# Effective Field Theory

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Nuclear Physics 2007 Long Range Plan
Joint Town Meeting on QCD
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# Effective Field Theories of QCD

ChPT
pions

HTL
large T

NRQCD
$c\bar{c}$ states

large density
HDET

unstable particle EFT
top quark

QCD

energetic hadrons
SCET

jets
SCET

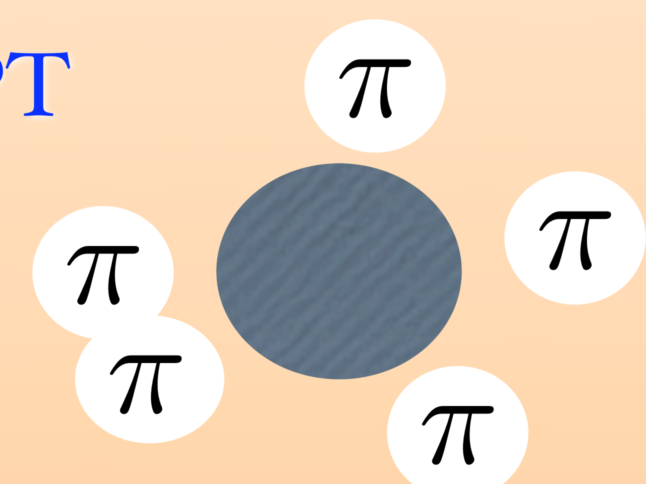
$b\bar{d}$ states
HQET

single nucleon
Heavy Hadron ChPT

nuclear forces
NNEFT

Arrows: Effective theories describe infrared limits of QCD

ChPT

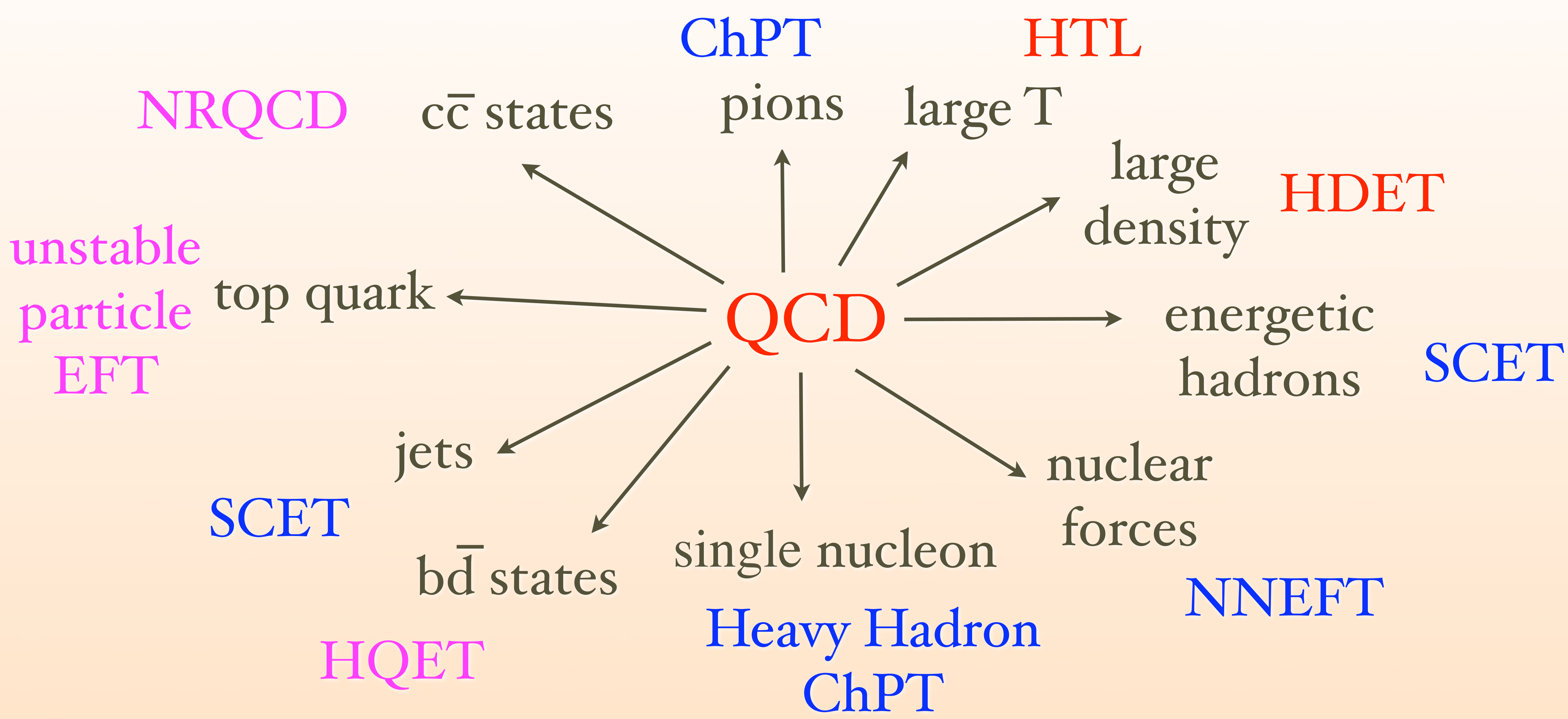



$$\frac{m_\pi}{\Lambda} \ll 1$$

$$\frac{p}{\Lambda} \ll 1$$

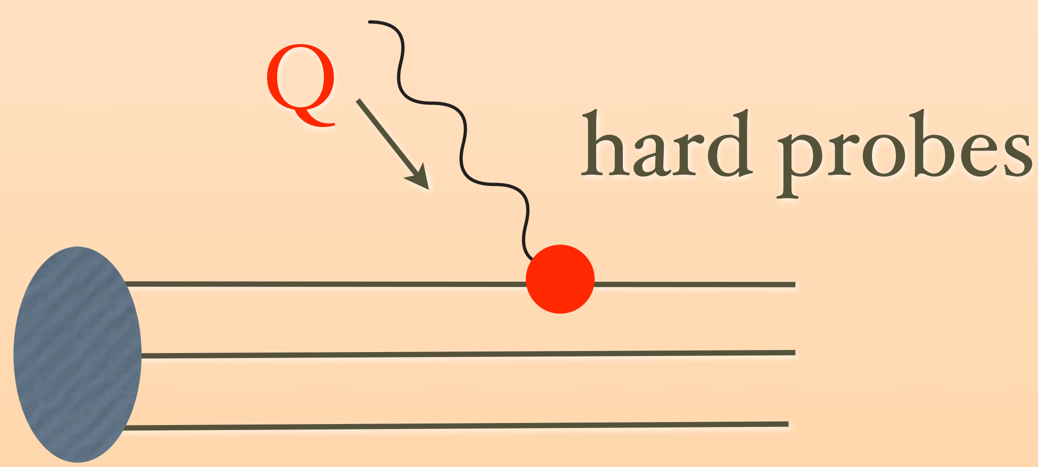
talks by D. Kaplan, D. Phillips, H. Griesshammer

# Effective Field Theories of QCD



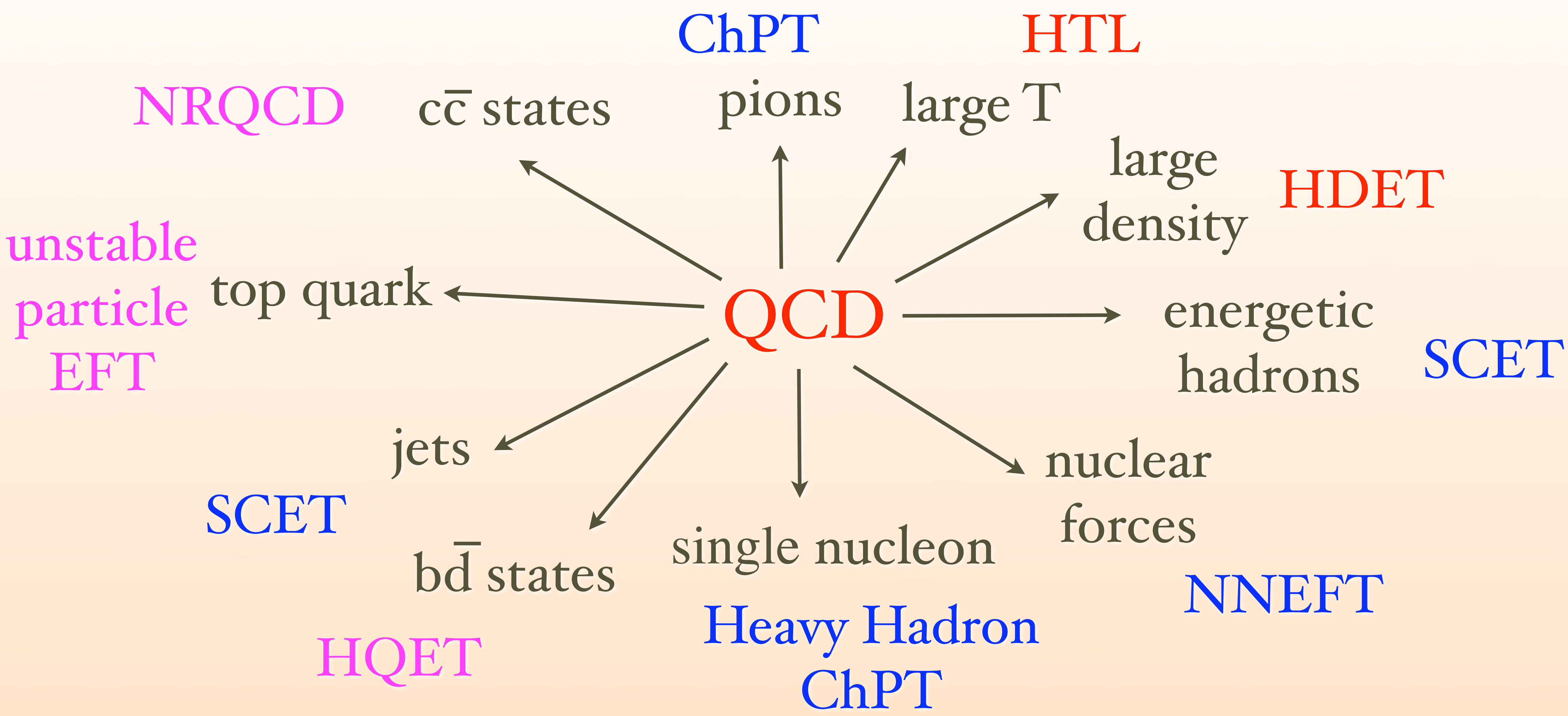

Arrows: Effective theories describe infrared limits of QCD

Soft-Collinear Effective Theory (SCET)

