*EIC Workshop: Electron-Nucleon Exclusive Reactions*
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# Orbital Angular Momentum and GPD moments

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**Overview:**

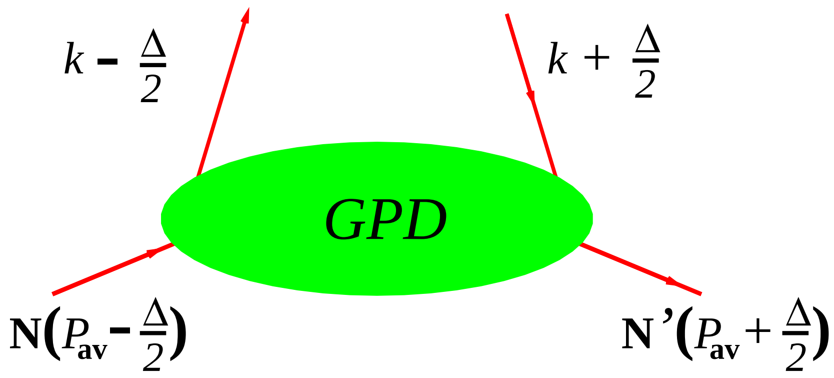
- introduction
- properties of GPDs: polynomiality
- form factors of energy momentum tensor
- results from chiral models
- stability, sign of $D$-term
- conclusions

# 1. Introduction

hard exclusive reactions allow to access

$$\text{Re/Im} \int \mathrm{d}x \, \frac{\text{GPD}(x, {\color{blue}\xi}, {\color{red}t})_{Q^2}}{x \pm {\color{blue}\xi} + i\epsilon}$$

$${\color{blue}\xi} = \frac{Q^2}{4Pq + Q^2}, \quad {\color{red}t} = \Delta^2, \; Q^2 = -q^2$$




# GPDs:

$$\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda {\color{blue}x}} \langle N({\color{blue}P'}) | \overline{\psi}_q(-\tfrac{\lambda n}{2}) \, n_\mu \gamma^\mu \, [-\tfrac{\lambda n}{2}, \tfrac{\lambda n}{2}] \psi_q(\tfrac{\lambda n}{2}) \, | N({\color{blue}P}) \rangle$$

$$= \overline{u}(p') \, n_\mu \gamma^\mu \, u(p) \; {\color{blue}H^q(x, \xi, t)} + \overline{u}(p') \, \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_N} \, u(p) \; {\color{blue}E^q(x, \xi, t)}$$

What can we learn from GPDs?

- Let us put $\xi \to 0$

momentum transfer only perpendicular to light-cone $t = \Delta^2 = -\vec{\Delta}_\perp^2$

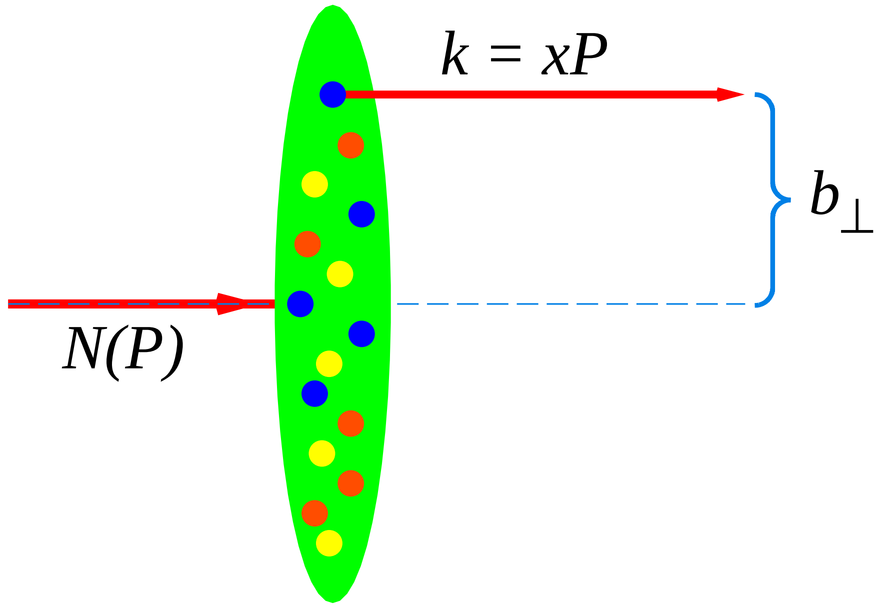
$$H^q(x, b_\perp) = \int \mathrm{d}^2\Delta_\perp \, e^{i\vec{\Delta}_\perp \vec{b}_\perp} \, H^q(x, 0, -\vec{\Delta}_\perp^2)$$

$b_\perp =$ impact parameter

Matthias Burkardt, 2000

exact interpretation:

