Transverse charge densities

Gerald A. Miller
University of Washington

Theme- interpret form factor as determining transverse charge and magnetization densities

Outline-

1. Why transverse density
2. Model independent neutron transverse charge density
3. Proton transverse magnetization density
4. Pion transverse charge density
5. Impact of going to higher values of Q^2

Transverse Charge Densities.
Gerald A. Miller, arXiv:1002.0355 [nucl-th]

What is charge density at the center of the neutron?

- **Neutron has no charge, but charge density need not vanish**
- **Is central density positive or negative?**

Fermi: n fluctuates to $p\pi^{-}$

p at center, pion floats to edge

One gluon exchange favors dud

Real question- how does form factor relate to charge density?

Meaning of form factor

- $G_E(Q^2)$ is NOT Fourier transform of charge density
- Relativistic treatment needed- wave function is frame-dependent, initial and final states differ, no density
- Light front coordinates, ∞ momentum frame

$$\text{“Time”}\ x^+ = (ct+z)/\sqrt{2} = (x^0+x^3)/\sqrt{2},\ \text{“evolution”}\ \mathrm{p}^- = (\mathrm{p}^0-\mathrm{p}^3)/\sqrt{2}$$

$$\text{“Space”}\ x^- = (x^0-x^3)/\sqrt{2},\ \text{“Momentum”}\ p^+ = (p^0+p^3)/\sqrt{2}$$

$$\text{“Transverse position, momentum},\ \mathbf{b},\mathbf{p}$$

These coordinates are used to analyze form factors, deep inelastic scattering, GPDs,TMDS

Relativistic formalism-kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

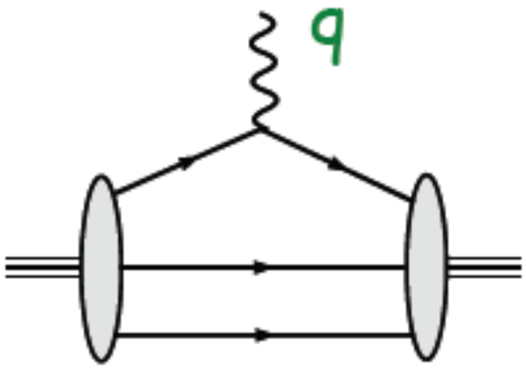
$$k^+ \rightarrow k^+, \ \ \mathbf{k} \rightarrow \mathbf{k} - k^+\mathbf{v}$$

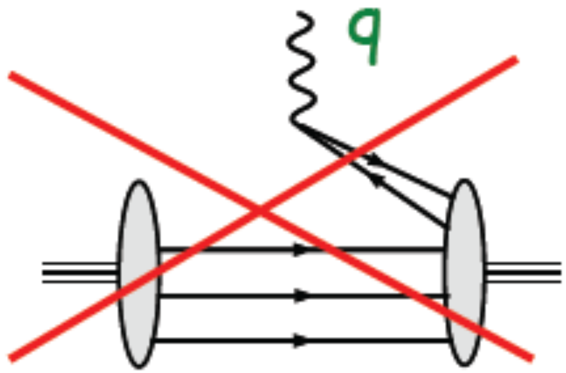
k^- such that k^2 not changed

Just like non-relativistic with k^+ as mass, take momentum transfer in perp direction, then density is 2 Dimensional Fourier Transform, also

$$q^+ = q^0 + q^3 = 0, -q^2 = Q^2 = \mathbf{q}^2$$

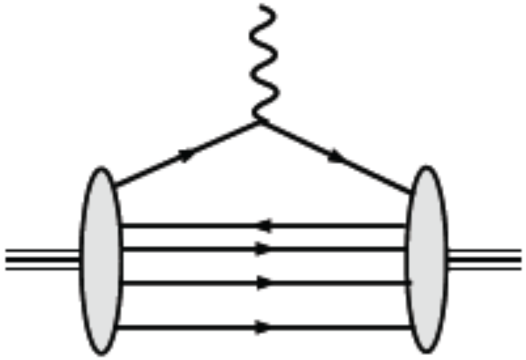
interpretation of FF as quark density





overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation



overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density

Absent in a Drell-Yan Frame

$$q^+ = q^0 + q^3 = 0$$

From Marc Vanderhaeghen

Model independent transverse charge density

$$J^+(x^-, \mathbf{b}) = \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b)$$ Charge Density

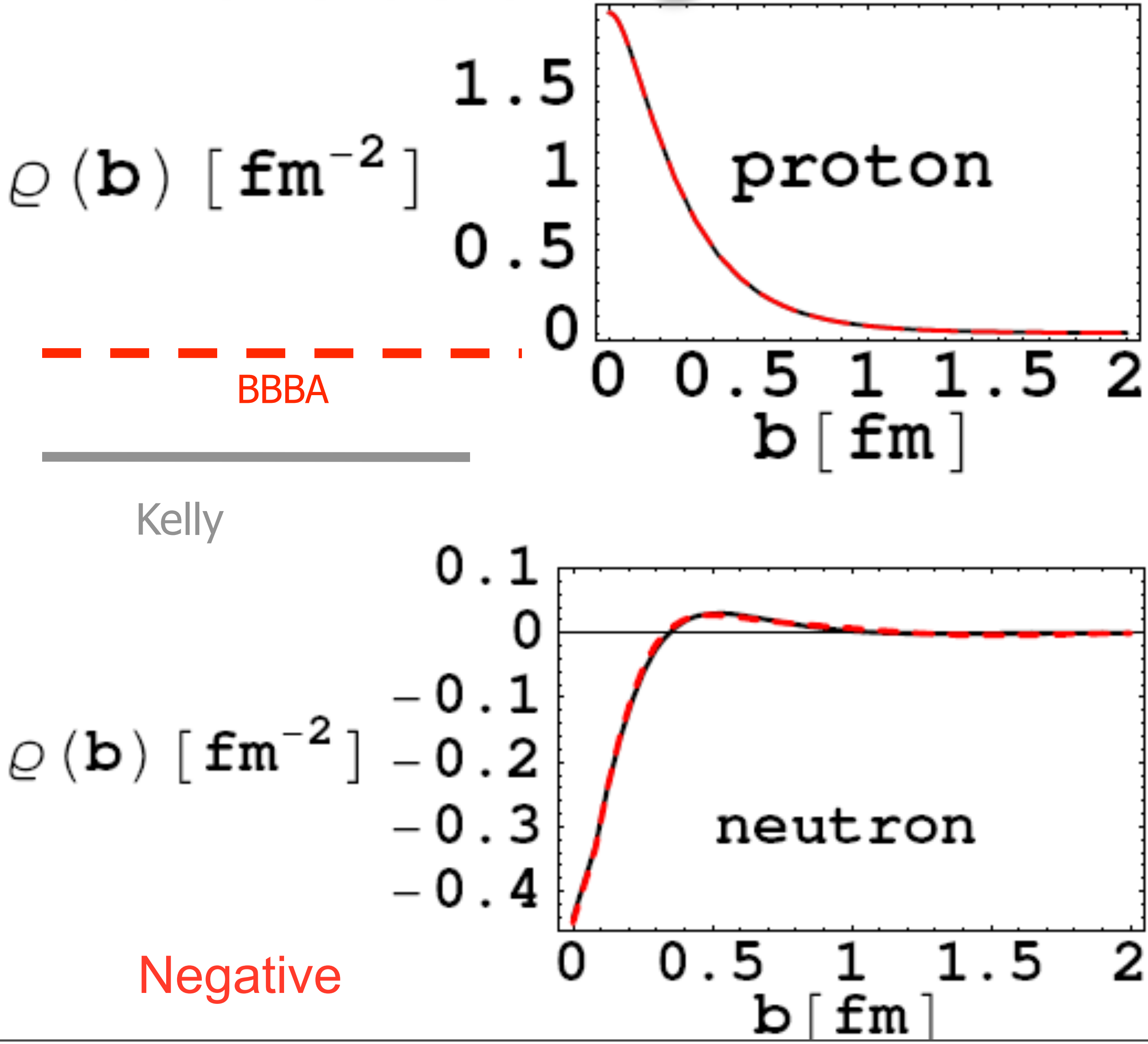
$$\rho_\infty(x^-, \mathbf{b}) = \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

