Jets from Massive Unstable Particles: Top-Mass Determination

Iain Stewart
MIT
Rutgers, March 2007

Based on work with:
Andre Hoang, Sean Fleming, & Sonny Mantry (hep-ph/0703207)
Outline

- Top mass measurements. Why do we want a precision $m_t$?
- Which mass? Observables & Issues
- Effective Field Theories for Top-Jets: SCET and HQET
- Factorization theorem for Jet Invariant Masses
- Summation of Large Logs $Q \gg m_t \gg \Gamma_t$
- Predictions and Phenomenology
- Summary
Motivation

The top mass is a fundamental parameter of the Standard Model

\[ m_t = 171.4 \pm 2.1 \text{ GeV} \] (already a 1% measurement!)

- Important for precision e.w. constraints
- Top Yukawa coupling is large. Top parameters are important for many new physics models
- Top is very unstable, it decays before it has a chance to hadronize. How does this effect jet observables involving top-quarks?

\[ \Gamma_t = 1.4 \text{ GeV} \] from \[ t \rightarrow bW \]

\[ \Lambda_{QCD} \]
Electroweak precision observables

\[ \delta M_W \propto m_t^2 \]
\[ \delta M_W \propto \ln(M_H) \]

\[ \sin^2 \theta_W \times \left( 1 + \delta(m_t, m_H, \ldots) \right) \]
\[ = 1 - \frac{m_W^2}{m_Z^2} \]

predictions for \( M_W \) and \( \sin^2 \theta_{\text{eff}} \)

- \( \delta m_t^{\text{exp}} = 0.1 \) GeV
- \( \delta m_t^{\text{exp}} = 2.0 \) GeV
- \( m_h = 115 \) GeV, \( \delta \Delta \alpha_{\text{had}} = 7 \times 10^{-5} \)

prospective exp. errors 68\% CL:

- LHC/LC
- GigaZ

Heinemeyer et al.
Mass of Lightest MSSM Higgs Boson

\[ m_h^2 \simeq M_Z^2 + \frac{G_F m_t^4}{\pi^2 \sin^2 \beta} \ln \left( \frac{m_{t_1} m_{t_2}}{m_t^2} \right) \]

<table>
<thead>
<tr>
<th>( \delta m_h )</th>
<th>LHC</th>
<th>LC</th>
</tr>
</thead>
<tbody>
<tr>
<td>needed ( \delta m_t )</td>
<td>4 GeV</td>
<td>0.2 GeV</td>
</tr>
<tr>
<td>expected ( \delta m_t )</td>
<td>1-2 GeV</td>
<td>~ 0.1 GeV</td>
</tr>
</tbody>
</table>

\[ \Delta m_h^{\text{exp}} \]

\( m_t = 175 \text{ GeV}, \tan \beta = 5 \)

theory prediction for \( m_h \)

- \( \delta m_t^{\text{exp}} = 2.0 \text{ GeV} \)
- \( \delta m_t^{\text{exp}} = 1.0 \text{ GeV} \)
- \( \delta m_t^{\text{exp}} = 0.1 \text{ GeV} \)

Heinemeyer et.al.
How is it the top-mass measured?

Best Tevatron Run II (preliminary, January 2007)

<table>
<thead>
<tr>
<th>Category</th>
<th>Mass (GeV/c²)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-Jets: CDF (943 pb⁻¹)</td>
<td>171.1 ± 4.3</td>
<td></td>
</tr>
<tr>
<td>Dilepton: CDF (1030 pb⁻¹)</td>
<td>164.5 ± 5.6</td>
<td></td>
</tr>
<tr>
<td>Dilepton: D0 (370 pb⁻¹)</td>
<td>178.1 ± 8.3</td>
<td></td>
</tr>
<tr>
<td>Lepton+Jets: CDF (940 pb⁻¹)</td>
<td>170.9 ± 2.5</td>
<td></td>
</tr>
<tr>
<td>Lepton+Jets: D0 (370 pb⁻¹)</td>
<td>170.3 ± 4.5</td>
<td></td>
</tr>
<tr>
<td>Tevatron (Run I/Run II, July 2006)</td>
<td>171.4 ± 2.1</td>
<td>χ²/dof = 10.6/10</td>
</tr>
</tbody>
</table>

pp → t¯tX

Γ

bW

q¯q'

e⁺ν

two b-jets + leptons

two b-jets + 2 jets+leptons

two b-jets + 4 jets
How is it the top-mass measured?

Within the SM:

\[ m_t > m_W + m_b \] dominant 2-body decay \( t \rightarrow Wb \) (\( Ws, Wd \) CKM suppressed)

Assuming unitarity of 3-generation CKM matrix:

\[ |V_{tb}| = 0.9990 \pm 0.0002 \text{ at } 90\% \text{ CL} \]

\[ B(t \rightarrow Wb) \sim 100\% \]

\[ t_{SM} = 1.4 \text{ GeV at } m_t = 175 \text{ GeV} \]

Top quark decays before top-flavored hadrons or \( tt \)-quarkonium bound states can form.

Top quark spin efficiently transferred to the final state.