**probability**




simultanous "measurement" of longitudinal momentum & transverse position

$\longrightarrow$ transverse nucleon imaging

has important applications: hadronic final state/event characteristics in high energy pp collisions (Frankfurt, Strikman, Weiss)

- Let us keep $\xi \neq 0$ and go to Mellin space

even Mellin moments $N = 2, 4, \ldots$

$$\int \mathrm{d}x\, x^{N-1}\, H^q(x,\xi,t) = h_0^{q(N)}(t) + h_2^{q(N)}(t)\,\xi^2 + \ldots + h_N^{q(N)}(t)\,\xi^N$$

$$\int \mathrm{d}x\, x^{N-1}\, E^q(x,\xi,t) = e_0^{q(N)}(t) + e_2^{q(N)}(t)\,\xi^2 + \ldots + e_N^{q(N)}(t)\,\xi^N$$

with $h_N^{q(N)}(t) = -\,e_N^{q(N)}(t)$ for spin $\frac{1}{2}$ particle

odd Mellin moments $N = 1, 3, \ldots$ : highest power is $\xi^{N-1}$

"polynomiality" $\longleftarrow$ Lorentz invariance, time-reversal, hermiticity

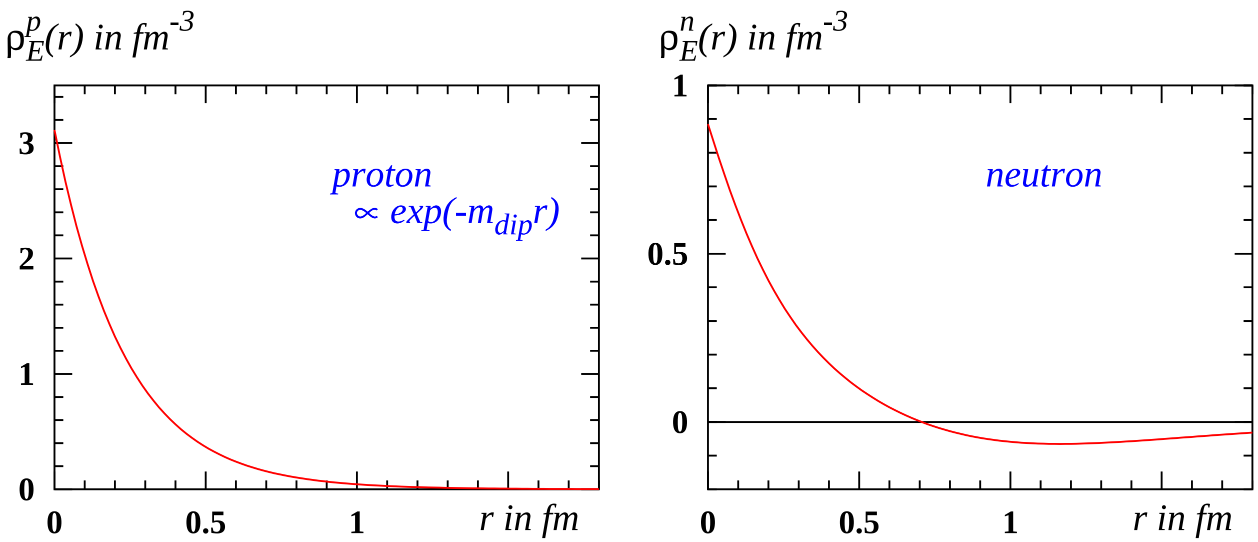
Remark: with all respect: always ask 2 ('mean') questions.
Here is the first: does your model fullfil polynomiality ...?

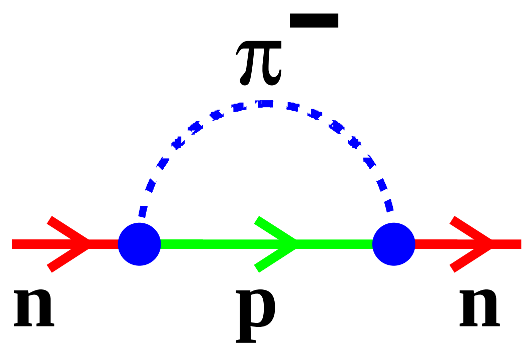
## $N = 1$: electromagnetic form factors Hofstadter et al, 1950s ...

$\sum_q e^q \int \mathrm{d}x\, H^q(x,\xi,t) = F_1(t)$ $\quad \sum_q e^q \int \mathrm{d}x\, E^q(x,\xi,t) = F_2(t)$ what did we learn?

e.g. $G_E(t) = F_1(t) + \frac{t}{4M_N^2} F_2(t) = \int \mathrm{d}^3 r\, \rho_E(r)\, e^{i\vec{q}\vec{r}}$ $\quad (t = -\vec{q}^{\,2}$, rel. corr.)