$$F_1 = \langle p^+, \mathbf{p}', \lambda | J^+(0) | p^+, \mathbf{p}, \lambda \rangle$$

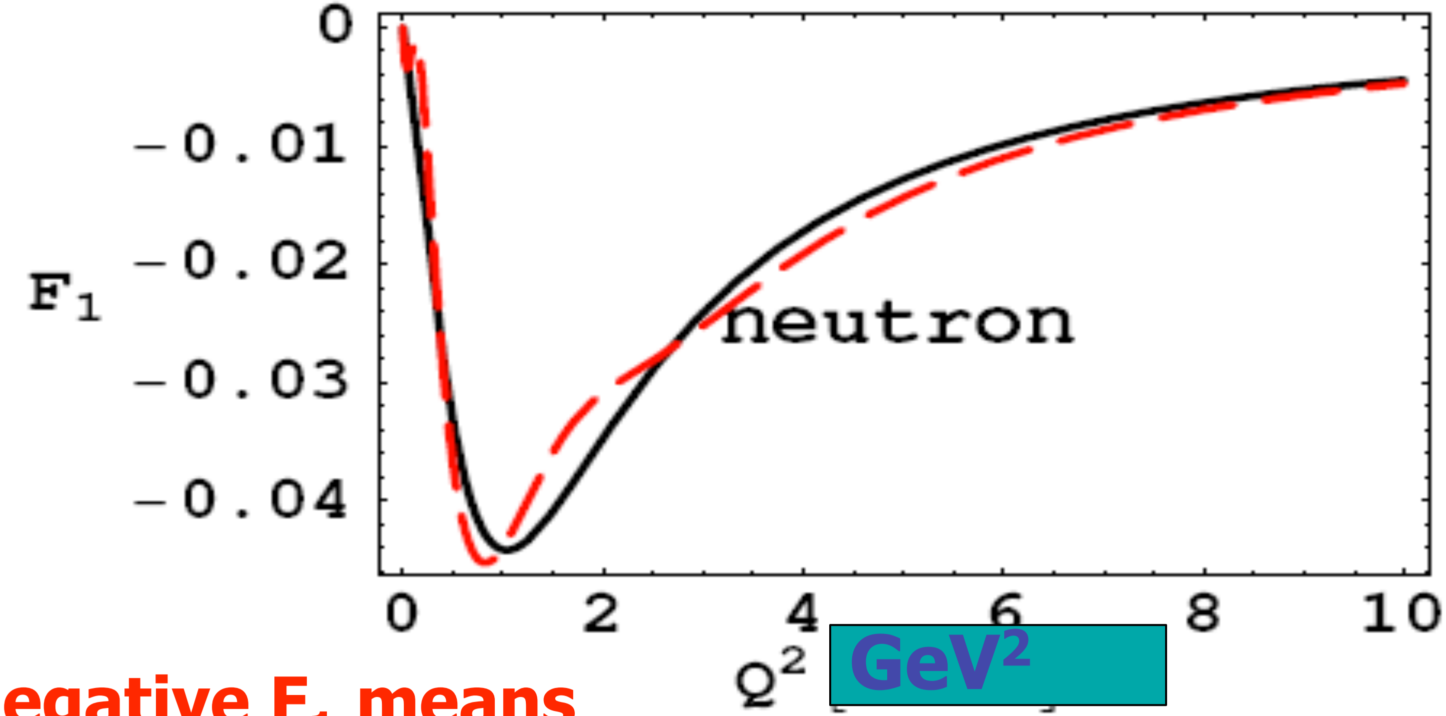
$$\rho(b) \equiv \int dx^- \rho_\infty(x^-, \mathbf{b}) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)$$

