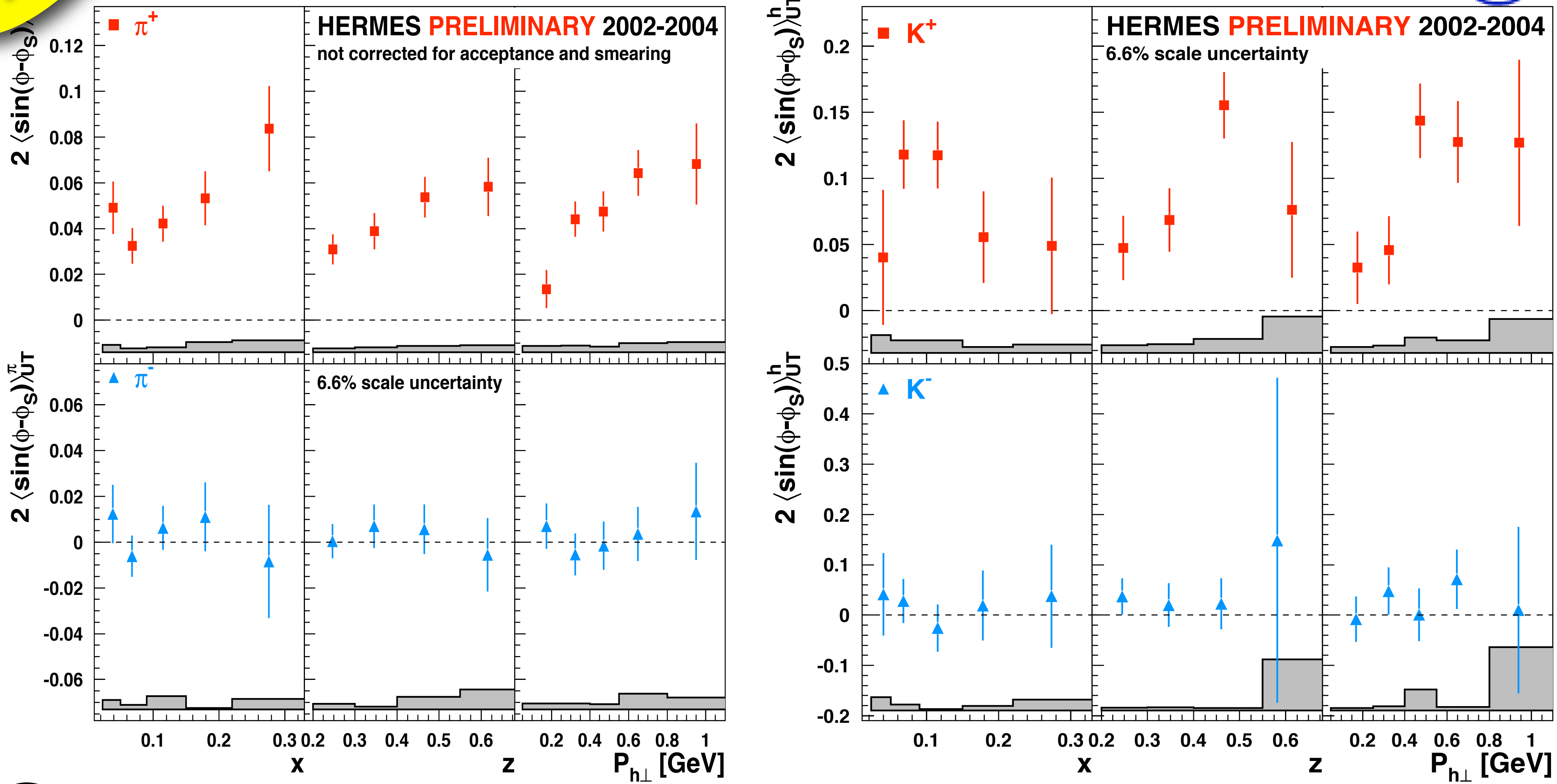
ISI for Drell-Yan vs. FSI for SIDIS ⇒ opposite sign predicted for Sivers SSA

- ***Need $\bar{q}$ Sivers or different q Sivers x, k_T - shapes in HERMES fits?***

- ***If ISI / FSI cancel at mid-rapidity, does their balance change at high η to yield sizable Sivers contribution to observed $\vec{p}p \rightarrow \pi^0 X$ SSA?***

NEW!