$G_E^p(t) \simeq 1/(1 - t/m_{\mathrm{dip}}^2)^2$, $\quad m_{\mathrm{dip}} = 0.84\,\mathrm{GeV}$, $\quad G_E^n(0) = 0$




**pion cloud:**
spontanous breaking
of chiral symmetry

continued interest: JLab Hall A $G_E^n(t)$, Hall C Q-weak

e.g. precise data at low $t$ constrain physics at TeV scale

Young, Carlini, Thomas, Roche, PRL99,122003(2007)

## $N=2$: energy-momentum tensor form factors

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = \qquad +M_2^Q(t) + \tfrac{4}{5}\, d_1^Q(t)\, \xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \tfrac{4}{5}\, d_1^Q(t)\, \xi^2$$ gluons analog

What we know:

$M_2^Q(0) \sim 0.5$ at few $\mathrm{GeV}^2$: quarks carry half of nucleon momentum (other half gluons)

What we would like to know: **Ji sum rule**

how quarks + gluons share nucleon spin

$$\sum_q \int \mathrm{d}x\, x\, (H^q + E^q)(x,\xi,t) = 2J^Q(t) \quad \text{and} \quad \lim_{t\to 0} J^Q(t) = J^Q(0)$$ Ji, 1997

Much interest in $J(t)$ at $t=0$ (fully deserved).
But what will we learn from $t$–dep.?
What is $d_1(t)$ good for?

## $N = 3, 4, 5, \ldots$: further form factors

of what? Of operators $\bar{\psi}\gamma_{\mu_1} D_{\mu_2} D_{\mu_2} \ldots D_{\mu_N} \psi$

scale dependent quantities,
each of them contains information
on some different aspect of nucleon

# 2. Energy momentum tensor $T^{\mu\nu}$

$T^{\mu\nu}$ fundamental object in field theory. In QCD $T^{Q,G}_{\mu\nu}$ both gauge invariant

$$\langle P'|T^{Q,G}_{\mu\nu}|P\rangle = \bar{u}(P')\Big[ M_2^{Q,G}(t)\,\frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu}\Big] u(p)$$

$$\underbrace{\qquad\qquad\qquad\qquad}$$

$$A^{Q,G}(t)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^{Q,G}(t)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M_N} + C^{Q,G}(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{M_N}$$

using Gordon identity
equiv. decomposition:

$M_2(t) = A(t)$

$2\,J(t) = B(t) + A(t)$

$\frac{1}{5}\,d_1(t) = C(t)$

$\bar{c}(t)$ because $\hat{T}^{Q,G}_{\mu\nu}$ not conserved separately, drops out from quark+gluon sum.

## Properties of form factors

total $T_{\mu\nu} = T^Q_{\mu\nu} + T^G_{\mu\nu}$ conserved

$M_2(t) = M^Q(t) + M^G_2(t)$, $J(t)$, $d_1(t)$ scale independent (like $F_1(t)$, $F_2(t)$)

with

$$M^Q_2(0) + M^G_2(0) = 1 \quad \text{quarks + gluons carry 100 \% of momentum}$$

$$J^Q(0) + J^G(0) = \frac{1}{2} \quad \text{quarks + gluons make up the nucleon spin}$$

$$d^Q_1(0) + d^G_1(0) \equiv \boldsymbol{d_1} \quad \text{what is that?}$$

## 'new characteristics'

$d_1 = d_1(0)$ equally fundamental like $M_N$, el. charge, $\mu$, $g_A$, ...
$d_1(t)$ dictates asymptotics of unpolarized GPDs
(Goeke, Polyakov & Vanderhaeghen, 2001)
What it means? Later.

## what we already know

$\boldsymbol{M_2^Q(0)} = \int \mathrm{d}x \sum_q x H^q(x,0,0) \equiv \int \mathrm{d}x \sum_q x f_1^q(x) \sim 0.5$ at $\mu^2 \sim$ few GeV$^2$,

half of momentum of (fast moving) nucleon carried by quarks, rest by gluons.

asymptotically $\mu \to \infty$ : $\quad M_2^Q(0) = \dfrac{3n_f}{16+3n_f}$ and $M_2^G(0) = \dfrac{16}{16+3n_f}$

Gross, Wilczek, 1974

$\boldsymbol{2J^Q(0)} = \int \mathrm{d}x \sum_q x(H^q + E^q)(x,0,0) =$ **?**

i.e. percentage of nucleon spin due to quarks?

asymptotically: $2J^Q(0) = \dfrac{3n_f}{16+3n_f}$ and $2J^G(0) = \dfrac{16}{16+3n_f}$ like $M_2$