Density is $u - \bar{u}, \; d - \bar{d}$

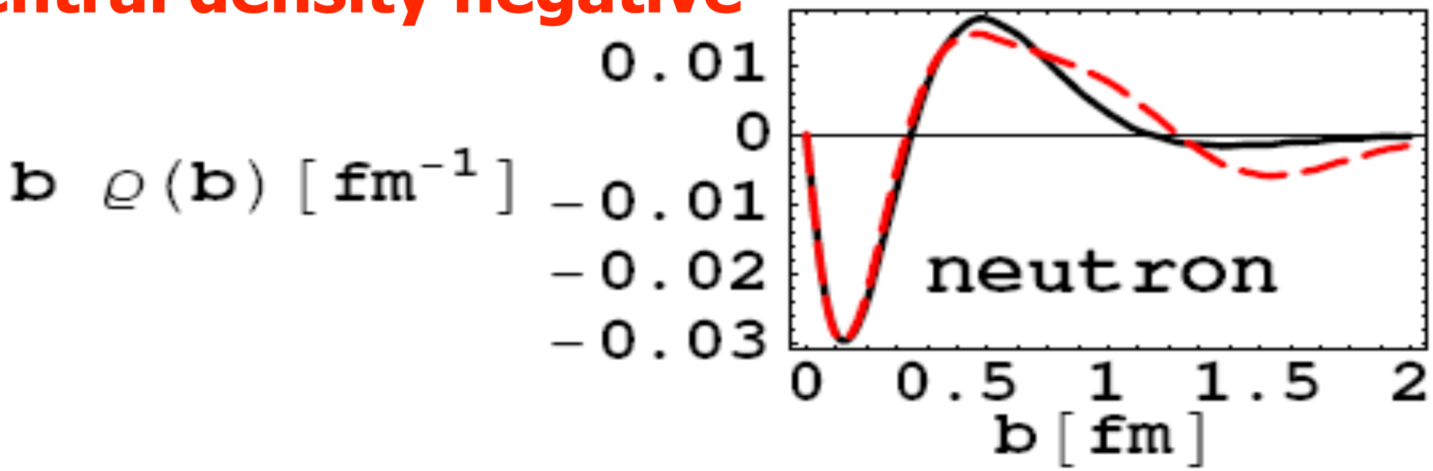
Transverse charge densities



Negative



Negative F_1 means central density negative

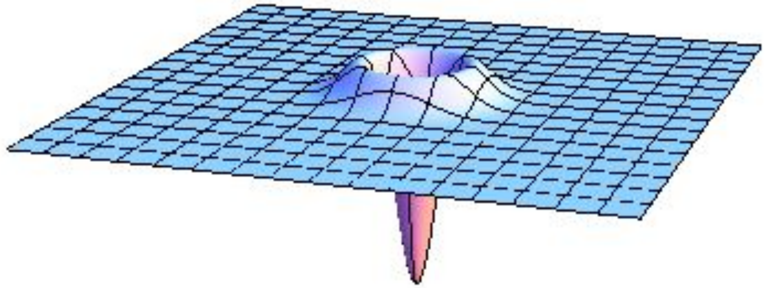


Neutron interpretation ρ(x,b)

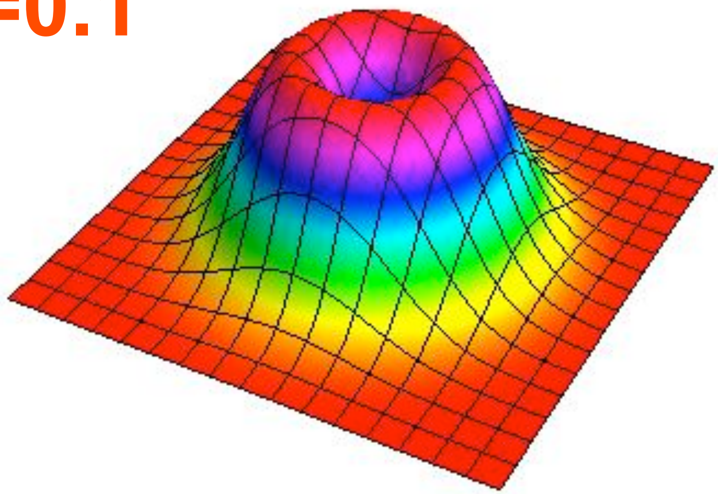
GAM, J. Arrington, PRC78,032201R '08

Using other people's models

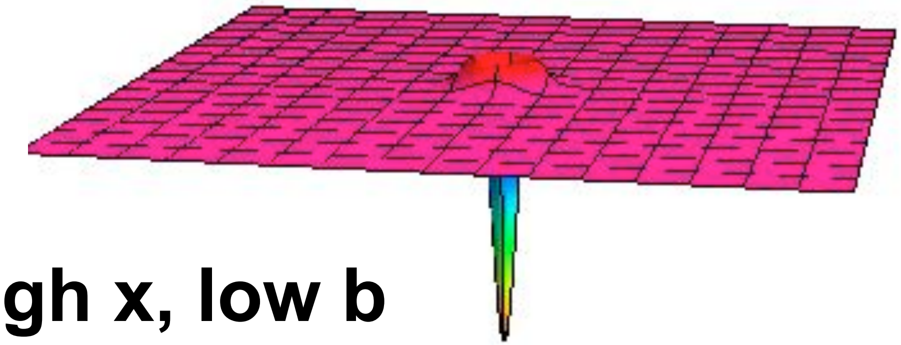
x=0.3



x=0.1



x=0.5



d or π^- dominates at high x, low b

Transverse Nucleon magnetization density

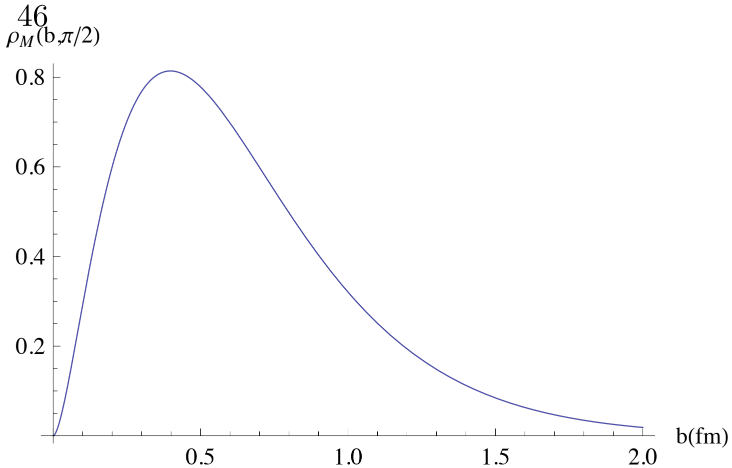
$$\vec{\mu}\cdot\vec{B} = \int d^3r\vec{j}\cdot\vec{A} = \frac{1}{2}\int d^3r\vec{j}\cdot(\vec{B}\times\vec{r}) = \frac{1}{2}\int d^3r(\vec{r}\times\vec{j})\cdot\vec{B}$$