Sivers Moments for Kaons from 2002–2004 Data



– Effect about **equal** for $K^- = s\bar{u}$ and $\pi^- = d\bar{u}$ → note: same antiquark ...

+ Effect *seems* ***larger*** for $K^+ = u\bar{s}$ than $\pi^+ = u\bar{d}$ at $x \approx 0.1$ ... !

→ significant ***antiquark*** Sivers functions? and strongly flavor-dependent?

The Tip of the Iceberg:
so much more to discover!

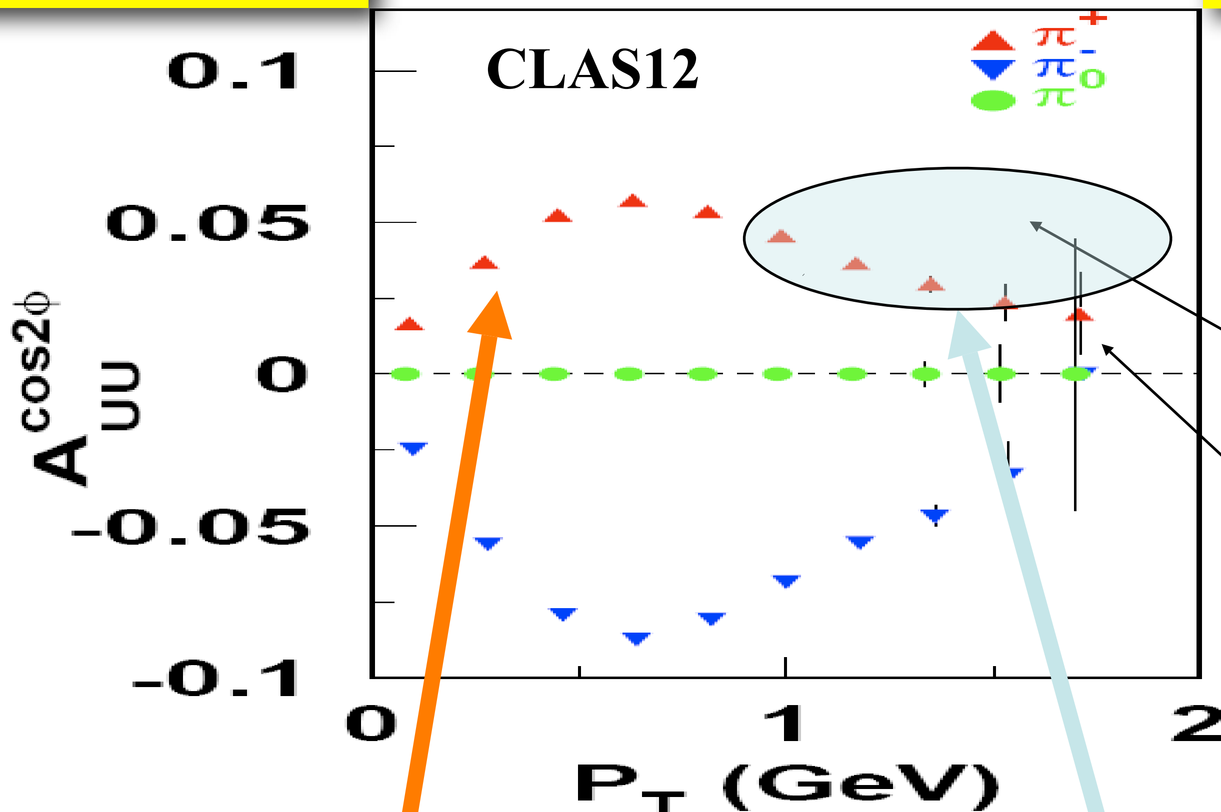
Jefferson Lab 12
Exploring the Nature of Matter