Ji, 1997

# 3. Physical meaning of EMT form factors

M.V.Polyakov, PLB 555, 57 (2003)

Go to Breit frame where $\vec{P}' = -\vec{P}$, i.e. $E' = E$ and $\Delta^\mu = (0, \vec{\Delta})$

$\vec{s} =$ spin vector of respective nucleon at rest

Define static EMT: $T^Q_{\mu\nu}(\vec{r}) = \frac{1}{2E}\int\frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3}\, e^{i\vec{\Delta}\vec{r}}\,\langle P'|\hat{T}^Q_{\mu\nu}|P\rangle$

Then:

$$J^Q(t) + \frac{2t}{3}\,J^{Q\prime}(t) \;=\; \int \mathrm{d}^3r\, e^{i\vec{r}\vec{\Delta}}\,\varepsilon^{ijk}\,s_i\,r_j\,T^Q_{0k}$$

$$d^Q_1(t) + \frac{4t}{3}\,d^{Q\prime}_1(t) + \frac{4t^2}{15}\,d^{Q\prime\prime}_1(t) \;=\; -\frac{M_N}{2}\int \mathrm{d}^3r\, e^{i\vec{r}\vec{\Delta}}\left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T^Q_{ij}$$

$$M_2(t) - \frac{t}{4M_N^2}\left(M_2(t) - 2J(t) + \frac{4}{5}\,d_1(t)\right) \;=\; \frac{1}{M_N}\int \mathrm{d}^3r\, e^{i\vec{r}\vec{\Delta}}\,T_{00}$$

gluon analog (last equation: quark+gluon)

$T_{00}(r)$ energy density

$T_{0j}(r) \to$ spatial angular momentum distribution

$$T_{ij}(r) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij}$$

$\boldsymbol{p(r)}$ distribution of *pressure* inside hadron
$\boldsymbol{s(r)}$ related to distribution of *shear forces* $\Big\} \longrightarrow$ **"mechanical properties"**

$$M_2(0) = \frac{1}{M_N}\int \mathrm{d}^3r\; T_{00}(r) = 1$$

$$J(0) = \int \mathrm{d}^3r\; \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(r) = \frac{1}{2}$$

$$d_1(0) = -\frac{M_N}{2}\int \mathrm{d}^3r\; \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right) T_{ij}(r) \equiv \mathbf{d_1}$$

(keep in mind: relativistic corrections, same resevations as for em form factors, ..., Miller)

## information in $T_{ij}(r)$

$$\partial^\mu T_{\mu\nu} = 0 \quad \Leftrightarrow \quad \nabla^i T_{ij}(\vec{r}) = 0$$

$$\longrightarrow \frac{2}{3}\, s'(r) + \frac{2}{r}\, s(r) + p'(r) = 0$$

$\longrightarrow$ "stability condition"

$$\boxed{\int\limits_0^\infty \mathrm{d}r\; r^2\, p(r) = 0}$$

remark: differential equations can be solved, e.g.

$$d_1(t) = \frac{15\, M_N}{2\, t} \int \mathrm{d}^3 r\; j_0(r\sqrt{-t})\, p(r)$$

$$\boxed{d_1 = \frac{5}{4}\, M_N \int \mathrm{d}^3 r\; r^2 p(r)}$$

## What do we know about $d_1$? It is negative! Why?

pion:

- soft-pion theorems $\frac{4}{5} d_1 = -M_2$ (M.V.Polyakov and C.Weiss, 1999)

nucleon

- chiral quark soliton model $d_1^Q = -4$ ('discovery of D-term', Petrov et al 1998)
- lattice QCD $d_1^Q < 0$ (LHPC 2003, QCDSF 2004)
- Skyrme model $d_1 = -6.60$ (Cebulla et al, 2008)

nucleus

- liquid drop model $\leftrightarrow$ "surface tension" and $d_1 < 0$ (M.V.Polyakov, 2003)
  (explicit calculations V.Guzey, M.Siddikov, 2006)