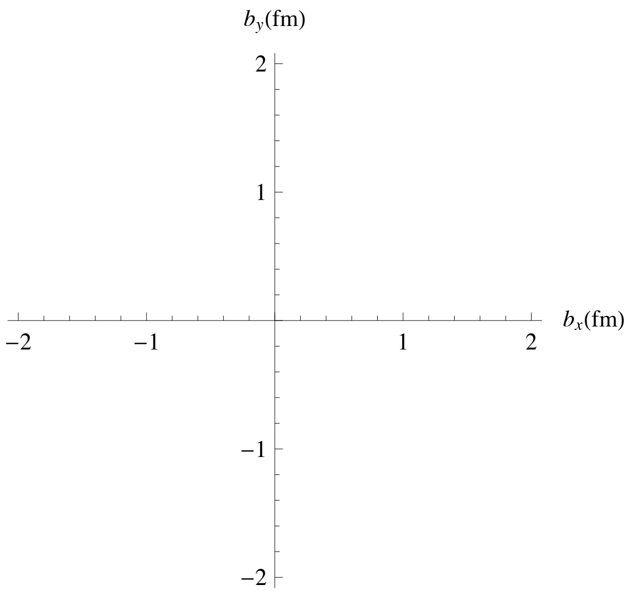
$\frac{1}{2}\int(\vec{r}\times\vec{j})$ is magnetization density, direction of $\vec{B}$

$\vec{B}$ in x-direction, calculate in IMF, integrate over x^-, matrix element in $|X\rangle$

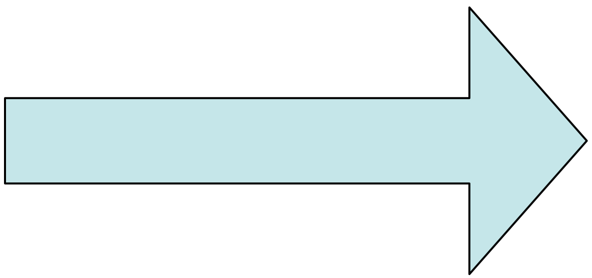
Magnetization density

$$\rho_M(\mathbf{b}) = \frac{\sin^2\phi}{2M}b\int\frac{Q^2dQ}{2\pi}F_2(Q^2)J_1(Qb)$$





Direction of magnetic field



Pion Transverse Charge Density

-GAM **Phys.Rev.C79:055204,2009**.

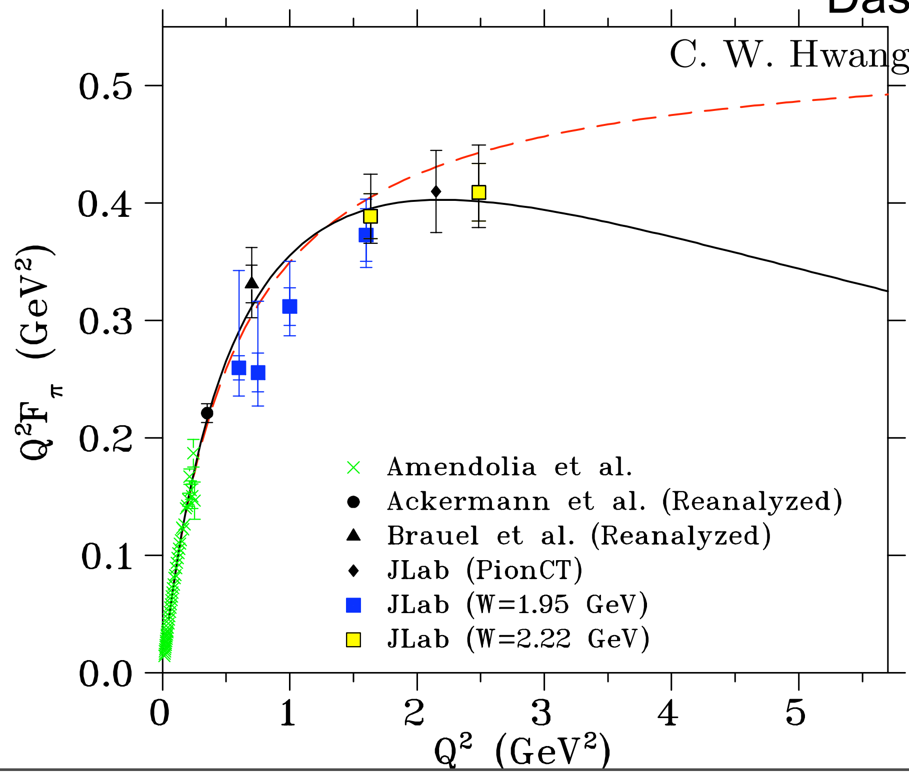
$$F_\pi(Q^2) = 1/(1 + R^2Q^2/6),$$

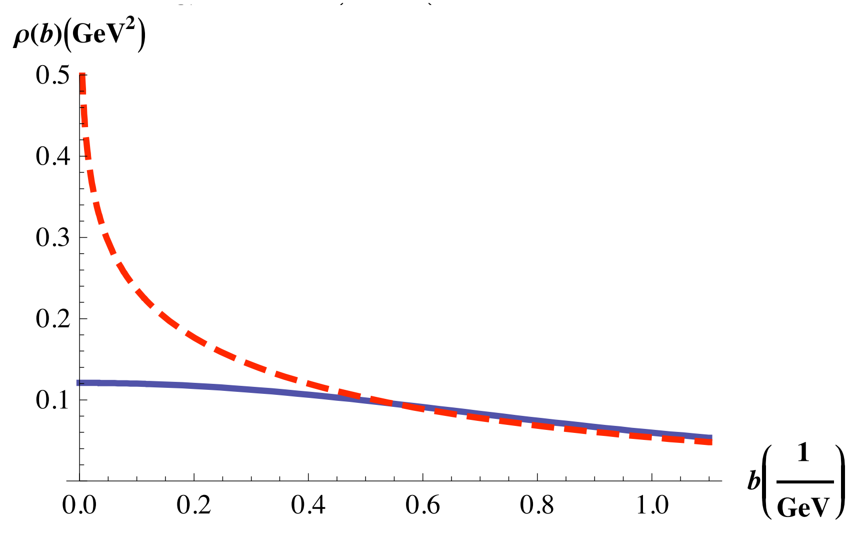
$$\rho(b) = \frac{3K_0\left(\frac{\sqrt{6}b}{R}\right)}{\pi R^2},$$