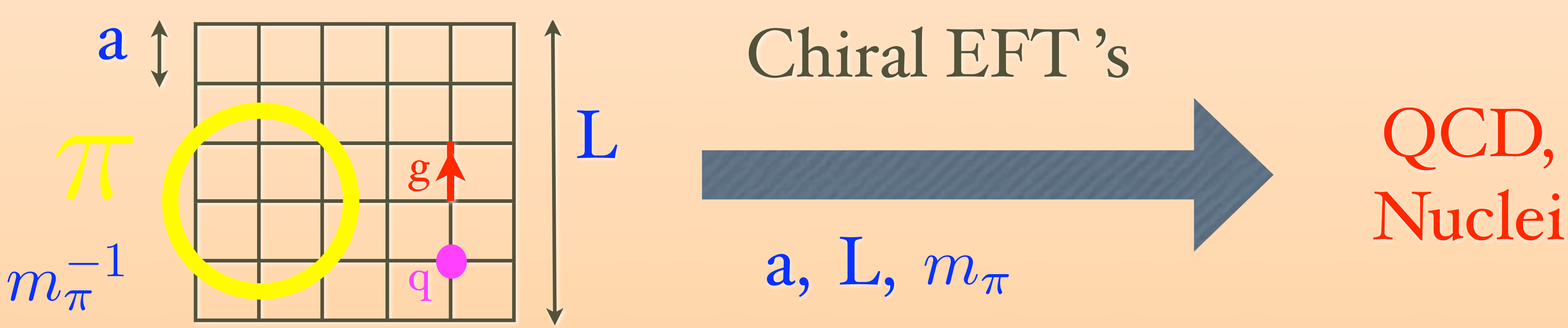



$$\frac{\Lambda}{Q} \ll 1$$

# Effective Field Theories of QCD




## Lattice QCD



QCD $Q$

↓ perturbative

SCET $\Lambda$

$$F(Q^2) = \frac{1}{Q^2} \int dx dy \, H(x,y) \, \phi(x) \, \phi(y)$$

QCD $\Lambda$

↓ non-perturbative

ChPT $m_\pi$

$$\mathcal{L} = \frac{f^2}{8} \mathrm{tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] + v \, \mathrm{tr}\left[m_q^\dagger \Sigma + m_q \Sigma^\dagger\right] + \mathcal{L}^{(4)}(L_i)$$

# Lattice & EFT - extrapolations

Symanzik Action: $S_{\text{Symanzik}} = S_0 + a\,S_1 + a^2\,S_2 + \ldots$

$a \ll \Lambda_{QCD}^{-1}$

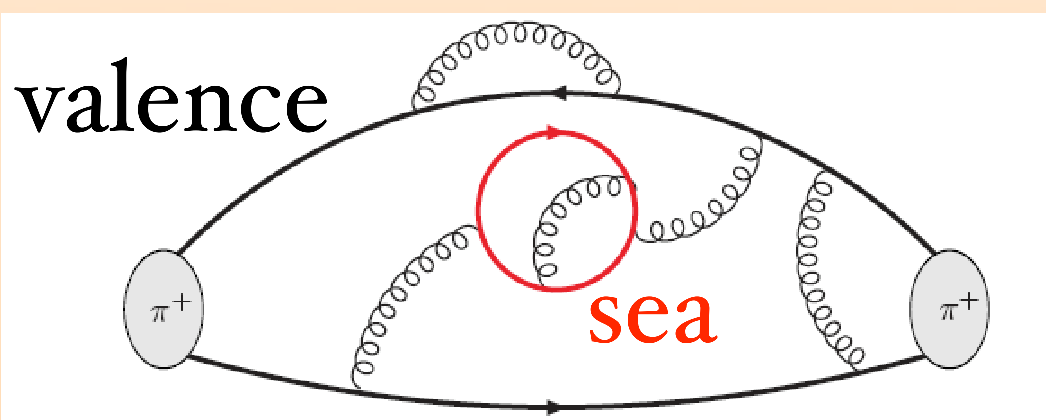
$S_0 = \overline{\psi}(\not{D} + m)\psi \qquad S_1 = a\,c_{SW}\,\overline{\psi}\sigma_{\mu\nu}G_{\mu\nu}\psi$

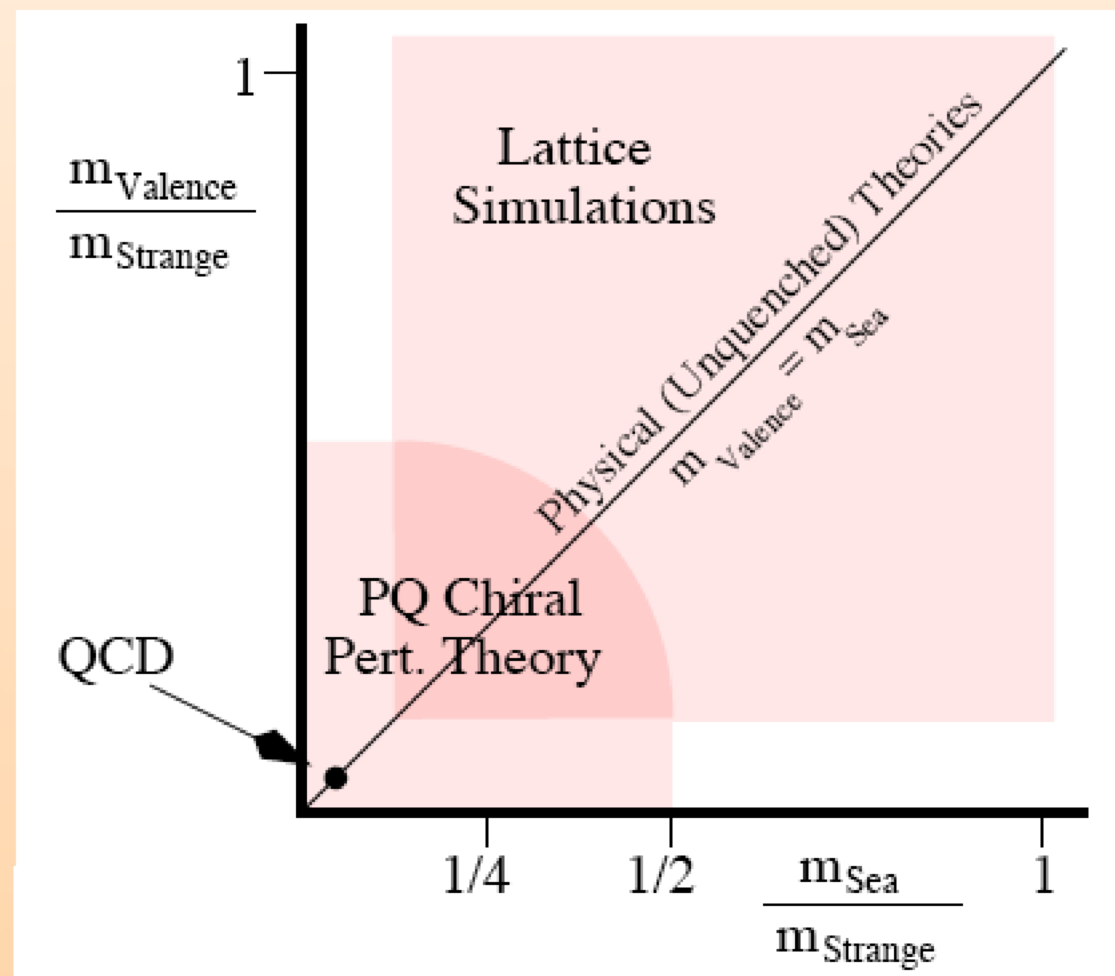
Solutions to fermion doubling generally lead to chiral symmetry breaking at finite $a$, with $m = 0$

reviewed by Tiburzi at Chiral Dynamics'06

Each lattice fermion has differing $\chi$EFT
Wilson, Twisted mass Wilson, Staggered,
Ginsparg-Wilson (Domain Wall), Hybrid (Mixed Action)