Typical final state signatures in top quark pair production:

- Dilepton (BR~5%, low background)
- \( e^+\nu \) (large bkgd)
- \( \text{MET} \) (BR~30%, moderate background)
- \( \text{Lepton+jets} \) (BR~30%, moderate background)
- \( \text{MET} \) (BR~46%, huge background)

Two b-jets + leptons

Two b-jets + 2 jets + leptons

Two b-jets + 4 jets
**Template Method (CDF II)**

- **Principle:** perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

\[
\chi^2 = \sum_{i=b,jets} \frac{(p_{T,i}^{\text{fit}} - p_{T,i}^{\text{meas}})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_{j}^{\text{UE,fit}} - p_{j}^{\text{UE,meas}})^2}{\sigma_j^2} + \frac{(M_{W} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{tW} - m_t^{\text{reco}})^2}{\Gamma_t^2} + \frac{(M_{bjj} - m_t^{\text{reco}})^2}{\Gamma_t^2}
\]

Usually pick solution with lowest \( \chi^2 \).

- **Build templates from MC for signal and background and compare to data.**

**Dynamics Method (D0 II)**

- **Principle:** compute event-by-event probability as a function of \( m_t \) making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

\[
P(x; m_t) = \frac{1}{\sigma} \int d^2 \sigma(y; m_t) dq_1 dq_2 f(q_1) f(q_2) W(x | y)
\]

- **Transfer function:** mapping from parton-level variables \( y \) to reconstructed-level variables \( x \).
Uncertainties

\[ m_t = 171.4 \pm 1.2 \text{(stat)} \pm 1.8 \text{(syst)} \text{ GeV} \]

(eg. reconstruction)

- determine parton momentum of daughters, combinatorics
- jet-energy scale: calorimeter response, uninstrumented zones, multiple hard interactions, energy outside the jet “cone”, underlying event (spectator partons)
  
- initial & final state radiation, parton distribution functions, b-fragmentation
- which jet algorithm? which Monte-Carlo?
- background (W+jets), b-tagging efficiency
- Statistics

W-mass helps

\[ m_t = 171.4 \pm 1.2 \text{(stat)} \pm 1.8 \text{(syst)} \text{ GeV} \]
Current Uncertainties

\[ m_t = 171.4 \pm 1.2 \text{ (stat)} \pm 1.8 \text{ (syst)} \text{ GeV} \]

Future -LHC: \[ pp \rightarrow t\bar{t}X \]

top factory, 8 million \[ t\bar{t} \] / year (at low luminosity)
\[ \delta m_t \sim 1 \text{ GeV} \] systematics dominated

Future -ILC: \[ e^+ e^- \rightarrow t\bar{t} \]

exploit threshold region
\[ \sqrt{s} \approx 2m_t \]

with high precision theory calculations
\[ \delta m_t \sim 0.1 \text{ GeV} \]
What mass is it? $m = 171.4 \pm 1.2 \text{ (stat) } \pm 1.8 \text{ (syst) GeV}$

- **pole mass?**
  - ambiguity $\delta m \sim \Lambda_{QCD}$, linear sensitivity to IR momenta
  - poor behavior of $\alpha_s$ expansion
  - not used anymore for $m_b, m_c$
    
    e.g. $m_b^{1S} = (4.70 \pm 0.04) \text{ GeV}$

- quark masses are Lagrangian parameters, use a suitable scheme

- **top $\overline{\text{MS}}$ mass?** No

  $m^{\text{pole}} - m^{\overline{\text{MS}}}(m) \sim 8 \text{ GeV}$

  some schemes are more appropriate than others
Theory Issues for $pp \rightarrow t\bar{t}X$

- jet observable
- suitable top mass for jets
- initial state radiation
- final state radiation
- underlying events
- color reconnection
- beam remnant
- parton distributions
- sum large logs $Q \gg m_t \gg \Gamma_t$
Theory Issues for $pp \rightarrow t\bar{t}X$

- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- underlying events
- color reconnection ★
- beam remnant
- parton distributions
- sum large logs $Q \gg m_t \gg \Gamma_t$ ★

Here we’ll study $e^+e^- \rightarrow t\bar{t}X$ and the issues ★

We’ll take this calculation seriously, it can be measured at a future ILC.
Goals  Use Effective Field Theory to:

- Connect jet observables and a Lagrangian mass parameter (define a short-distance top-mass that is suitable for measurement with jets)
- Prove factorization: separation of length scales & dynamics
- Simultaneously treat top production and top decay
- Quantify non-perturbative and perturbative effects, universality, hopefully reduce experimental uncertainties
Measure what observable?

Hemisphere Invariant Masses

\[ M^2_t = \left( \sum_{i \in a} p_i^\mu \right)^2 \]

\[ M^2_{\bar{t}} = \left( \sum_{i \in b} p_i^\mu \right)^2 \]
Measure what observable?

Hemisphere Invariant Masses

\[ M_t^2 = \left( \sum_{i \in a} p_i^\mu \right)^2 \]

\[ \bar{M}_{\bar{t}}^2 = \left( \sum_{i \in b} p_i^\mu \right)^2 \]

Peak region:

\[ s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2 \]

\[ \bar{s}_{\bar{t}} \equiv \bar{M}_{\bar{t}}^2 - m^2 \sim m\Gamma \ll m^2 \]
Invariant Mass Distribution

\[
\frac{d^2 \sigma}{dM_t^2 \, dM_t^2}
\]

\[s_t \equiv M_t^2 - m^2 \sim m \Gamma \ll m^2\]
Invariant Mass Distribution

\[ \frac{d^2 \sigma}{dM_t^2 \, d\hat{M}_t^2} \]

\[ s_t \equiv M_t^2 - m^2 \sim m \Gamma \ll m^2 \]

- A first guess might be that the shape is a Breit Wigner

\[ \frac{m \Gamma}{s_t^2 + (m \Gamma)^2} = \left( \frac{\Gamma}{m} \right) \frac{1}{\hat{s}_t^2 + \Gamma^2} \]

\[ \hat{s}_t \equiv \frac{s_t}{m} \sim \Gamma \]
Invariant Mass Distribution

\[ \frac{d^2 \sigma}{dM_t^2 \ dM_{\bar{t}}^2} \]