Singular - varies as log (b) small b, log(log(b)) in pQCD

Dashed- monopole fit, solid rel. cqm:

C. W. Hwang, Phys. Rev. D **64**, 034011 (2001).





How will higher Q^2 better determine transverse charge magnetization

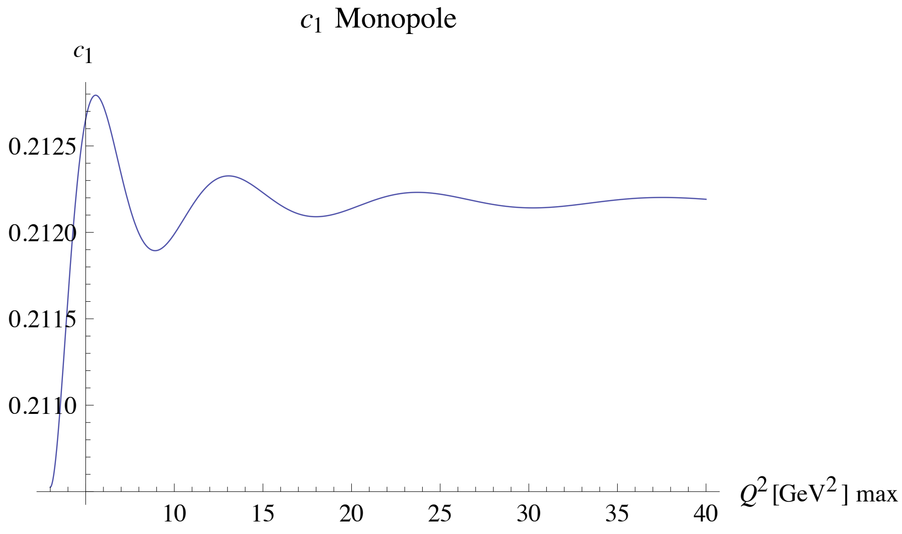
- Well designed procedure for 3 D Fourier transform relations-Friar Negele Kelly ...
- They used spherical Bessel, now Bessel or other orthogonal functions

$$\rho(b) = \sum_n c_n \, J_0(b \frac{x_{0n}}{R}), \; b \leq R \;, \; x_{0n} \text{ location of } n\text{'th zero}$$

$$c_n = \frac{2x_{0n}}{J_1(x_{0n})} \int_0^\infty \frac{QdQ}{2\pi} \frac{F(Q^2) J_0(QR)}{x_{0n}^2 - Q^2 R^2}$$

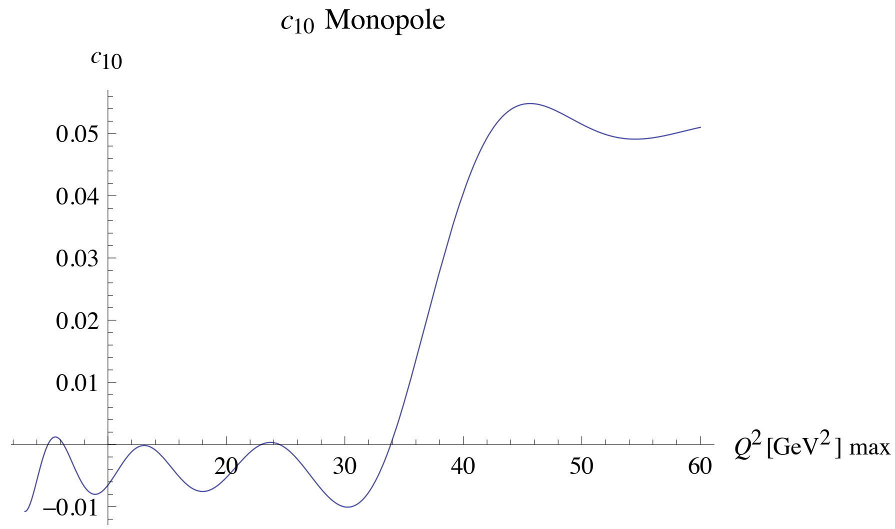
If know all c_n, know transverse density, integrate to max value of Q. See how high you need to go, pion=monopole