Partially Quenched

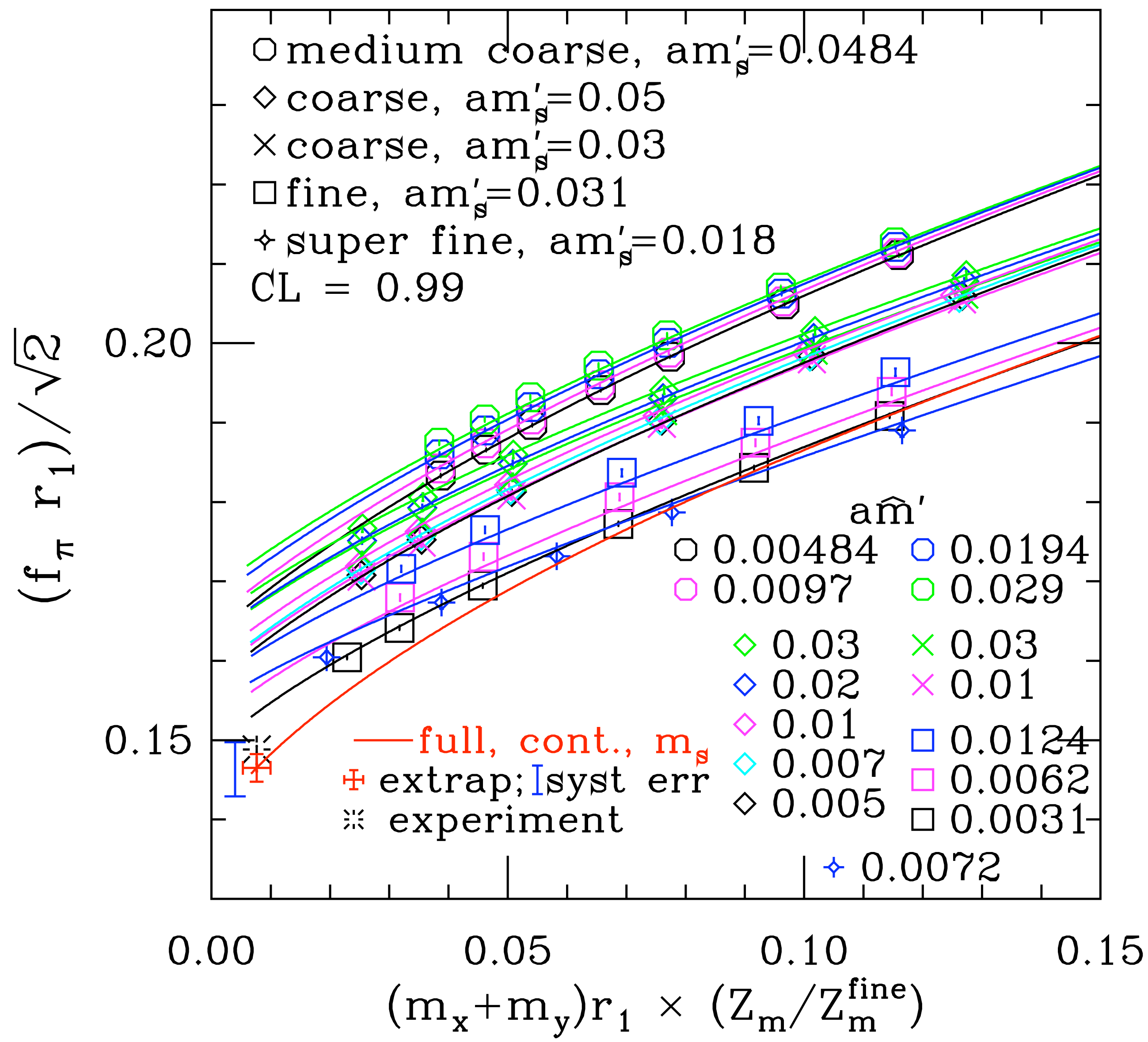


# Decay constants $f_K, f_\pi$

MILC

from C.Bernard




## Staggered Fermions

unquenched, fast, light $m_\pi$

but 4 “tastes” for each flavor,

issue: $\det(\not{D} + m) \to \det(\not{D} + m)^{1/4}$

lots of parameters in ChPT

$$f_\pi = 128.6 \pm 0.4 \pm 3.0 \text{ MeV}$$

nuclear beta-decay gives

$$f_\pi = 130.7 \pm 0.1 \pm 0.4 \text{ MeV}$$

$$\left.\frac{f_K}{f_\pi}\right|_{\text{MILC}} = 1.210 \pm 0.004 \pm 0.013$$

## Domain Wall valence

NPLQCD (‘06)

$$\frac{f_K}{f_\pi} = 1.218 \pm 0.002 \,{}^{+0.011}_{-0.024}$$

# And **Low Energy Constants** (preliminary)

$$\begin{aligned} 2L_6 - L_4 &= 0.5(1)(2) \\ 2L_8 - L_5 &= -0.1(1)(1) \\ L_4 &= 0.1(2)(3) \\ L_5 &= 2.0(3)(2) \end{aligned}$$

at chiral scale $m_\eta$

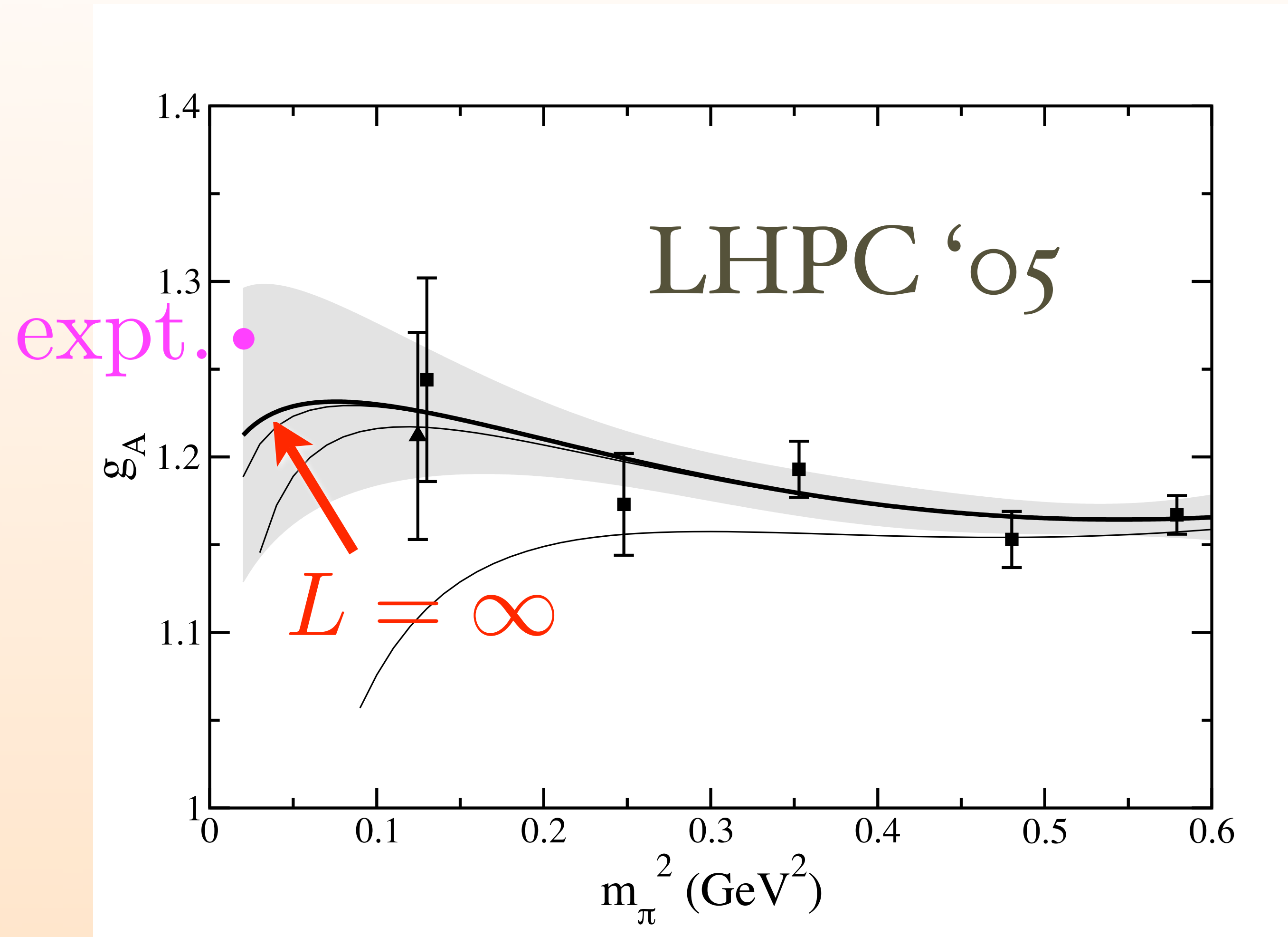
units $10^{-3}$

# Nucleon axial charge $g_A$

Staggered Sea quarks

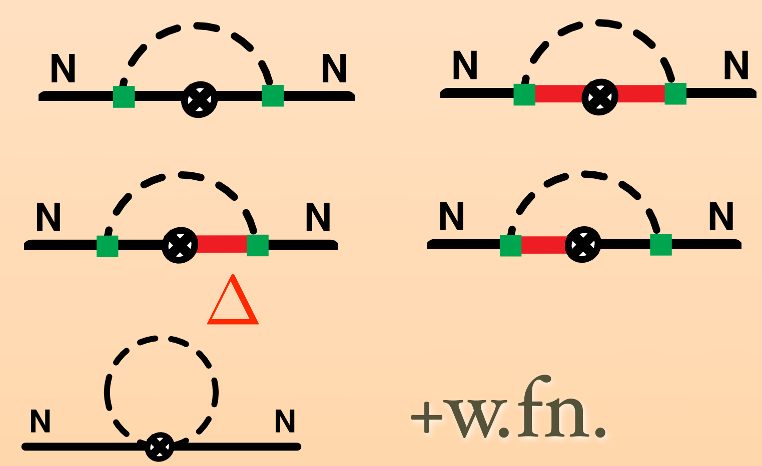
Domain Wall valence

(see talks by Negele, Orginos)