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\[ \hat{s}_t \equiv \frac{s_t}{m} \sim \Gamma \]

- Since \( \Gamma \gg \Lambda_{\text{QCD}} \) we can calculate it and see. Answer: not quite. Our guess is a bit too naive.
\[ Q \gg m \gg \Gamma \sim \hat{S}_{t,\bar{t}} \]
$Q \gg m$  \hspace{1cm} SCET = Soft Collinear Effective Theory

(Bauer, Pirjol, I.S.; Fleming, Luke)

Top quarks are collinear.
Soft radiation btwn. jets.
\( Q \gg m \)  

**SCET = Soft Collinear Effective Theory**  
(Bauer, Pirjol, I.S.; Fleming, Luke)

Top quarks are collinear. 
Soft radiation btwn. jets.

\( m \gg \Gamma \sim \hat{s}_t, \bar{t} \)  

**HQET = Heavy Quark Effective Theory**  
(Isgur, Wise, ...)

Fluctuations \( \ll m \), tops act like static boosted color source

unstable particle EFT
Beneke, Chapovsky, Signer, Zanderighi
Brief Intro to SCET

<table>
<thead>
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<th>Degrees of Freedom</th>
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<td>SCET ( \lambda \sim m/Q \ll 1 )</td>
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<td>( n )-collinear ( (\xi_n, A_n^\mu) )</td>
</tr>
<tr>
<td>( \bar{n} )-collinear ( (\xi_{\bar{n}}, A_{\bar{n}}^\mu) )</td>
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<td>Crosstalk: soft ( (q_s, A_s^\mu) )</td>
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\( (+, -, \perp) \)

quark fields

gluon fields

light-cone coordinates
Soft - Collinear EFT

A formalism for jets.

\[ p^2 = p^+ p^- + p_\perp^2 \]

eg. \( e^+ e^- \rightarrow 2 \text{ jets} \)

\[ \lambda \sim \frac{\Delta}{Q} \]

\[ m_X^2 \sim \Delta^2 \]

\[ \Lambda^2 \ll \Delta^2 \ll Q^2 \]

Jet constituents: \( p^\mu \sim \left( \frac{\Delta^2}{Q}, Q, \Delta \right) \sim Q(\lambda^2, 1, \lambda) \)
**Brief Intro to SCET**

### Degrees of Freedom

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<td>(n)-collinear (n) (A_n^\mu) (p_n^\mu \sim Q(\lambda^2, 1, \lambda))</td>
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<tr>
<td>(\bar{n})-collinear (\xi_n, A_n^\mu) (p_n^\mu \sim Q(1, \lambda^2, \lambda))</td>
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**LO collinear Lagrangian:**

\[
\mathcal{L}_{qn}^{(0)} = \bar{\xi}_n \left[ i n \cdot D_s + g n \cdot A_n + (i \slashed{\partial}_c - m) W_n \frac{1}{\bar{n} \cdot \slashed{P}} W_n^\dagger (i \slashed{\partial}_c^\perp + m) \right] \frac{\bar{\eta}}{2} \xi_n
\]

**eikonal soft couplings**

**collinear Wilson line**

\[
W_n = P \exp \left( i g \int_0^\infty ds \bar{n} \cdot A_n(s\bar{n}) \right)
\]
Ultrasoft - Collinear Factorization

Multipole Expansion:

\[ \mathcal{L}_c^{(0)} = \bar{\xi}_n \left\{ n \cdot iD_{us} + gn \cdot A_n + i\mathcal{P}_1 \frac{1}{i\bar{n} \cdot D_c} i\mathcal{P}_1 \right\} \frac{\hat{\phi}}{2} \xi_n \]

usoft gluons have eikonal Feynman rules and induce eikonal propagators
Ultrasoft - Collinear Factorization

Multipole Expansion:

\[ \mathcal{L}_c^{(0)} = \bar{\xi}_n \left\{ n \cdot iD_{us} + gn \cdot A_n + i\mathcal{P}_\perp \frac{1}{i\bar{n} \cdot D_c} i\mathcal{P}_\perp \right\} \frac{\hat{\phi}}{2} \xi_n \]

Ultrasoft gluons have eikonal Feynman rules and induce eikonal propagators

Field Redefinition:

\[ \xi_n \rightarrow Y \xi_n, \quad A_n \rightarrow YA_nY^\dagger \]

\[ n \cdot D_{us}Y = 0, \quad Y^\dagger Y = 1 \]

\[ Y(x) = P \exp \left( ig \int_{-\infty}^{0} ds \ n \cdot A_{us}(x+ns) \right) \]

choice of \( \pm \infty \) here is irrelevant if one is careful
Ultrasoft - Collinear Factorization

Multipole Expansion:
\[ \mathcal{L}_c^{(0)} = \bar{\xi}_n \left\{ n \cdot iD_{us} + gn \cdot A_n + i\mathcal{P}_{\perp} \frac{1}{i\bar{n} \cdot D_c} i\mathcal{P}_{\perp} \right\} \frac{\bar{n}}{2} \xi_n \]