Out[297]=



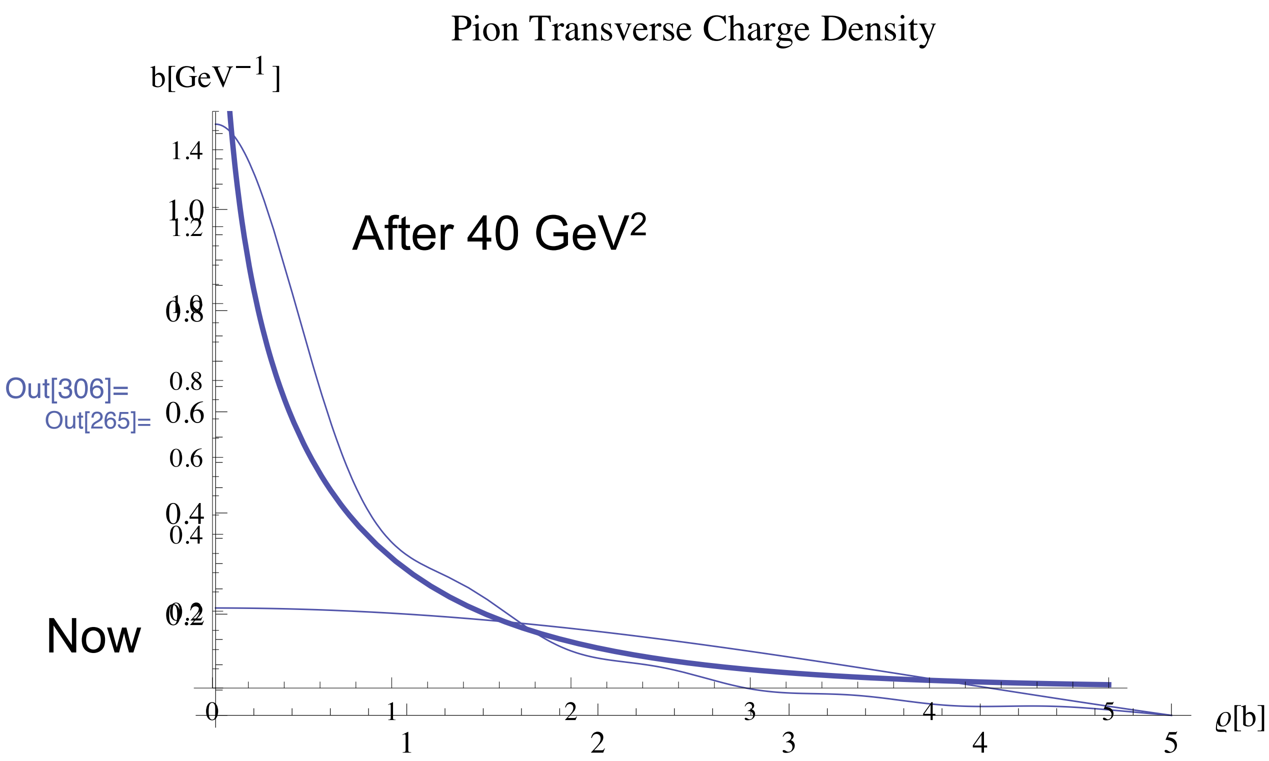
c_1 is known now

Out[298]=



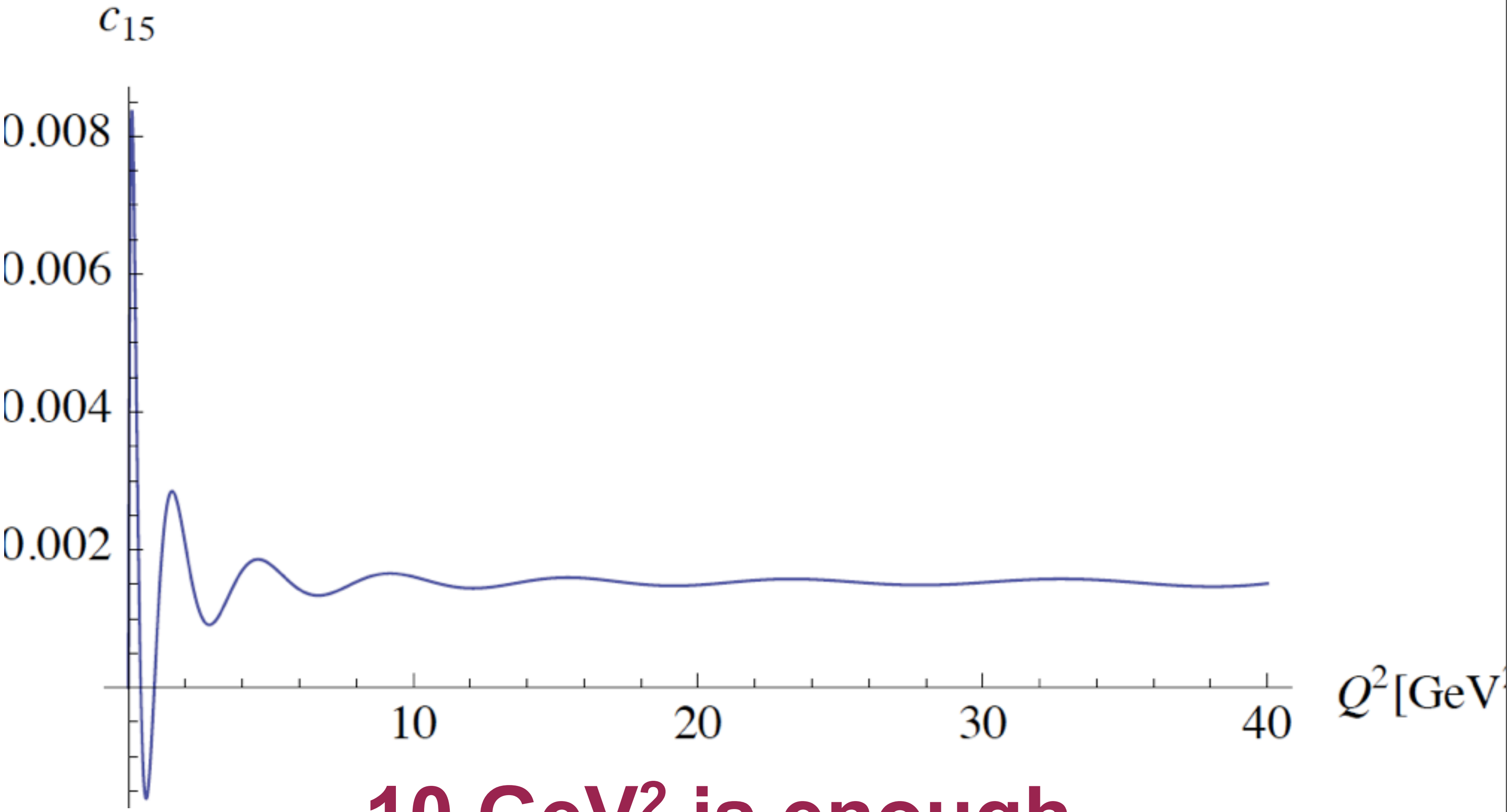
c_{10} needs 40 GeV^2

Model independent for pion



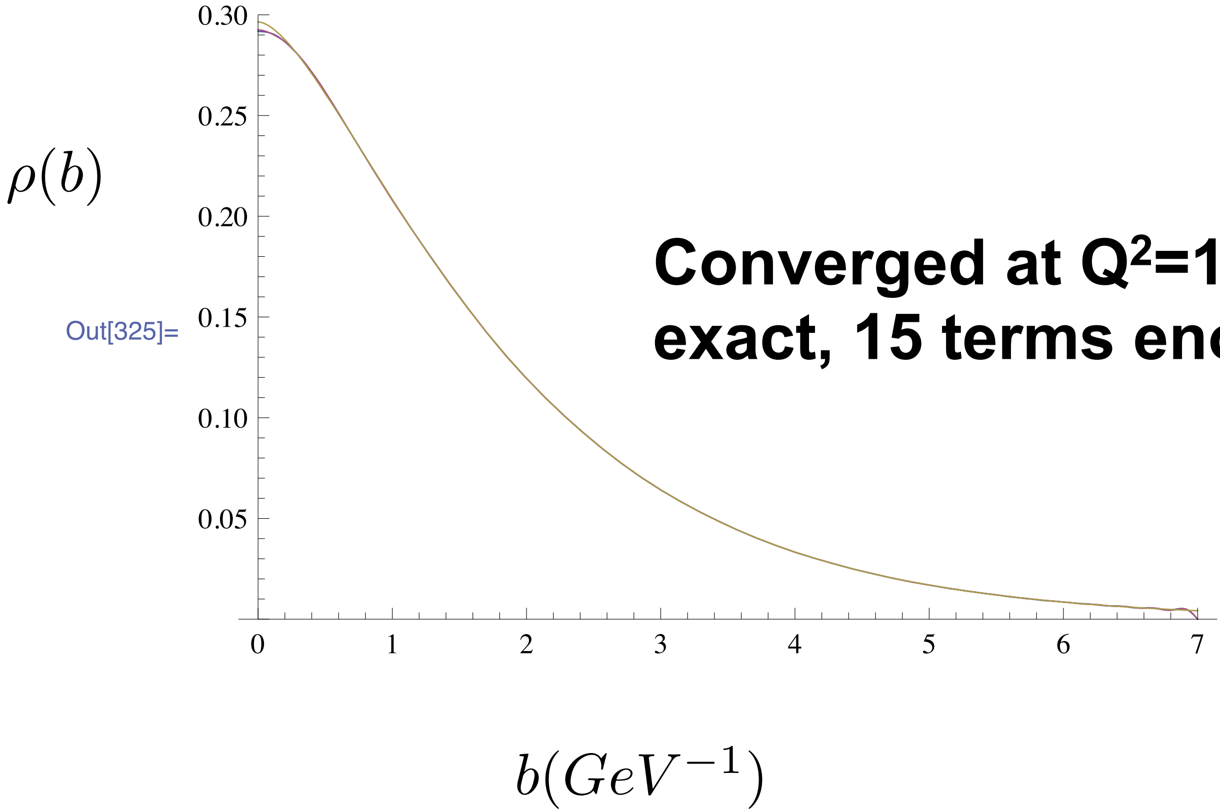
Blue curve is model exact -singular

Dipole form factor proton -rapid convergence



10 GeV^2 is enough

Model independent - dipole form factor



Converged at Q^2=10 GeV^2, exact, 15 terms enough

Summary

- Transverse charge and magnetization densities are the only sane way to interpret form factors
- Proton transverse charge density is determined now
- Going to Q^2=40 GeV^2, can determine other transverse densities: pion charge, proton magnetization and neutron charge, magnetization in model independent way

Spares follow

Return of the cloudy bag model