Chiral Extrap. at finite volume

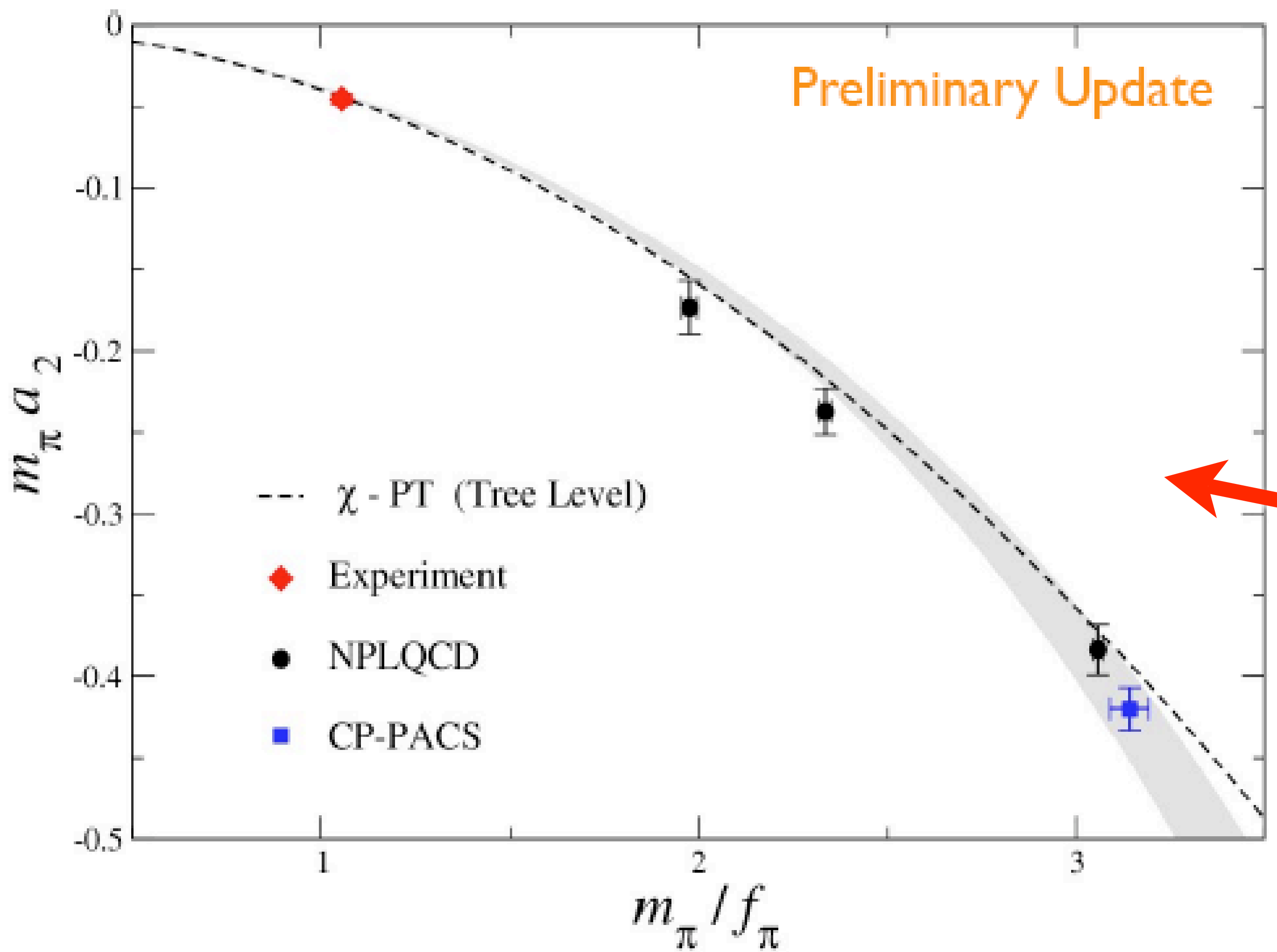
$$\Gamma^{L}_{NN} = g_A - i\frac{4}{3f^2}\Big[4g_A^3\ J_1(m_\pi,0,\mu) + 4\left(g_{\Delta N}^2 g_A + \frac{25}{81}g_{\Delta N}^2 g_{\Delta\Delta}\right)\ J_1(m_\pi,\Delta,\mu) + \frac{3}{2}g_A\ R_1(m_\pi,\mu) - \frac{32}{9}g_{\Delta N}^2 g_A\ N_1(m_\pi,\Delta,\mu)\Big]$$
$$+ \frac{m_\pi^2}{3\pi^2 f^2}\Big[g_A^3\mathbf{F_1} + \left(g_{\Delta N}^2 g_A + \frac{25}{81}g_{\Delta N}^2 g_{\Delta\Delta}\right)\mathbf{F_2} + g_A\mathbf{F_3} + g_{\Delta N}^2 g_A\mathbf{F_4}\Big]$$




Beane & Savage, '04 Detmold & Lin, '05

# Extrapolations for $I = 2$ $\pi\pi$ scattering

Chen, O'Connell,
Van De Water, Walker-Loud '06




NPLQCD:
Isospin 2 pion scattering length: Domain-wall valence quarks on staggered sea quarks.

S. Beane, P. Bedaque, K. Orginos, M. Savage PRD73 (2006)

Experimental point NOT used to constrain fit

$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 + l_{\pi\pi}(\mu)\right] - \frac{m_\pi^2}{(4\pi f_\pi)^2}\frac{\tilde{\Delta}_{PQ}^4}{6m_\pi^4}\right\}$$

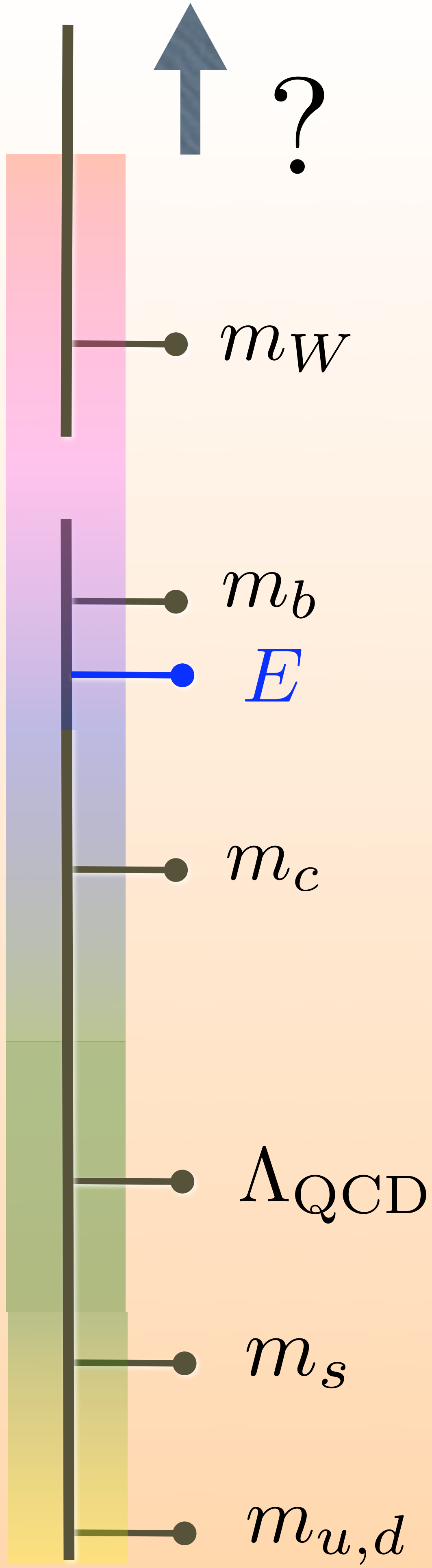
form independent of sea quarks

$\Rightarrow$ $l_{\pi\pi}(\mu)$ perturbatively insensitive to sea quarks and lattice spacing

# Soft - Collinear Effective Theory

power counting parameter $\lambda = \dfrac{\Lambda}{Q} \ll 1$

Bauer, Fleming, Luke, Pirjol, Stewart



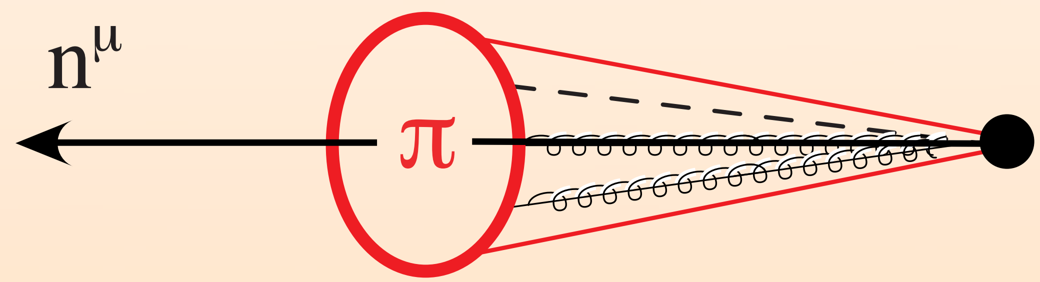

# Electromagnetic Probes

$e^- p \to e^- X \qquad p\bar{p} \to X \ell^+ \ell^-$

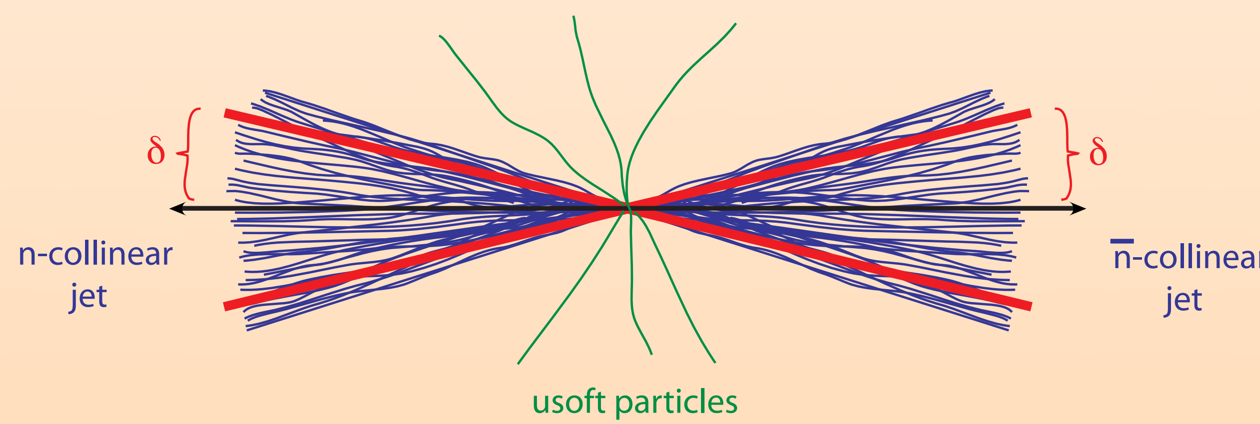
$e^- \gamma \to e^- \pi^0 \qquad \gamma^* M \to M' \qquad \Upsilon \to X\gamma$