Ultrasoft gluons have eikonal Feynman rules and induce eikonal propagators

Field Redefinition:
\[ \bar{\xi}_n \rightarrow Y \xi_n, \quad A_n \rightarrow Y A_n Y^\dagger \]
\[ n \cdot D_{us} Y = 0, \quad Y^\dagger Y = 1 \]

gives:
\[ \mathcal{L}_c^{(0)} = \bar{\xi}_n \left\{ n \cdot iD_{us} + \ldots \right\} \frac{\bar{n}}{2} \xi_n \rightarrow \bar{\xi}_n \left\{ n \cdot iD_c + i\mathcal{P}_\perp \frac{1}{i\bar{n} \cdot D_c} i\mathcal{P}_\perp \right\} \frac{\bar{n}}{2} \xi_n \]

Moves all ultrasoft gluons to operators, simplifies cancellations
Brief Intro to SCET

Degrees of Freedom

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<td>$n$-collinear ($\xi_n, A^\mu_n$)</td>
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LO collinear Lagrangian:

$$\mathcal{L}_{qn}^{(0)} = \bar{\xi}_n \left[ i \bar{n} \cdot D_s + g n \cdot A_n + (i \not\!{D}_c - m) W_n \frac{1}{\bar{n} \cdot \mathcal{P}} W_n \not\!{D}_c + m \right] \frac{\eta}{2} \xi_n$$

Production Current:

$$\bar{n} \not\!{\psi} \Gamma^\mu \psi \rightarrow (\bar{\xi}_n W_n) \omega \Gamma^\mu (W_{\bar{n}}^\dagger \xi_{\bar{n}}) \bar{\omega} = (\bar{\xi}_n W_n) \omega Y_n^\dagger \Gamma^\mu Y_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}}) \bar{\omega}$$
Matching and Running

QCD → SCET → HQET

Integrate out Hard Modes

Factorize Jets, Integrate out energetic collinear gluons

Evolution and decay of top close to mass shell

Soft Cross-Talk
 Brief Intro to unstable boosted HQET

fluctuations beneath the mass

one HQET for top

one HQET for antitop

\[ p^\mu = m v_+^\mu + k^\mu \]

collinear, but with smaller overall scale

<table>
<thead>
<tr>
<th>bHQET [\Gamma/m \ll 1]</th>
<th>( n )-ucollinear ((h_{v+}, A_{v+}^\mu))</th>
<th>( k^\mu \sim \Gamma(\lambda^2, 1, \lambda) )</th>
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<tr>
<td>( n )-ucollinear ((h_{v-}, A_{v-}^\mu))</td>
<td>( k^\mu \sim \Gamma(1, \lambda^2, \lambda) )</td>
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</tr>
<tr>
<td>same soft ((q_s, A_s^\mu))</td>
<td>( p_s^\mu \sim (\Delta, \Delta, \Delta) )</td>
<td></td>
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</table>

\[ \mathcal{L}_+ = \bar{h}_{v+} (i v_+ \cdot D_+ - \delta m + \frac{i}{2} \Gamma) h_{v+}, \quad \mathcal{L}_- = \bar{h}_{v-} (i v_- \cdot D_- - \delta m + \frac{i}{2} \Gamma) h_{v-} \]

mass scheme choice

\[ \delta m = m_{\text{pole}} - m \]

our observable is inclusive in top decay products

a) \( \bar{t} \rightarrow Wb \rightarrow Zb \)
We are ready
to derive the
Factorization Theorem
In QCD: The full cross-section is

\[
\sigma = \sum_X (2\pi)^4 \delta^4(q - p_X) \sum_{i=a,v} L_{\mu\nu}^i \langle 0|J_i^{\nu\dagger}(0)|X\rangle\langle X|J_i^\mu(0)|0\rangle
\]

a restricted set of states: \(s_t \equiv M_i^2 - m^2 \sim m\Gamma \ll m^2\)

lepton tensor, \(\gamma \) & \(Z\) exchange

by using EFT’s we will be able to move these restrictions into the operators
In QCD: The full cross-section is a restricted set of states:

\[ s_t \equiv M_i^2 - m^2 \sim m\Gamma \ll m^2 \]

\[ \sigma = \sum_X (2\pi)^4 \delta^4(q - p_X) \sum_{i=a,v} L_{\mu\nu}^i \langle 0 | J_{i\mu}^\dagger(0) | X \rangle \langle X | J_{i\mu}^\dagger(0) | 0 \rangle \]

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by using EFT’s we will be able to move these restrictions into the operators

In SCET:

\[ J_{i\mu}^\dagger(0) = \int d\omega \, d\bar{\omega} \, C(\omega, \bar{\omega}, \mu) \, J_{i(0)\mu} (\omega, \bar{\omega}, \mu) \]

Wilson coefficient SCET current

Momentum conservation:

\[ \rightarrow C(Q, Q, \mu) \]

\[ (\bar{\xi}_n \, W_n)_{\omega} \, Y_n^\dagger \Gamma^\mu \, Y_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}} \]

\[ \equiv \bar{\chi}_n, \omega \, Y_n^\dagger \Gamma^\mu \, Y_{\bar{n}} \chi_{\bar{n}, \bar{\omega}} \]
SCET cross-section: $|X\rangle = |X_nX_{\bar{n}}X_s\rangle$

$$\sigma = K_0 \sum_{\bar{n}} \sum_{X_nX_{\bar{n}}X_s} (2\pi)^4 \delta^4(q - P_{X_n} - P_{X_{\bar{n}}} - P_{X_s}) \langle 0|\bar{Y}_{\bar{n}} Y_n|X_s\rangle \langle X_s|Y_n^\dagger \bar{Y}_{\bar{n}}^\dagger |0\rangle \times |C(Q, \mu)|^2 \langle 0|\hat{n}_{X_n,\omega'}|X_n\rangle \langle X_n|\bar{n}_{X_{\bar{n}},\omega}|0\rangle \langle 0|\bar{n}_{X_{\bar{n}},\bar{\omega}'}|X_{\bar{n}}\rangle \langle X_{\bar{n}}|\hat{n}_{X_{\bar{n}},\bar{\omega}}|0\rangle$$