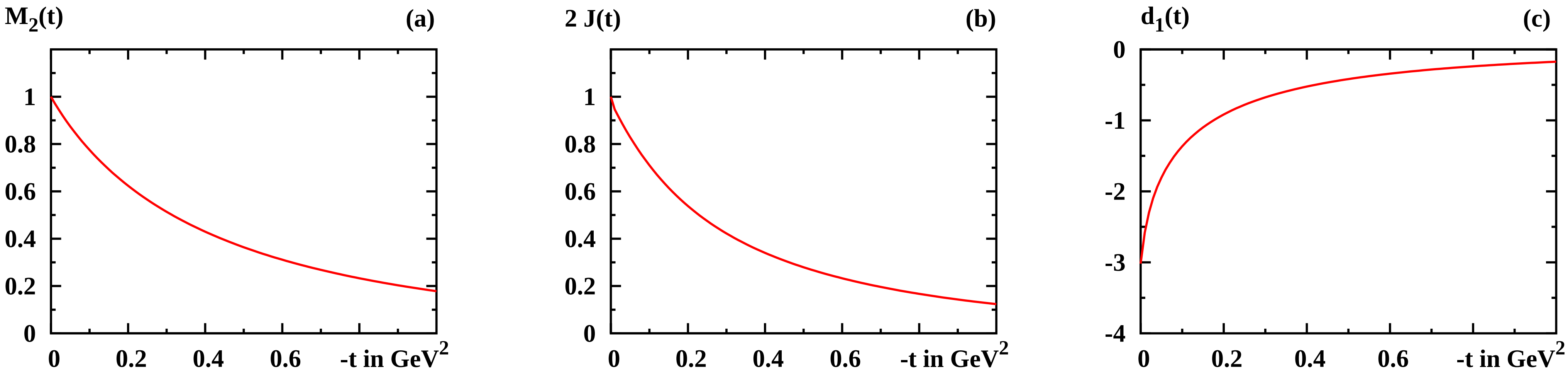
## 4. How do these form factors look like? Some insight through

chiral quark soliton model $\mathcal{L}_{\text{eff}} = \overline{\Psi}(i\partial\!\!\!/ - M\,\exp(i\gamma_5\tau^a\pi^a/f_\pi))\Psi$

Diakonov, Petrov, Pobylitsa 1988

- from instanton model of QCD vacuum, $\rho_{\text{av}}/R_{\text{av}} \sim \frac{1}{3}$ Diakonov, Petrov 1984
  ($\rho_{\text{av}}$ = instanton size, $R_{\text{av}}$ = instanton separation)
- $M = 350\,\text{MeV}$, cutoff $\rho_{\text{av}}^{-1} \sim 600\,\text{MeV}$, no adjustable parameters!
- nucleon = soliton of chiral field $U$, in limit number of colors $N_c \to$ large
  Witten 1979
- successful phenomenology within accuracy of (10-30)%
- main advantage: FIELD THEORY!
  $\rightarrow$ analytical proofs: sum rules, positivity, **polynomiality**

# Results for form factors



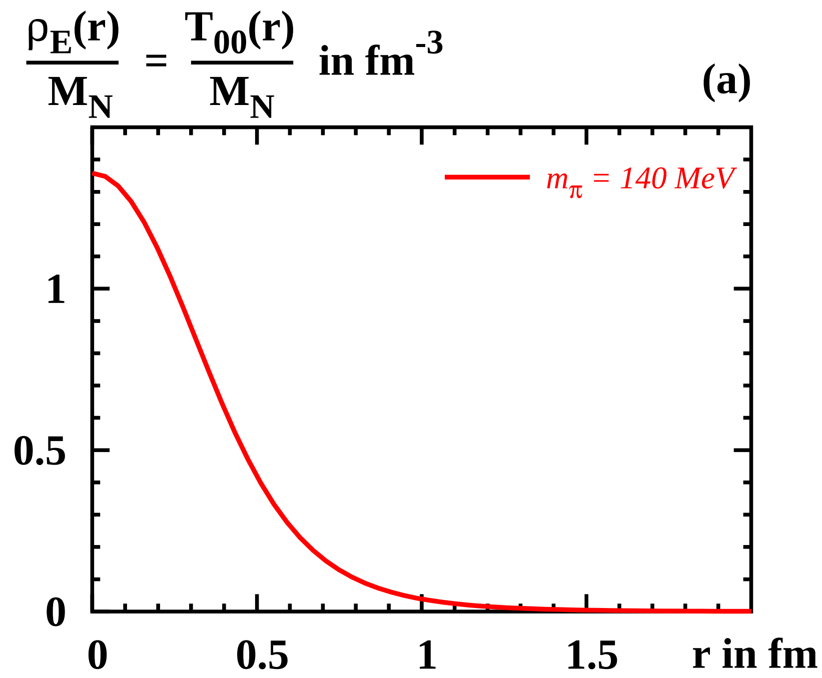

$M_{\text{dip}}(M_2) = 0.91$ GeV $\qquad M_{\text{dip}}(J) = 0.75$ GeV $\qquad M_{\text{dip}}(d_1) = 0.65$ GeV

for $|t| \lesssim 1\ \text{GeV}^2$ reasonable approximation $\boldsymbol{F(t) \approx \dfrac{F(0)}{(1 - t/M_{\text{dip}}^2)^2}}$ with

vs. electromagnetic form factors, for example, $G_E^p(t)$ with $M_{\text{dip}} \approx 0.91$ GeV

$\Rightarrow$ $\boldsymbol{M_2(t)}$ similar to em form factors, $\boldsymbol{J(t)}$ and $\boldsymbol{d_1(t)}$ different
Instructive: need to extrapolate from data at $t < 0$ to get $J^Q(0)$!