$e^+ e^- \to \text{jets} \qquad e^+ e^- \to J/\Psi X$

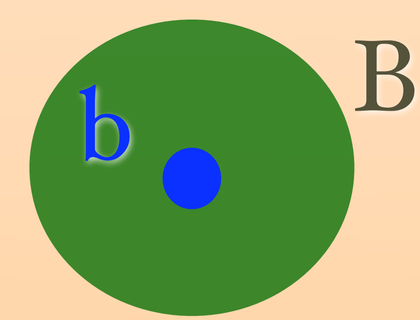
Energetic pion (proton) with collinear constituents




energetic jets



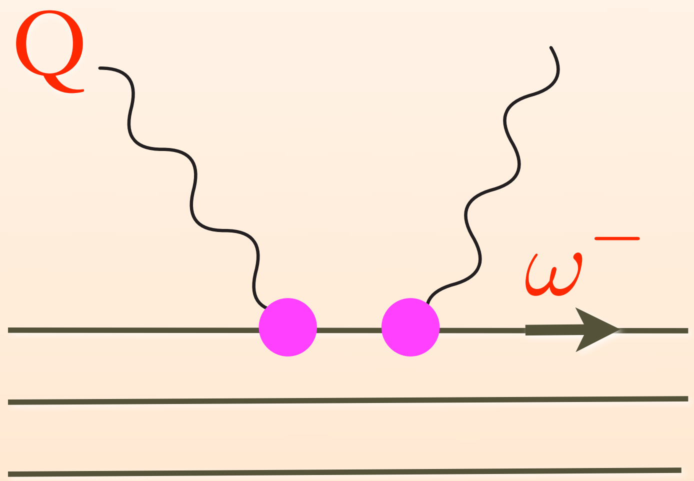

B - decays by weak interactions



# Basic Building Blocks in SCET are collinear "parton" fields

(can be derived starting from QCD and removing hard fluctuations)

matrix elements of these building blocks probe properties of hadrons




eg. Fourier Transform of standard twist-2 quark p.d.f

$$\Big\langle p \Big| (\bar{\xi}_n W) \bar{n}\!\!\!/ \, \underbrace{\delta(w^- - \hat{P}^-) \underbrace{(W^\dagger \xi_n)}_{\chi_n}}_{\chi_{n,\omega^-}} \Big| p \Big\rangle = f_q(\omega^-/p^-)$$

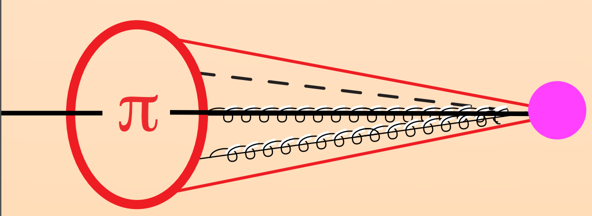
forward m.elt.

parton field with momentum $\omega^-$

get GPD from:

$$\Big\langle p \Big| \bar{\chi}_{n,\omega} \bar{n}\!\!\!/ \chi_{n,\omega'} \Big| p' \Big\rangle$$

gluon p.d.f. $\Big\langle p \Big| \mathrm{tr}\, \mathcal{B}^{\perp}_{\mu} \mathcal{B}^{\mu}_{\perp,\omega} \Big| p \Big\rangle = f_g(\omega^-/p^-)$




hard interaction creates a pion or proton

twist-2 pion distribution $\Big\langle \pi(p_\pi) \Big| \bar{\chi}_n \, \bar{n}\!\!\!/ \gamma_5 \, \chi_{n,\omega} \Big| 0 \Big\rangle = \phi_\pi\Big(\frac{\omega^-}{p_\pi^-}\Big)$

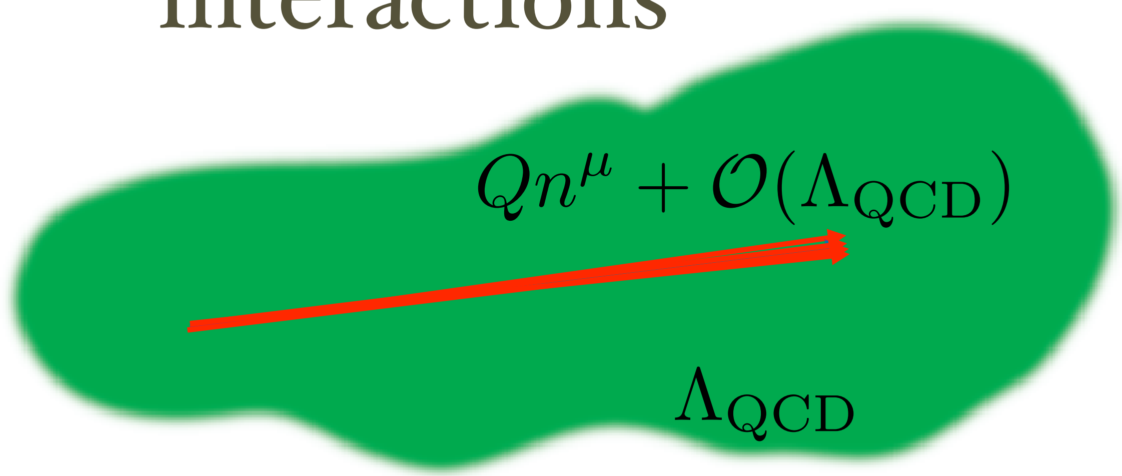
twist-2 proton distribution $\Big\langle p \Big| \bar{\chi}^i_{n,\omega} \bar{\chi}^j_{n,\omega'} \bar{\chi}^k_n \Gamma^{ijk} \Big| 0 \Big\rangle = \phi_p\Big(\frac{\omega^-}{p^-}, \frac{\omega'^-}{p^-}\Big)$

At leading order SCET reproduces well known factorization theorems.

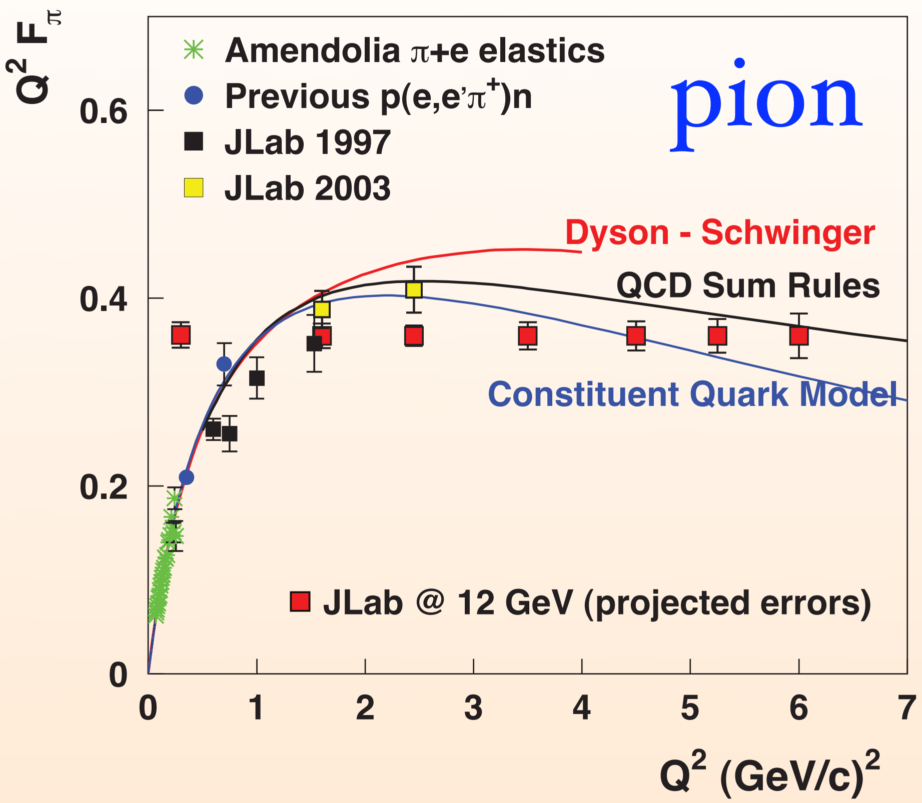
Systematically improvable. So we can attack power corrections in a whole host of processes.

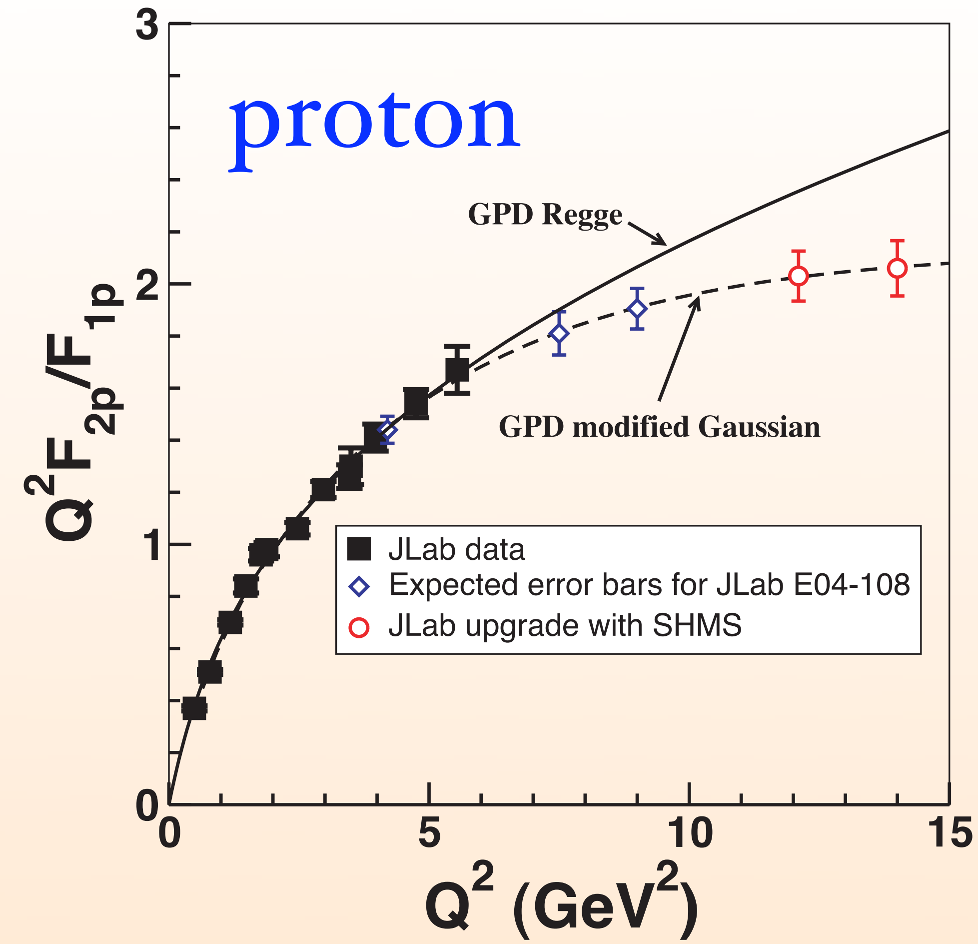
$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