- In a model nucleon:bare nucleon + pion cloud - parameters adjusted to give negative definite F_1, pion at center causes negative central transverse charge density
- Boosting the matrix element of J^0 to the infinite momentum frame changes G_E to F_1

Rinehimer and Miller
PRC80,015201, 025206

Generalized transverse densities

$$\mathcal{O}_q^\Gamma(px, \mathbf{b}) = \int \frac{dx^- e^{ipxx^-}}{4\pi} q_+^\dagger(0, \mathbf{b}) \Gamma q_+(x^-, \mathbf{b})$$

$$\rho^\Gamma(b) = \int dx \sum_q e_q \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \mathcal{O}_q^\Gamma(p^+ x, \mathbf{b}) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

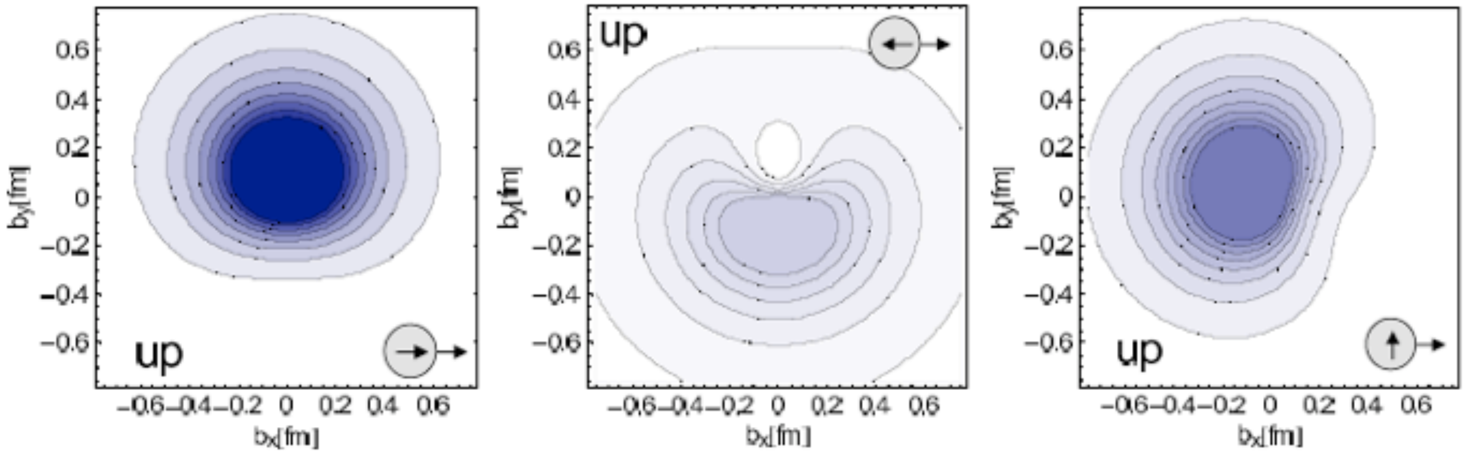
$\int dx$ sets $x^- = 0$, get $q_+^\dagger(0, \mathbf{b}) \Gamma q_+(0, \mathbf{b})$ **Density!**

$\Gamma = \frac{1}{2}(1 + \mathbf{n} \cdot \boldsymbol{\gamma} \gamma^5)$ gives spin-dependent density