Collins asymmetry & Boer-Mulders Effect

(PR12-06-112)

Transversely polarized quarks in the unpolarized nucleon

$$A_{UU}^{\cos 2\phi} \propto h_1^{\perp} H_1^{\perp}$$



$s_T(p \times k_T) \leftrightarrow h_1^{\perp}$

$\sin(\phi_C) = \cos(2\phi_h)$

In the perturbative limit $1/P_T^2$ behavior expected (F.Yuan)

quark-scalar diquark model (L.Gamberg)

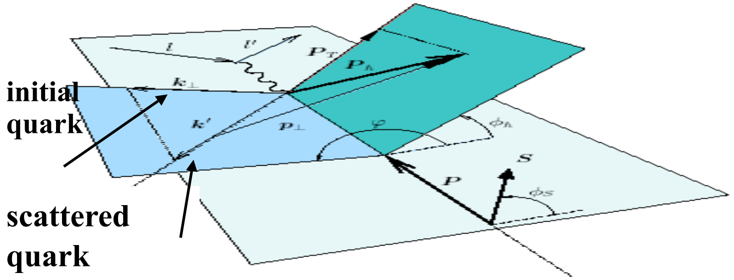
4<Q^2<5 (2000h @ 11 GeV with 10^{35}sec^{-1}cm^{-2})