A field theory for
Soft & Collinear
interactions



# Form Factors




$$F_\pi(Q^2) \quad = \frac{f_\pi^2}{Q^2}\int_0^1 dxdy\, H(x,y)\,\phi_\pi(x)\phi_\pi(y)$$

$$+\ldots$$

$$F_1^p(Q^2) = \frac{f_p^2}{Q^2}\int dx_i dy_j\, H(x_i,y_j)\,\phi_p(x_i)\phi_p(y_j)$$

$$+\ldots$$

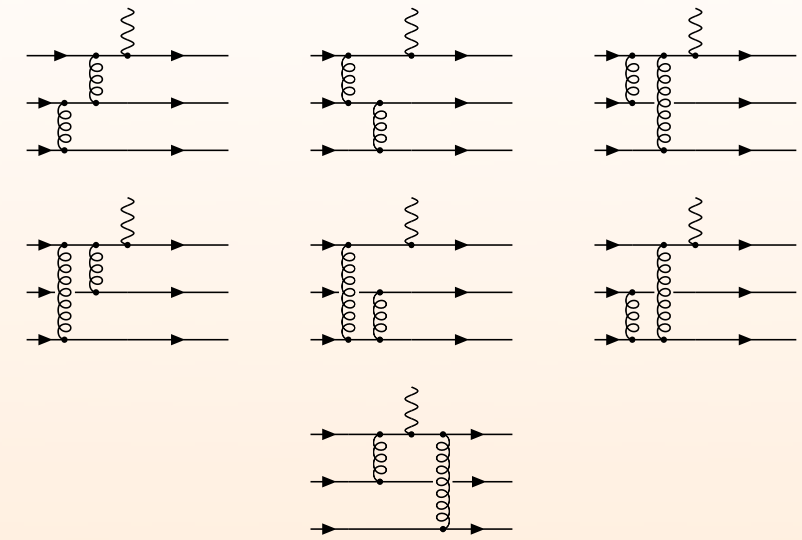
but $\quad Q^2 F_2^p/F_1^p \neq \text{constant}$

SCET can be used to systematically compute these (+...) power corrections

As we learned, so far Q^2 was not big enough, but with 12 GeV upgrade these corrections become quite interesting

# Endpoint Singularites

$$\int_0^1 \mathrm{d}x \, \frac{\phi_\pi(x)}{x^2} = \int_0 \frac{dx}{x} = ?$$



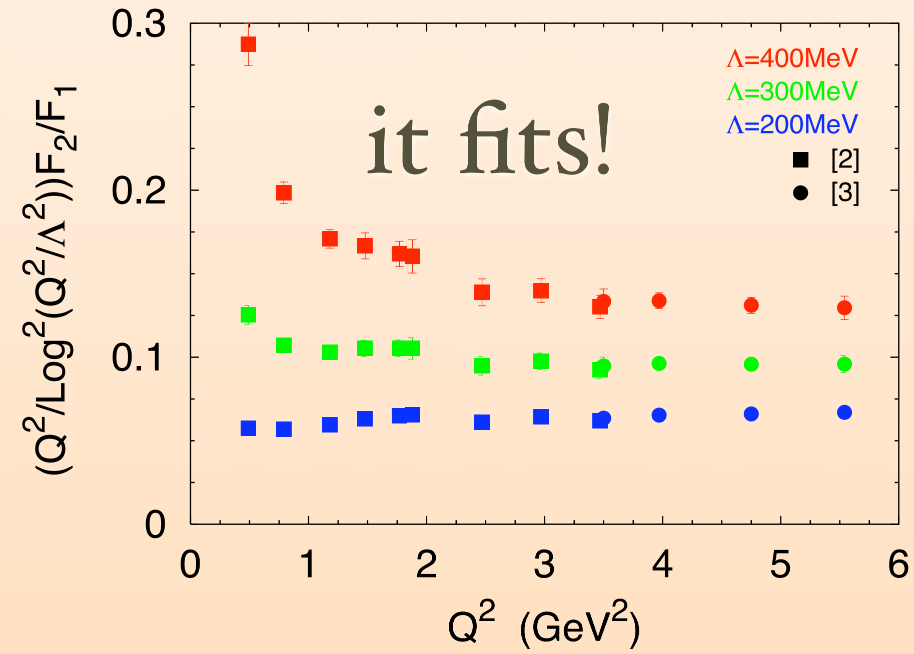
At subleading order one often encounters endpoint singularities

Is this a breakdown of factorization?

Belitsky, Ji, Yuan ('03)

$$\frac{Q^2 F_2^p}{F_1^p} \sim \ln^2\left(\frac{Q^2}{\Lambda^2}\right)$$

from singularity




**Resolution:** need rapidity, $\zeta$ , dependent distribution functions

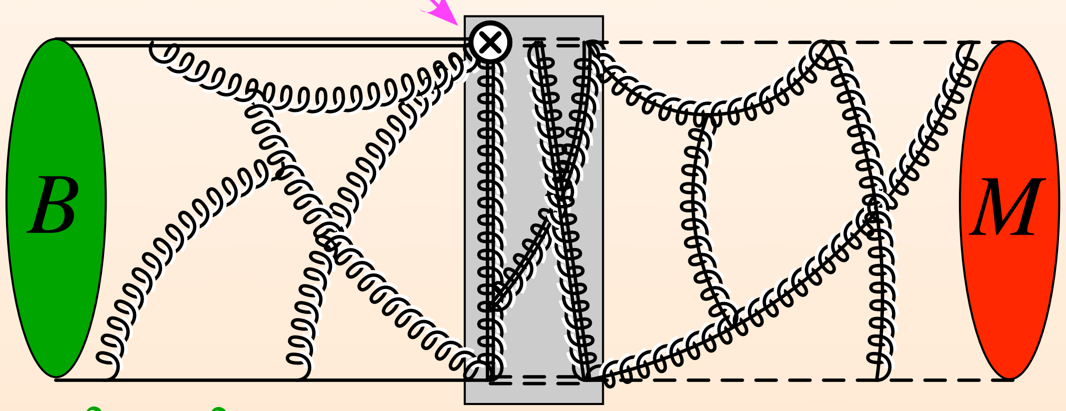
$\phi_\pi(x, \zeta)$ Manohar, I.S. ('06)

similar thing is known to happen for $k_\perp$ dependent p.d.f's

$f_q(x, k_\perp, \zeta)$ for a review see Collins (hep-ph/0304122)

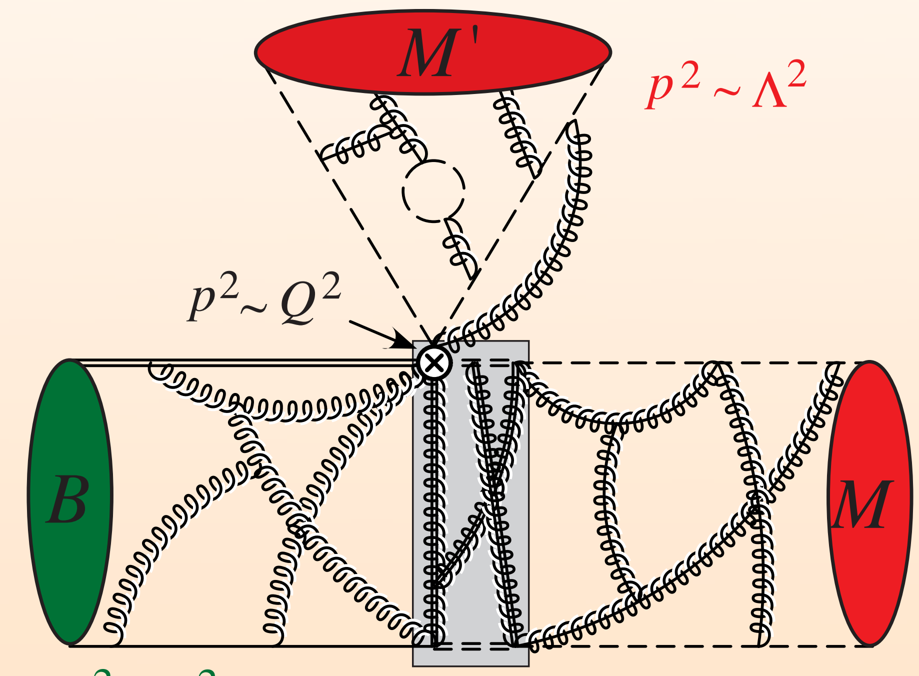
# Test SCET in B-Physics where Q is a bit bigger

## Form Factors




$B \to \pi\pi$

important for measuring CP-violation




## Single Jet Production

$$\frac{d\Gamma}{dE_\gamma} = \left|C(m_b,\mu)\right|^2 \int dk^+ \mathrm{Im} J_P(k^+,\mu) S(2E_\gamma - m_b + k^+,\mu)$$