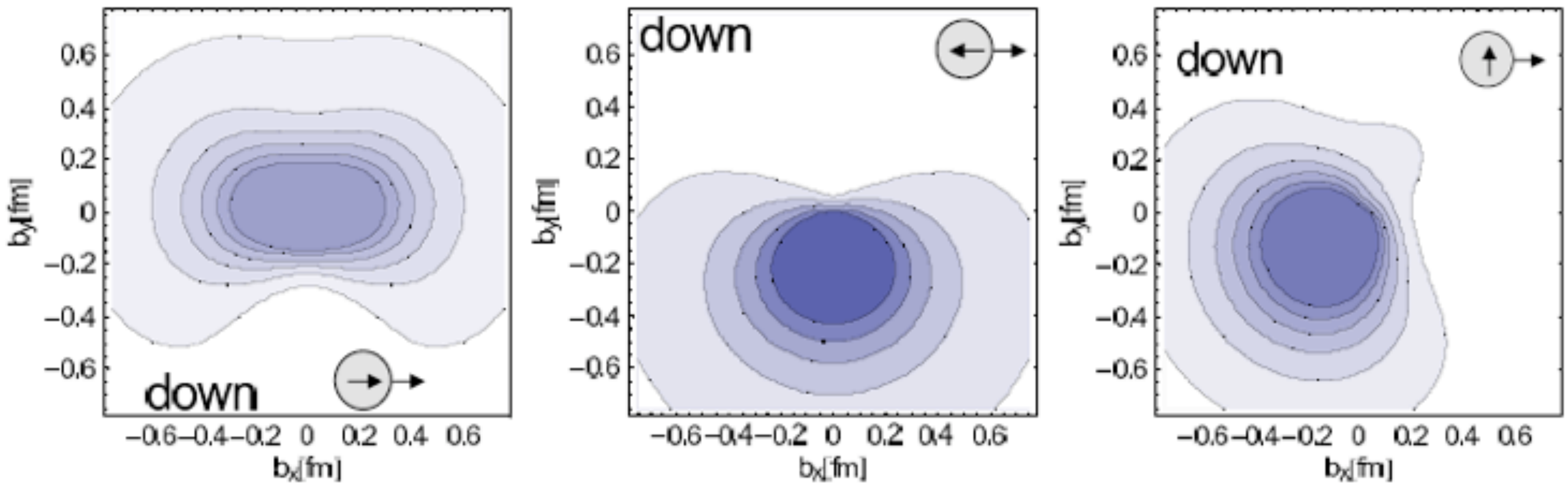
Local operators calculable on lattice M. Göckeler et al PRL98,222001 $\widetilde{A}''_{T10} \sim \mathrm{sdd}$ spin-dependent density

Schierholtz, 2009 -this quantity is not zero, proton is not round

Spin dependent densities-transverse-Lattice QCDSF, Zanotti, Schierholz...



This is not zero!



Transverse Momentum Distributions - momentum space density

In a state of fixed momentum

$\Phi_q^\Gamma(x, \mathbf{K})$ give probability of quark of given 3-momentum

$h_{1T}^\perp$ gives momentum-space spin-dependent density

measurable experimentally

hard to calculate on lattice because - gauge link

Relation or not between GPD and TMD

GPD :

$$\langle P', S'| \int \frac{dx^-}{4\pi} \bar{q}(-\frac{x^-}{2}, \mathbf{0}) \gamma^+ q(\frac{x^-}{2}, \mathbf{0}) e^{ix\bar{p}^+ x^-} |P, S\rangle_{\, x^+ = 0}$$

$$= \frac{1}{2\bar{p}^+} \bar{u}(P', S') \left(\gamma^+ H_q(\xi, t) + i \frac{\sigma^{+\nu}\Delta_\nu}{2M} E_q(x, \xi, t) \right) u(P, S)$$

TMD :

$$\Phi_q^\Gamma(x = \frac{k^+}{P^+}, \mathbf{k}) = \langle P, S| \int \frac{d\zeta^- d^2\zeta}{2(2\pi)^3} e^{ik\cdot\zeta} \bar{q}(0)\Gamma q(\zeta)|P, S\rangle_{\zeta^+=0}$$

GPD: nucleons have different momenta, but FT local in coordinate space if integrate over x

TMD: nucleons have same momenta, operator is local in momentum space

Both can be obtained Wigner distribution operator

$$W_q^\Gamma(\zeta^-, \boldsymbol{\zeta}, k^+, \mathbf{k})$$

$$= \frac{1}{4\pi}\int d\eta^- d^2\eta e^{ik\cdot\eta}\bar{q}(\zeta^- - \frac{\eta^-}{2}, \boldsymbol{\zeta} - \frac{\boldsymbol{\eta}}{2})\Gamma q(\zeta^- + \frac{\eta^-}{2}, \boldsymbol{\zeta} + \frac{\boldsymbol{\eta}}{2})$$

$$H_q(x, \xi, t) = \langle P', S'| \int \frac{d^2\mathbf{k}}{(2\pi)^2} W_q^{\gamma^+}(\zeta^- = 0, \zeta = 0, k^+, \mathbf{k})|P, S\rangle$$

$$\Phi_q^\Gamma(x, \mathbf{k}) = \langle P, S| \int \frac{d\zeta^-}{(2\pi)^2} W_q^\Gamma(\zeta^-, \zeta, k^+, \mathbf{k})|P, S\rangle$$

Summary

- **Form factors, GPDs, TMDs, understood from unified light-front formulation**
- **Neutron central transverse density is negative- consistent with Cloudy Bag Model**
- **Proton is not round- lattice QCD spin-dependent- density is not zero**
- **Experiment can whether or not proton is round by measuring** $h_{1T}^{\perp}$

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The Proton

Cloudy Bag Model~1980

PHYSICAL REVIEW D VOLUME 24, NUMBER 1 1 JULY 1981

Cloudy bag model of the nucleon

A. W. Thomas and S. Théberge
TRIUMF, University of British Columbia, Vancouver, British Columbia, Canada V6T2A3

Gerald A. Miller
Institute for Nuclear Theory and Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 28 January 1981)

A previously derived model in which a baryon is treated as a three-quark bag that is surrounded by a cloud of pions is used to compute the static properties of the nucleon. The only free parameter of the model is the bag radius which is fixed by a fit to pion-nucleon scattering in the (3,3)-resonance region to be about 0.8 fm. With the model so determined the computed values of the root-mean-square radii and magnetic moments of the neutron and proton, and g_A, are all in very good agreement with the experimental values. In addition, about one-third of the Δ-nucleon mass splitting is found to come from pionic effects, so that our extracted value of α_s is smaller than that of the MIT bag model.

Many successful predictions

One feature- pion penetrates to the bag interior