Non-perturbative TMD

Perturbative region

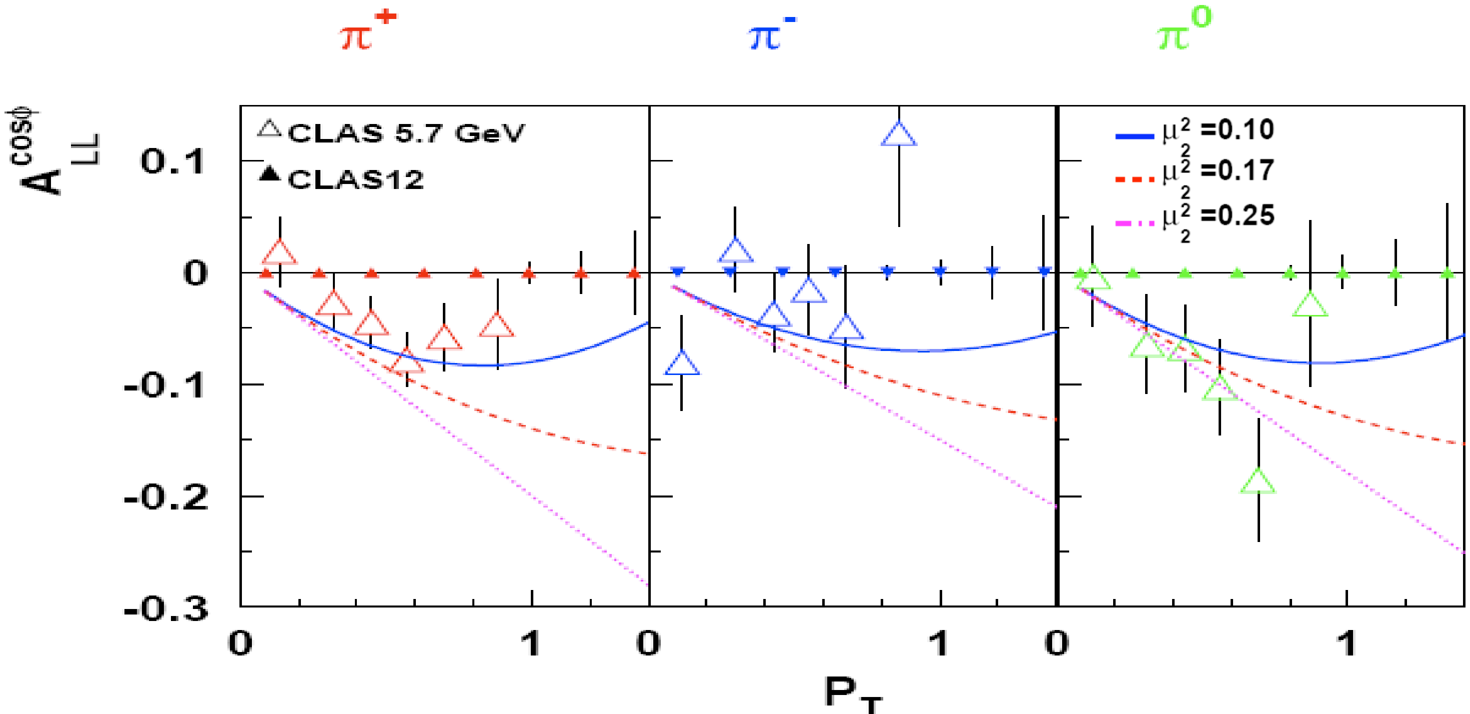
$$\Lambda_{QCD} \ll P_T \ll Q$$

- BM cos2ϕ moment, sensitive to spin-orbit correlations: the only leading twist azimuthal moment for unpolarized target
- P_T-dependence of BM asymmetry allows studies of transition from non-perturbative to perturbative description (Unified theory by Ji et al).
- More info will be available from SIDIS (HERMES,COMPASS,ZEUS,EIC) and DY (RHIC,GSI)



A_1-P_T-dependence

hep-ph/0608048
μ_0^2=0.25GeV^2
μ_D^2=0.2GeV^2



$$\sigma_0 = \frac{1+(1-y)^2}{xy^2}\,\frac{1}{\mu_D^2 + z^2\mu_0^2}\exp\left(-\frac{P_{hT}^2}{\mu_D^2 + z^2\mu_0^2}\right)\sum_q e_q^2\, f_1^q(x)\, D_q^h(z)$$

$$\Delta\sigma_{LL}^{\cos\phi_h} = -4\,\frac{\sqrt{1-y}}{xy}\,\frac{z\,\mu_2^2\, P_{hT}}{Q\,(\mu_D^2 + z^2\mu_2^2)^2}\exp\left(-\frac{P_{hT}^2}{\mu_D^2 + z^2\mu_2^2}\right)\sum_q e_q^2\, g_1^q(x)\, D_q^h(z)$$

$$f_1^q(x,k_\perp) = f_1^q(x)\,\frac{1}{\pi\mu_0^2}\exp\left(-\frac{k_\perp^2}{\mu_0^2}\right),$$

$$D_q^h(z,p_\perp) = D_q^h(z)\,\frac{1}{\pi\mu_D^2}\exp\left(-\frac{p_\perp^2}{\mu_D^2}\right)$$

$$g_1^q(x,k_\perp) = g_1^q(x)\frac{1}{\pi\mu_2^2}exp(-\frac{k_\perp^2}{\mu_2^2})$$

P_T-dependence of the cosϕ moment of double spin asymmetry is consistent with ***significant difference in k_T-distributions of polarized and unpolarized quarks***

TMD's: Spin-Orbit Effects in QCD

A great deal has been learned!

- **Collins** effect isolated for the first time at **HERMES**
  - **sign** of effect supports **$^3P_0$ picture** of color string breaking
  - **favored/disfavored** Collins functions of **opposite sign**
  - result **confirmed** by new data from **BELLE** & **COMPASS**
- **Sivers** effect is ***non-zero*** in DIS: quark orbital motion!
  - successful global analysis of **HERMES** (H) & **COMPASS** (D)
  - large **antiquark** contributions to **orbital *L*** indicated ... !
- Theoretical fusion betw **TMD** (low p_T) and **twist-3** (hi p_T) descriptions of **single-spin asymmetries**

Hadron Structure: Outlook

SURA QCD Workshop

A great deal has been learned / is on its way from

JLab, RHIC, HERMES, COMPASS, FNAL-E906, ...

about
- the internal spin and flavor ***landscape*** of hadrons,
- the formation of ***hadronic final-states*** in QCD ...

but not nearly enough to test / understand QCD in any conclusive way

If these **key questions** are to be answered, and the **study of QCD** is to move forward, **new facilities** are needed.