all-orders
SCET cross-section: \[ |X⟩ = |X_nX_\bar{n}X_s⟩ \]

\[ \sigma = K_0 \sum_{\tilde{n}} \sum_{X_nX_\bar{n}X_s} (2\pi)^4 \delta^4(q-P_{X_n} - P_{X_\bar{n}} - P_{X_s}) \langle 0|\bar{Y}_\bar{n}Y_n|X_s⟩ \langle X_s|Y_n^\dagger \bar{Y}_\bar{n}^\dagger|0⟩ \]

\[ \times |C(Q, \mu)|^2 \langle 0|\hat{n}_{X_n,\omega'}|X_n⟩ \langle X_n|\bar{x}_{n,\omega}|0⟩ \langle 0|\bar{x}_{\bar{n},\bar{\omega}'}|X_\bar{n}⟩ \langle X_\bar{n}|\hat{n}_{X_\bar{n},\bar{\omega}}|0⟩ \]

QCD \rightarrow \text{all-orders}

SCET \rightarrow \text{one-loop}

gives \[ C(Q, \mu) = 1 + \frac{\alpha_s C_F}{4\pi} \left[ 3 \log \frac{-Q^2 - i0}{\mu^2} - \log^2 \frac{-Q^2 - i0}{\mu^2} - 8 + \frac{\pi^2}{6} \right] \]
Specify hemisphere invariant masses for the jets:

total soft momentum is the sum of momentum in each hemisphere

\[ K_{X_s} = \kappa_s^a + \kappa_s^b \]

\[ \hat{P}_a \left| X_s \right\rangle = \kappa_s^a \left| X_s \right\rangle, \quad \hat{P}_b \left| X_s \right\rangle = \kappa_s^b \left| X_s \right\rangle \]

hemisphere projection operators
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hemisphere projection operators

**Insert:**

\[ 1 = \int dM_t^2 \, \delta((p_n + k_s^a)^2 - M_t^2) \int dM_t^2 \, \delta((p_n + k_s^b)^2 - M_t^2) \]
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total soft momentum is the sum of momentum in each hemisphere

\[ K_{X_s} = k_s^a + k_s^b \]

\[ \hat{P}_a |X_s\rangle = k_s^a |X_s\rangle, \quad \hat{P}_b |X_s\rangle = k_s^b |X_s\rangle \]

hemisphere projection operators

Insert:

\[ 1 = \int dM_t^2 \delta \left( (p_n + k_s^a)^2 - M_t^2 \right) \int dM_t^2 \delta \left( (p_n + k_s^b)^2 - M_t^2 \right) \]

... Some Algebra ...
SCET factorization Theorem:

\[ \frac{d^2\sigma}{dM_t^2 \, dM_{t'}^2} = \sigma_0 \, H_Q(Q, \mu) \int_{-\infty}^{\infty} d\ell^+ \, d\ell^- \, \bar{J}_n(s_t - Q\ell^+, \mu) \, J_n(s_{t'} - Q\ell^-, \mu) \, S_{\text{hemi}}(\ell^+, \ell^-, \mu) \]

Hard Function

\[ H_Q(Q, \mu) = |C(Q, \mu)|^2 \]

Top Jet Function

Anti-top Jet Function

Soft radiation Function

universal

deep in

we’re here

\( Q \) \hspace{1cm} \( n \) \hspace{1cm} \( \bar{n} \)

\( m_t \)

\( \Gamma_t \)
Soft function is nonperturbative, but universal, it also appears in massless dijets

\[ S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a}) \delta(\ell^- - k_s^{-b}) \langle 0 | \bar{Y}_n Y_n(0) | X_s \rangle \langle X_s | Y_n^+ \bar{Y}_n^T(0) | 0 \rangle \]

Jet function:

\[ J_n(Q r_n^+ - m^2) = \frac{-1}{2\pi Q} \int d^4 x \ e^{i r \cdot x} \ \text{Disc} \langle 0 | T\{\bar{\chi}_n, Q(0) \hat{\gamma} \chi_n(x)\} | 0 \rangle \]

is perturbative
\[
\frac{d^2 \sigma}{dM_t^2 \, dM_{\tilde{t}}^2} = \sigma_0 \, H_Q(Q, \mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, J_n(s_t - Q \ell^+, \mu) J_{\tilde{n}}(s_{\tilde{t}} - Q \ell^-, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

\[
\hat{s}_t = s_t/m \ll m
\]

match onto HQET

Integrate out mass scale

**SCET jet fn.**

**Wilson coefficient**

**HQET jet fn.**

\[
J_n(m \hat{s}, \Gamma, \mu_m) = T_+(m, \mu_m) \, B_+(\hat{s}, \Gamma, \mu_m)
\]

**Matching:**

\[
T_{\pm}(\mu, m) = 1 + \frac{\alpha_s C_F}{4\pi} \left( \ln^2 \frac{m^2}{\mu^2} - \ln \frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6} \right)
\]
\[
\left( \frac{d^2 \sigma}{dM_t^2 \, dM_l^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m(m, \frac{Q}{m}, \mu_m, \mu) \\
\times \int_{-\infty}^{\infty} d\ell^+ \, d\ell^- \, B_+ \left( \hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu \right) \, B_- \left( \hat{s}_t - \frac{Q\ell^-}{m}, \Gamma, \mu \right) \, S_{\text{hemi}}(\ell^+, \ell^-, \mu).
\]