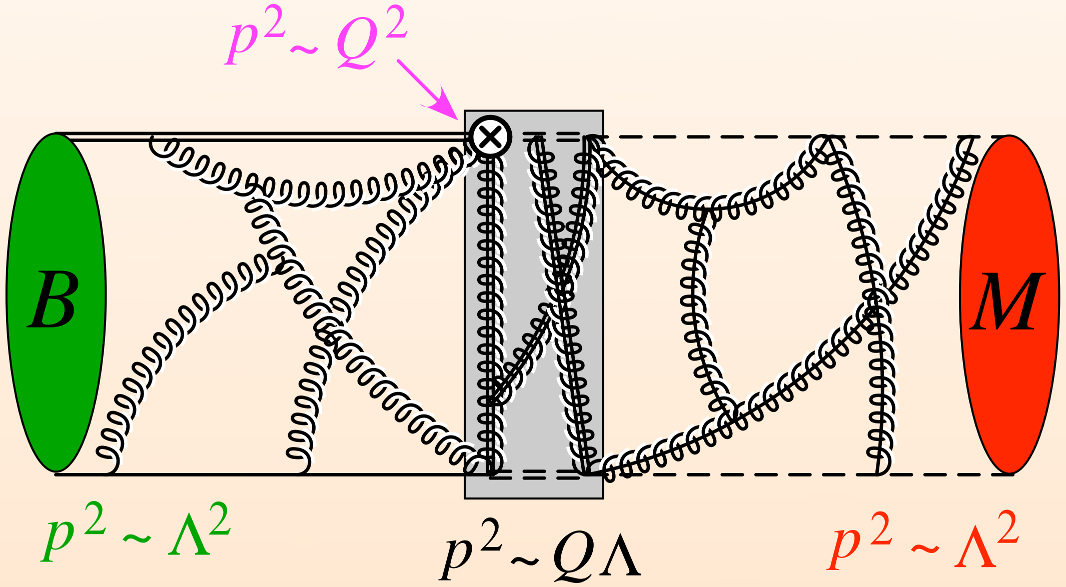
$$J_Q(k^+) = Disc \int d^4x \, e^{-ik\cdot x} \langle 0|T\bar{\chi}_{n,Q}(x)\bar{\not{n}}\, \chi_n|0\rangle$$

jet of energy Q

# Test SCET in B-Physics where Q is a bit bigger

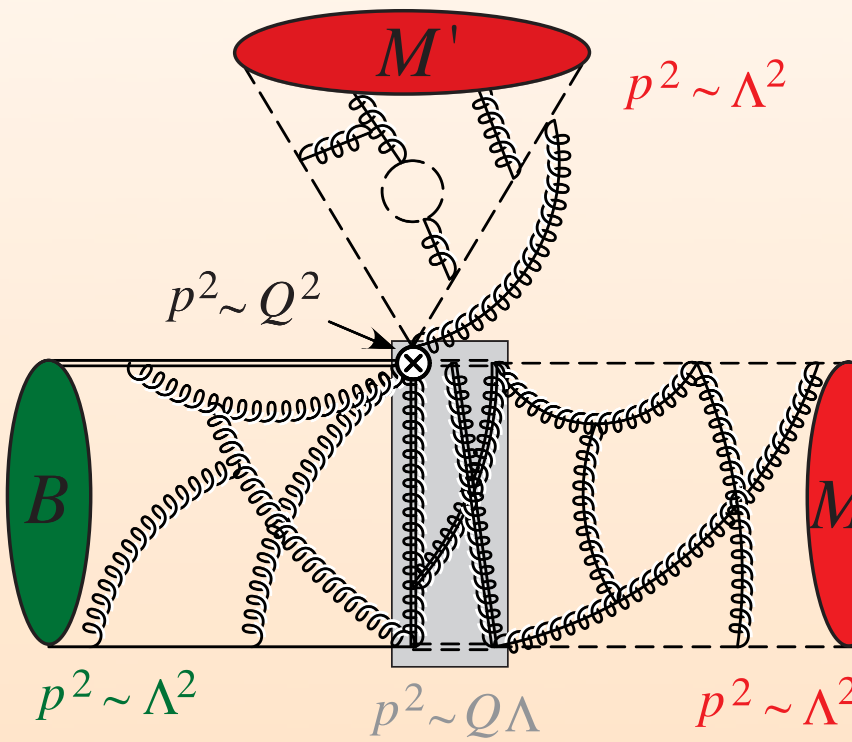
## Form Factors

$B \to \pi \ell \bar{\nu}$




$B \to \pi\pi$

important for measuring CP-violation




## Single Jet Production

$B \to X_s \gamma$

$B \to X_u \ell \bar{\nu}$

$$\frac{d\Gamma}{dE_\gamma} = \left|C(m_b,\mu)\right|^2 \int dk^+ \mathrm{Im} J_P(k^+,\mu) S(2E_\gamma - m_b + k^+, \mu)$$

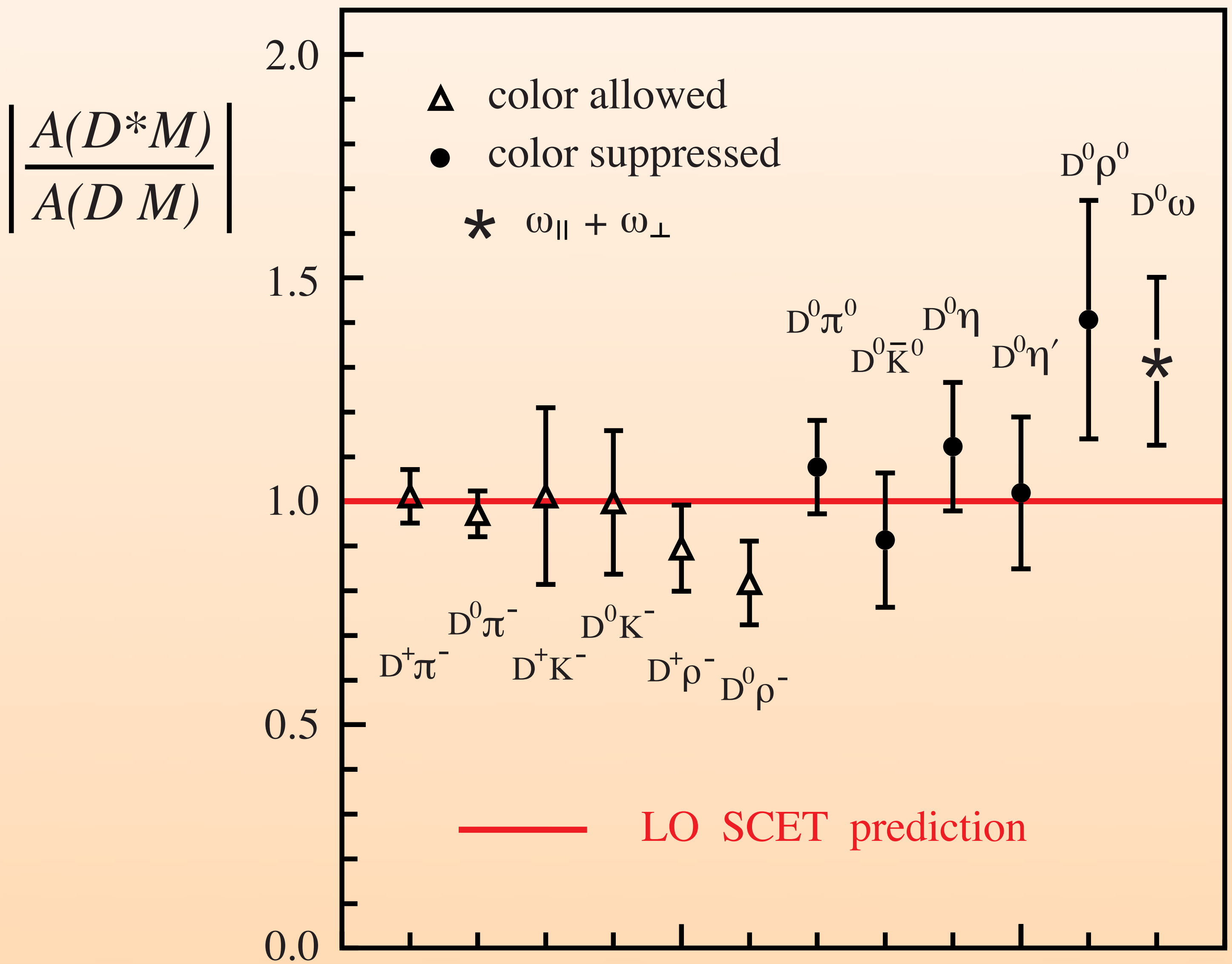
$$J_Q(k^+) = Disc \int d^4x\, e^{-ik\cdot x} \langle 0 | T \bar{\chi}_{n,Q}(x) \bar{n}\!\!\!/\, \chi_n | 0 \rangle$$

jet of energy Q

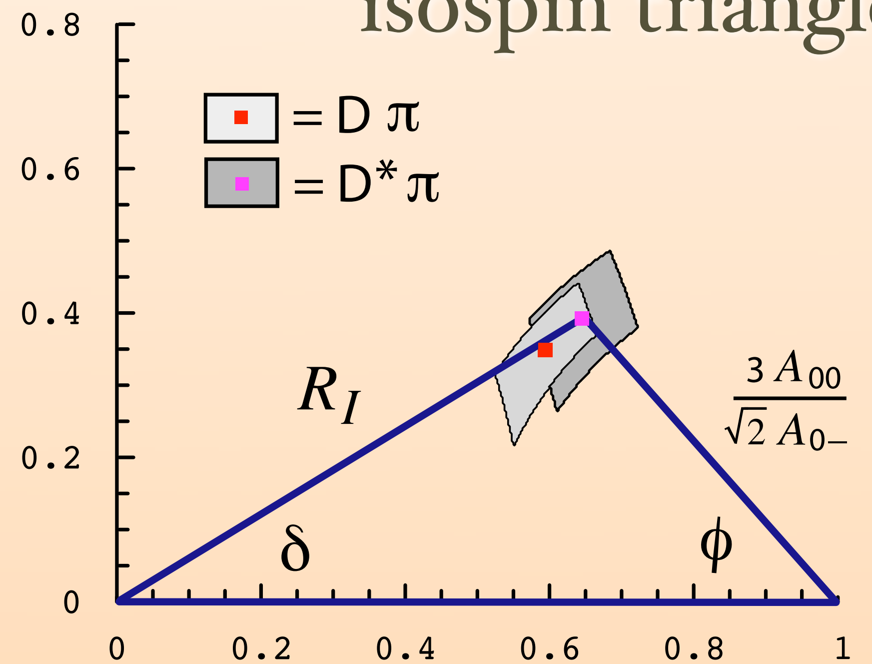
# $\bar{B}^0 \to D^0\pi^0$ Factorization, Comparison to Data

(Cleo, Belle, Babar)

Mantry, Pirjol, I.S.