# Energy distribution




- $\rho_E(0) = 1.7$ GeV fm$^{-3}$ $= 3.0 \times 10^{15}$g/cm$^3$
  $\sim 13\times$nuclear matter equilibrium density

- distributed 'similar' to electric charge

- chiral limit: $\rho_E(r) \sim 3\left(\frac{3g_A}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large $r$

- $\langle r_E^2\rangle \equiv \dfrac{\int d^3\mathbf{r}\; r^2 \rho_E(r)}{\int d^3\mathbf{r}\; \rho_E(r)} = 0.7$ fm$^2$ similar to proton electric charge radius

- leading non-analytic term $\langle r_E^2\rangle = \langle \mathring{r}_E^{\,2}\rangle - \dfrac{81\, g_A^2}{64\pi f_\pi^2 M_N}\, m_\pi +$ higher orders

- for $m_\pi \to 0$ nucleon 'grows' (range of pion cloud increases)

- reasonable & consistent picture

# Compare: energy vs. electric density

in Skyrme model

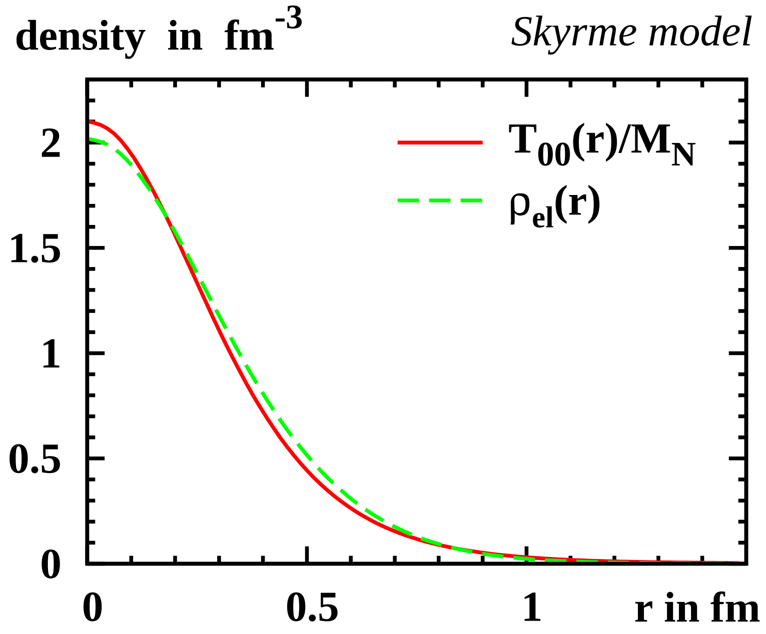
- baryon number density

  = isoscalar charge density

- distributions similar at intermediate $r$
- significant differences at large $r$

  $\rho_E(r) \propto \frac{1}{r^6}$ vs. $\rho_{el}(r) \propto \frac{1}{r^9}$

  different chiral physics!



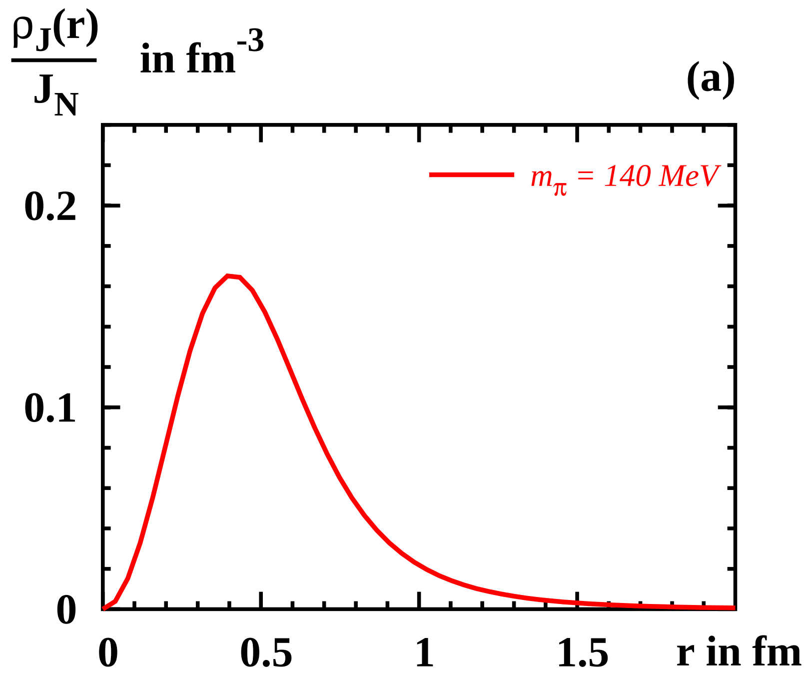

Picture in model.
And, in nature?