At tree level:

\[
B_{\pm}^{\text{tree}}(\hat{s}, \Gamma) = \frac{-1}{8\pi N_c m} \, (-2N_c) \, \text{Disc} \left( \frac{i}{v_{\pm} \cdot k + i\Gamma/2} \right) = \frac{1}{4\pi m} \, \text{Im} \left( \frac{-2}{v_{\pm} \cdot k + i\Gamma/2} \right) \\
= \frac{1}{\pi m} \, \frac{-\Gamma}{\hat{s}^2 + \Gamma^2}. \quad \text{our Breit-Wigner}
\]

- B.W. receives calculable perturbative corrections
- cross-section depends on non.pert. soft function, not just B.W.'s
- the B.W. is only a good approx. for collinear top & gluons
- in the fact. thm. we remove largest component of soft momentum from the inv.mass. to get the argument for the B.W.
A Short-Distance Top-Mass for Jets

- First, why not \( \overline{\text{MS}} \)? \( \delta \overline{m} \sim \alpha_s \overline{m} \gg \Gamma \)

when we switch to a short-distance mass scheme we must expand in \( \alpha_s \)

\[
B_+(\hat{s}, \mu, \delta \overline{m}) = \frac{1}{\pi \overline{m}} \left\{ \frac{\Gamma}{\left[ \frac{(M_t^2 - \overline{m}^2)^2}{\overline{m}^2} + \Gamma^2 \right]} + \frac{(4 \hat{s} \Gamma) \delta \overline{m}}{\left[ \frac{(M_t^2 - \overline{m}^2)^2}{\overline{m}^2} + \Gamma^2 \right]^2} \right\}
\]

\[
\sim \frac{1}{\overline{m} \Gamma}
\]

\[
\sim \frac{\alpha_s}{\Gamma^2}
\]

not a correction! it swamps the 1st term
A Short-Distance Top-Mass for Jets

- First, why not $\overline{\text{MS}}$? \[ \delta \overline{m} \sim \alpha_s \overline{m} \gg \Gamma \]

when we switch to a short-distance mass scheme we must expand in $\alpha_s$

\[ B_+(\hat{s}, \mu, \delta \overline{m}) = \frac{1}{\pi \overline{m}} \left\{ \frac{\Gamma}{[(M_t^2 - \overline{m}^2)^2 + \Gamma^2]} + \frac{(4 \hat{s} \Gamma) \delta \overline{m}}{[(M_t^2 - \overline{m}^2)^2 + \Gamma^2]^2} \right\} \]

\[ \sim \frac{1}{(\overline{m} \Gamma)} \]

\[ \sim \frac{\alpha_s}{\Gamma^2} \]

not a correction! it swamps the 1st term

- Jet mass scheme $m_J(\mu)$ \[ \delta m \sim \hat{s}_t \sim \hat{s}_\overline{t} \sim \Gamma \]

define the scheme by holding the B.W. peak position fixed

\[ \frac{dB_+(\hat{s}, \mu, \delta m_J)}{d\hat{s}} \bigg|_{\hat{s}=0} = 0 \]

\[ m_J(\mu) = m_{\text{pole}} - \delta m_J \]

\[ = m_{\text{pole}} - \Gamma \frac{\alpha_s(\mu)}{3} \left[ \ln \left( \frac{\mu}{\Gamma} \right) + \frac{3}{2} \right] \]
We can define a short distance mass scheme, $\delta m$, for jets by demanding that the peak of the jet function does not get shifted by perturbation theory.
There is no theoretical obstacle to measuring this jet mass to accuracy better than $\Lambda_{QCD}$.
\[
\left( \frac{d^2 \sigma}{dM_t^2 \, dM_t^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m \left( \frac{Q}{m_J}, \mu_{m}, \mu \right) \\
\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, \tilde{B}_+ \left( \hat{s}_t - \frac{Q\ell^+}{m_J} \right) \, \tilde{B}_- \left( \hat{s}_t - \frac{Q\ell^-}{m_J} \right) \, S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]
\[ \left( \frac{d^2 \sigma}{dM_t^2 \, dM_{l_t}^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m \left( m_J, \frac{Q}{m_J}, \mu_m, \mu \right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, \tilde{B}_+ \left( \hat{s}_t - \frac{Q \ell^+}{m_J}, \Gamma, \mu \right) \tilde{B}_- \left( \hat{s}_t - \frac{Q \ell^-}{m_J}, \Gamma, \mu \right) S_{\text{hemi}}(\ell^+, \ell^-, \mu) \]

Evolution and decay of top quark close to mass shell

Final cross-section with short-dist. mass

Non-perturbative Cross talk

Let's first study the phenomenological implications.
Let's first study the phenomenological implications.

I will then come back to prove that the summation of large logs does not significantly affect this phenomenology.

despite the large hierarchy! $Q \gg m \gg \Gamma$
Plots and Analysis
Plots and Analysis

- Soft function is nonperturbative. Can be modeled

\[ S_{\text{hemi}}^{M1}(\ell^+, \ell^-) = \frac{\theta(\ell^+)\theta(\ell^-)}{\Lambda^2} \mathcal{N}(a, b) \left( \frac{\ell^+\ell^-}{\Lambda^2} \right)^{a-1} \exp \left( -\frac{(\ell^+)^2 - (\ell^-)^2 - 2b\ell^+\ell^-}{\Lambda^2} \right) \]

and extracted from massless dijets using universality.
massless dijet event shapes

fit soft fn.

\[ a = 2, \quad b = -0.4 \]
\[ \Lambda = 0.55 \text{ GeV} \]

Figure 1: Heavy jet mass (a) and \( C \)–parameter (b) distributions at \( Q = M_2 \) with and without power corrections included.