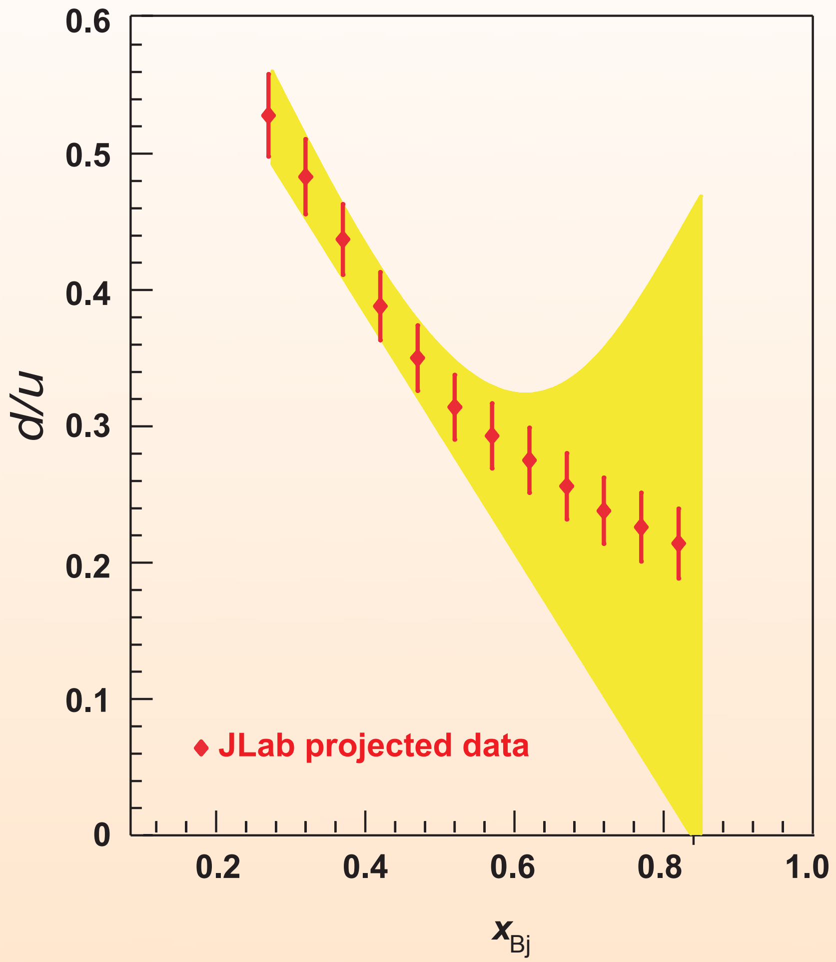


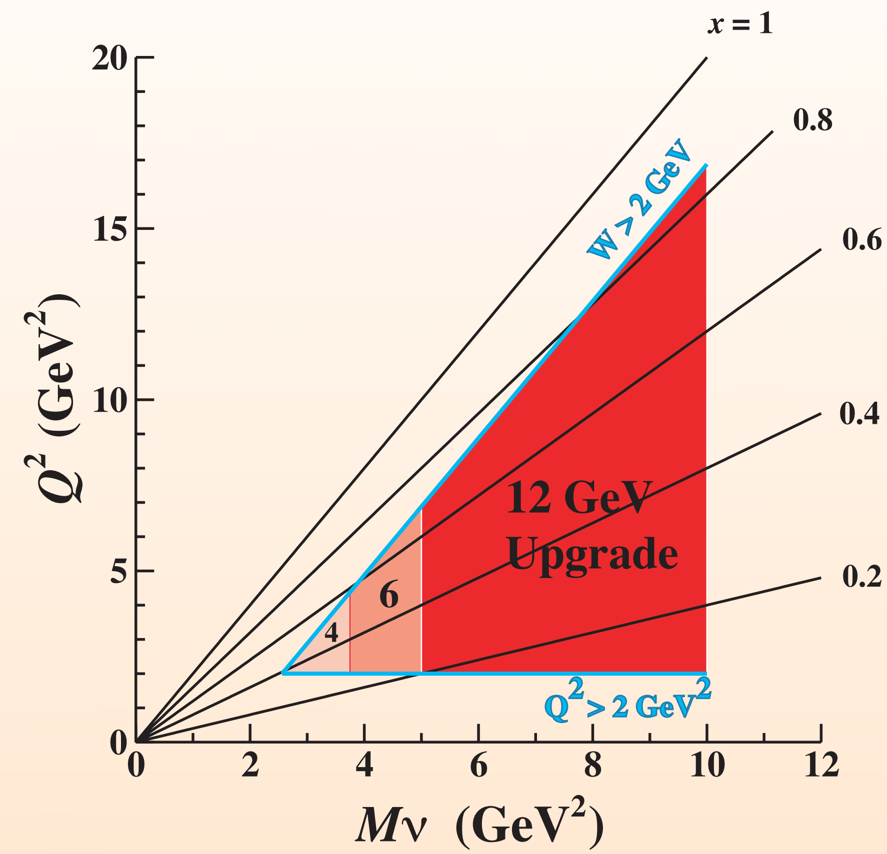

isospin triangle




$$\delta(D\pi) = 30.4 \pm 4.8^\circ$$
$$\delta(D^*\pi) = 31.0 \pm 5.0^\circ$$

# DIS for large x




Jlab at 12 GeV may well probe this region,
will there be surprises?

a Jet!

$$m_X^2 = \frac{Q^2(1-x)}{x} + m_p^2$$

For $x \sim \mathcal{O}(1)$, $m_X^2 \sim Q^2$, standard DIS

For $x \sim 1 - \Lambda/Q$, $\Lambda^2 \ll m_X^2 \ll Q^2$, endpoint DIS

For $x \sim 1 - \Lambda^2/Q^2$, $\Lambda^2 \sim m_X^2$, not deep inelastic

# Recent work in SCET

- Factorization theorem clarified (role of nonperturbative effects)

Manohar
Chay and Kim
Becher, Neubert, Peczak
Chen, Idilbi, Ji

- Resum large logs, $\alpha_s \ln(1-x)$, directly in momentum space. (No Landau pole problem.)

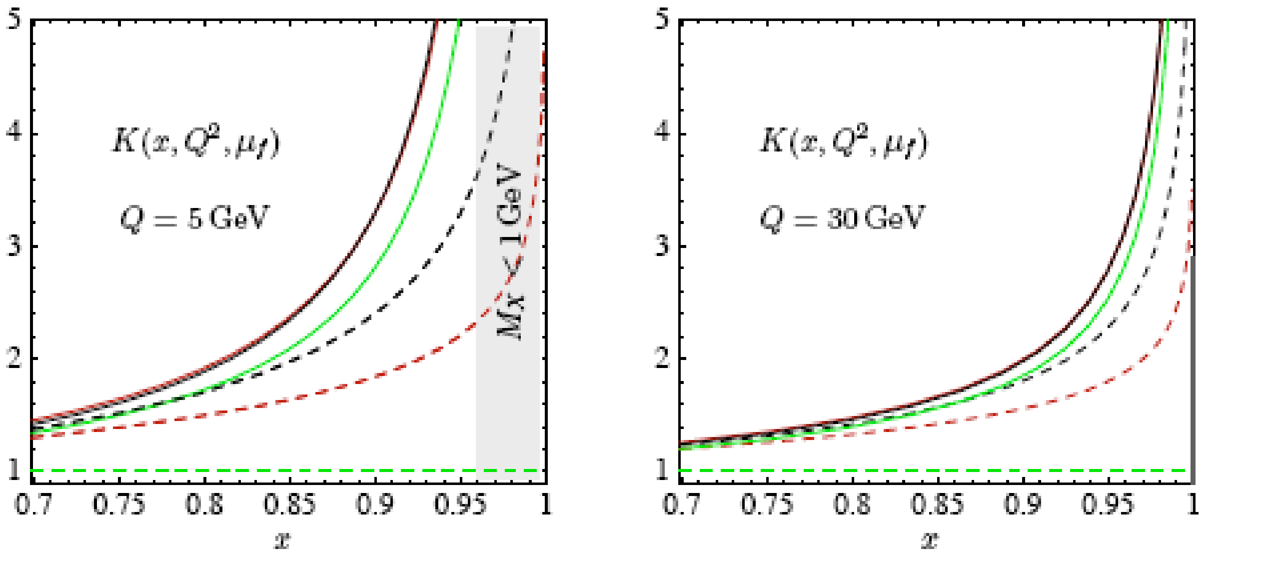
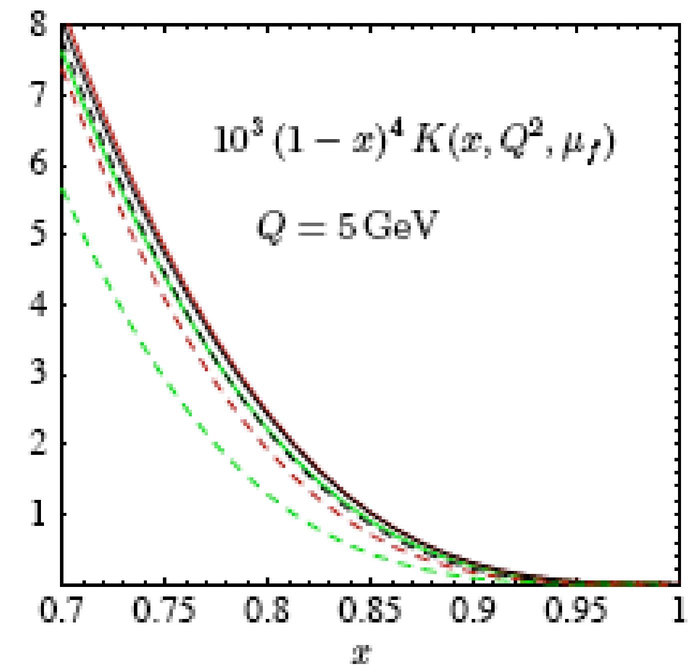



Figure 5: Comparison between fixed-order (dashed) and resummed results (solid) for the $K$ factor. The green curves are the LO result, red NLO, black NNLO. For the resummed result, we set $\mu_h = Q$, $\mu_i = M_X$, $\mu_f = Q$, and $b(\mu_f) = 4$. The fixed-order result is obtained by setting all scales equal to $\mu_f$.



# The END