# Angular momentum distribution

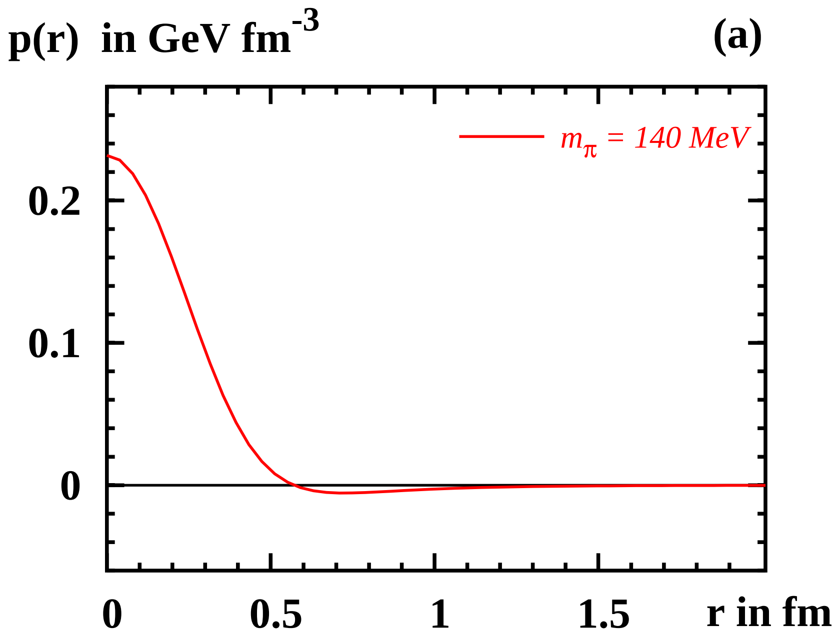
- $\rho_J(r) \propto r^2$ at small $r$
- $\langle r_J^2 \rangle = 1.3 \text{ fm}^2$

  2 times larger than $\langle r_E^2 \rangle$ or $\langle r_{em}^2 \rangle$
- in chiral limit: $\rho_J(r) \sim \frac{1}{r^4}$ at large $r$

  such that $\langle r_J^2 \rangle$ diverges



# Pressure




- $p(0) = 0.23\,\text{GeV/fm}^3 = 4 \cdot 10^{34}\,\text{N/m}^2$
  $\sim$ (10–20) $\times$ (pressure in neutron star)

- $p(0) \times$ (typical hadronic area $1\,\text{fm}^2$)
  $\sim\ 0.2\,\text{GeV/fm} \sim \dfrac{1}{5} \times$ {string tension}

- chiral limit: $p(r) \sim -\left(\frac{3g_A}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large $r$

- consequence: derivative $d_1'(0) = -\dfrac{3\,g_A^2\,M_N}{32\,\pi f_\pi^2\,m_\pi} + \dots$ diverges in chiral limit

- $r < 0.57\,\text{fm}$: $\boldsymbol{p(r) > 0}$ $\Rightarrow$ **repulsion** $\leftrightarrow$ quark core, Pauli principle

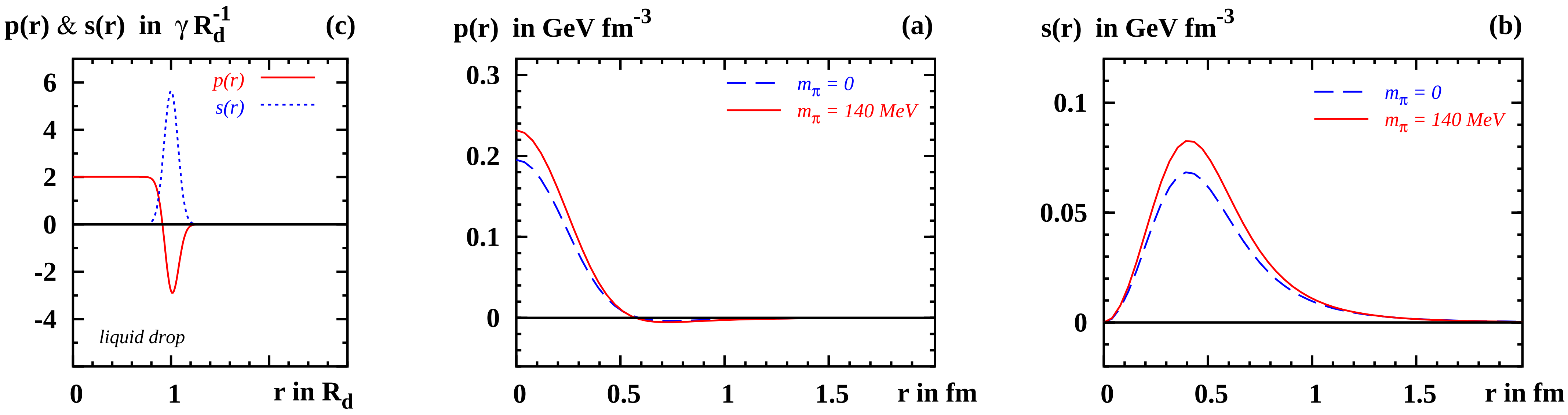
- $r > 0.57\,\text{fm}$: $\boldsymbol{p(r) < 0}$ $\Rightarrow$ **attraction** $\leftrightarrow$ pion cloud, binding forces