Figure 2: Comparison of the QCD predictions for the heavy jet mass (a) and \( C \)–parameter (b) distributions with the data at different center-of-mass energies (from bottom to top): \( Q/\text{GeV} = 35, 44, 91, 133, 161, 172, 183, 189 \), based on the shape function.
and thrust too

\[ T = \max_\hat{t} \frac{\sum_i |\hat{t} \cdot p_i|}{Q} \]
So we can use it to predict the top-invariant mass distribution

\[
\frac{d^2 \sigma}{dM_t \, dM_\tilde{t}} = 4M_t M_\tilde{t} \, \sigma_0^H \int_{-\infty}^{\infty} d\ell^+ d\ell^- \tilde{B}_+ \left( \frac{\hat{s}_t - Q\ell^+}{m_J} , \Gamma, \mu \right) \tilde{B}_- \left( \frac{\hat{s}_\tilde{t} - Q\ell^-}{m_J} , \Gamma, \mu \right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

\[
\hat{s}_t = 2M_t - 2m_J , \quad \hat{s}_\tilde{t} = 2M_\tilde{t} - 2m_J ,
\]

Start with lowest order

\[
\tilde{B}_+ (\hat{s}_t) = \frac{2}{(m_J \Gamma)} \frac{1}{(\hat{s}_t/\Gamma)^2 + 1} , \quad \tilde{B}_- (\hat{s}_\tilde{t}) = \frac{2}{(m_J \Gamma)} \frac{1}{(\hat{s}_\tilde{t}/\Gamma)^2 + 1}
\]
Non-perturbative effects *shift* the peak position, and broadens the distribution.

Simple soft model: very narrow Gaussian centered at $\ell^\pm = \ell_0^\pm$

peak occurs at $M_{t,\bar{t}} \sim m_J + Q\ell_0^\pm / (2m_J)$
Nonperturbative Peak & Width Shifts with $Q$

- **Peak Position versus $Q/m$**
  - $M_t^\text{peak}$ (GeV)
  - $m_J = 172$

- **Peak Width versus $Q/m$**
  - Linear growth with $Q$!

**Linear growth with $Q$!**
This can be understood analytically:

Mean of distribution: \( 2L \gg Q\Lambda \)

\[
F^{(1)} \equiv \frac{1}{m_J^2 \Gamma^2} \int_{-L}^{L} ds_t \frac{\hat{s}_t}{2} \int_{-\infty}^{\infty} ds_i \; F(M_t, M_i) = \int_{-\infty}^{\infty} d\ell^+ \int_{-L}^{L} ds_t \frac{\hat{s}_t}{2} \tilde{B}_+ \left( \hat{s}_t - \frac{Q\ell^+}{m_J} \right) \int_{-\infty}^{\infty} d\ell^- S_{\text{hemi}}(\ell^+, \ell^-) \\
\simeq \frac{1}{2} \int_{-\infty}^{\infty} d\ell^+ \int_{-L}^{L} ds_t \left( \hat{s}_t + \frac{Q\ell^+}{m_J} \right) \tilde{B}_+ (\hat{s}_t) \int_{-\infty}^{\infty} d\ell^- S_{\text{hemi}}(\ell^+, \ell^-) \\
= \frac{Q}{2m_J} S_{\text{hemi}}^{(1,0)}.
\]

slope is \( S_{\text{hemi}}^{(1,0)} = \int d\ell^+ d\ell^- \ell^+ S_{\text{hemi}}(\ell^+, \ell^-) \)
This can be understood analytically:

Mean of distribution:

\[ F^{(1)} = \frac{1}{m_J^2 \Gamma^2} \int_{-L}^{L} ds_t \int_{-\infty}^{\infty} d\tilde{s}_t \frac{\hat{s}_t}{2} F(M_t, M_{\tilde{t}}) = \int_{-\infty}^{\infty} d\ell^+ \int_{-L}^{L} ds_t \frac{\hat{s}_t}{2} \tilde{B}_+ \left( \hat{s}_t - \frac{Q \ell^+}{m_J} \right) \int_{-\infty}^{\infty} d\ell^- S_{\text{hemi}}(\ell^+, \ell^-) \]

\[ \approx \frac{1}{2} \int_{-\infty}^{\infty} d\ell^+ \int_{-L}^{L} ds_t \left( \hat{s}_t + \frac{Q \ell^+}{m_J} \right) \tilde{B}_+ (\hat{s}_t) \int_{-\infty}^{\infty} d\ell^- S_{\text{hemi}}(\ell^+, \ell^-) \]

\[ = \frac{Q}{2m_J} S_{\text{hemi}}^{(1,0)}. \]