# Intuition on pressure and shear forces

In a 'liquid drop':

$$p(r) = p_0\,\theta(R_d - r) - \tfrac{1}{3}\,p_0 R_d\,\delta(R_d - r)$$

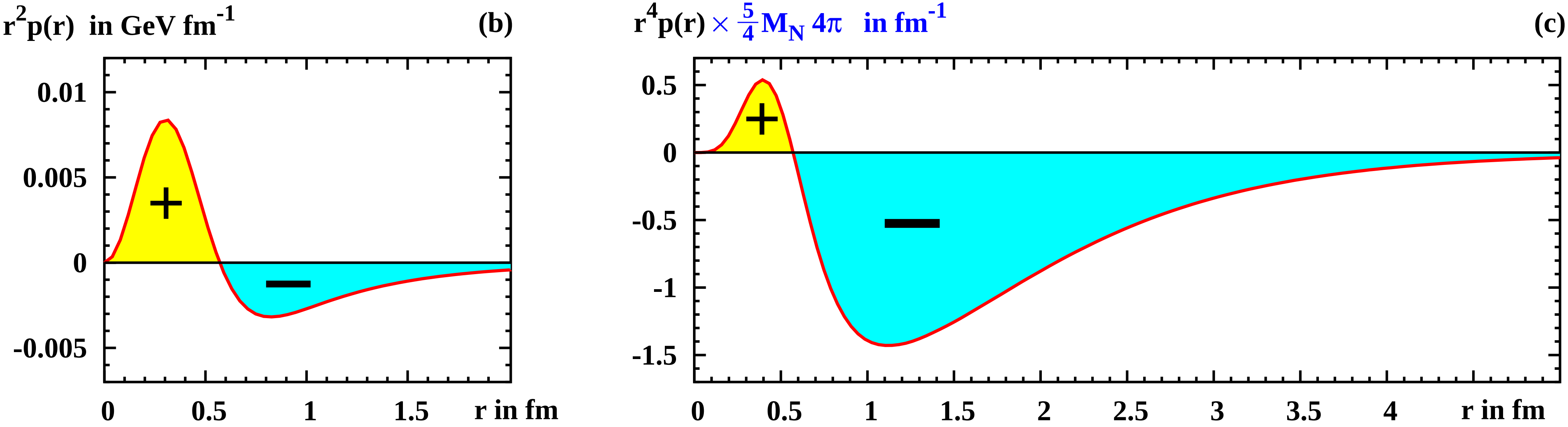
$s(r) = \gamma\,\delta(R_d - r)$ with surface tension $\gamma = \frac{1}{2}\,p_0 R_d$ (Kelvin, 1858)




nucleon does not resemble much a liquid drop (no 'sharp edge')

concept more useful for nuclei

# 3.7. Stability & sign of D-term




- $\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$ ✓ (second 'mean' question: do you have a stable nucleon?)
- $\int_0^\infty \mathrm{d}r\, r^4 p(r) < 0$, **of course!**
- $d_1 < 0$ **natural consequence of stability!**

(Advertisement: at JLab, in May, talk by Manuel Mai on $d_1$ in $Q$-balls! Fascinating!!!)

# Conclusions

- GPDs promise new insights Ji sum rule, impact parameter space, what else?
  (coming from DIS Duke-workshop: in models pretzelosity → orbital ang. mom.?)

- **form factors of energy momentum tensor!**
  (and only infinitesimal fraction of information content of GPDs)

- information content: energy density, spatial distribution of angular momentum,
  distribution of strong forces, meaning of $d_1$

- illustrated in chiral models, application ($t$-extrapolation, lattice),
  and vision have seen: distribution of energy density, strong forces
  connection: sign of $d_1$ vs. stability of nucleon, picture in model

- **how does it look like in nature with quarks and gluons** . . . ?

**Thank you !!!**