Peak of distribution:

\[ 0 = \frac{1}{m_J^2 \Gamma^2} \int_{-\infty}^{\infty} d\tilde{s}_t \frac{dF(M_t, M_{\tilde{t}})}{d\tilde{s}_t} = \int_{-\infty}^{\infty} d\ell^+ \tilde{B}_+ \left( \hat{s}_t - \frac{Q \ell^+}{m_J} \right) \int_{-\infty}^{\infty} d\ell^- S_{\text{hemi}}(\ell^+, \ell^-) \]

\[ = \int_{-\infty}^{\infty} d\ell^+ \left( \hat{s}_t - \frac{Q \ell^+}{m_J} \right) \tilde{B}_+''(0) + \frac{1}{3!} \left( \hat{s}_t - \frac{Q \ell^+}{m_J} \right)^3 \tilde{B}_+^{(4)}(0) + \ldots \right] \int_{-\infty}^{\infty} d\ell^- S_{\text{hemi}}(\ell^+, \ell^-) \]

\[ M_t^{\text{peak}} \approx m_J + Q/(2m_J) S_{\text{hemi}}^{(1,0)}. \]

slope is \[ S_{\text{hemi}}^{(1,0)} = \int d\ell^+ d\ell^- \ell^+ S_{\text{hemi}}(\ell^+, \ell^-) \]
If for some (eg. experimental) reason the universality of the soft function was not applicable then we would need to fit the soft function as well:

\[ S(I^+) \]

Shape Function Models \{a,b\}

\{1,0.9\} \{2,0.9\} \{3,0.9\}
\{1, 0.0\} \{2, 0.0\} \{3, 0.0\}
\{1, 0.9\} \{2, 0.9\} \{3, 0.9\}

\[ F^{(1)} \]

Moment \( F^{(1)} \) versus moment \( S^{(1,0)} \)

\[ d\sigma \]

\[ dM_t \]

\[ M_t^{\text{peak}} \]

\[ S^{(1,0)} \]
Finally, other observables can be projected out from ours.

**Thrust** \( T = \max \frac{\sum_i |\hat{t} \cdot p_i|}{Q} \)

2 massive particles: \( T = \sqrt{Q^2 - 4m^2}/Q = 1 - 2m^2/Q^2 + \mathcal{O}(m^4/Q^4) \)

Insert: \( 1 = \int dT \, \delta \left( 1 - T - \frac{M_t^2 + M_{\tilde{t}}^2}{Q^2} \right) \)

\[
\frac{d\sigma}{dT} = \sigma_0^H(\mu) \int_{-\infty}^{\infty} ds_t \, ds_{\tilde{t}} \, \tilde{B}_+ \left( \frac{s_t}{m_J}, \Gamma, \mu \right) \tilde{B}_- \left( \frac{s_{\tilde{t}}}{m_J}, \Gamma, \mu \right) S_{\text{thrust}} \left( 1 - T - \frac{(2m_J^2 + s_t + s_{\tilde{t}})}{Q^2}, \mu \right)
\]

\[
S_{\text{thrust}}(\tau, \mu) = \int_0^{\infty} d\ell^+ \, d\ell^- \, \delta \left( \tau - \frac{(\ell^+ + \ell^-)}{Q} \right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]
Thrust Distribution

\[ 1 - T = \frac{2m_f^2}{Q^2} \]

FIG. 8: Plot of the thrust distribution. The graph shows the distribution of thrust as a function of the variable \( (1 - T) \), with the thrust values normalized on the vertical axis. The equation \( 1 - T = \frac{2m_f^2}{Q^2} \) is used to calculate the thrust distribution. The peak of the distribution is at \( T = 0.080 \), indicating the maximum thrust value.
What about using a Jet Algorithm?

If all soft radiation is grouped into the jets (inclusive mode) then the factorization theorem is the same, but has a different soft function.
What about using a Jet Algorithm?

If all soft radiation is grouped into the jets (inclusive mode) then the factorization theorem is the same, but has a different soft function.
Log resummation

from renormalization of UV divergences in the effective field theories, which induce anomalous dimensions.
Log resummation

Matching Functions

$H(Q, \mu_h)$

Top-down running

$U_H$

Bottom-up running

$U_{J_-}$ $U_{J_+}$ $U_S$

Scales

$Q$

$m$

$\Delta$

$\Gamma \sim s/m$

$\Lambda_{QCD}$

Product Running

$C(m, \mu_m)$

Product Running

$S(\Delta, \mu_\Delta)$

Convolution Running

$B_{\pm}(\Gamma, \mu_\Gamma)$

$\mu \frac{d}{d\mu} B_{\pm}(\hat{s}, \mu) = \int d\hat{s}' \gamma_{B_{\pm}}(\hat{s} - \hat{s}') B_{\pm}(\hat{s}', \mu)$
SCET Log resummation

**top-down:**

$$\mu \frac{d}{d\mu} H_Q(Q, \mu) = \gamma_{H_Q}(Q, \mu) H_Q(Q, \mu)$$

$$H_Q(Q, \mu) = U_{H_Q}(\mu, \mu_h) H_Q(Q, \mu_h)$$

$$U_{H_Q}(\mu, \mu_Q) = \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_Q)} \right]^{-16\pi C_F}{\beta_0^6 \alpha_s(\mu_Q)} + \frac{6C_F}{\beta_0} \left( \frac{\mu}{\mu_Q} \right){}_{-8C_F/\beta_0} \left( \frac{\mu_Q}{Q} \right){}_{8C_F/\beta_0} \ln \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_Q)} \right)$$

- Product of soft and collinear jet functions run locally all the way down to the low scale.
- This local running only affects the normalization of the distribution.
SCET Log resummation

Matching Functions

Top-down running

Bottom-up running

Scales

Q

m

Δ

Γ \sim s/m

Λ_{QCD}

Bottom-up:

\mu \frac{d}{d\mu} J_{n,\bar{n}}(s, \mu) = \int ds' \gamma_{J_{n,\bar{n}}}(s-s') J_{n,\bar{n}}(s', \mu)

J_n(s, \mu) = \int ds' \ U_{J_n}(s-s', \mu, \mu_m) J_n(s', \mu_m)

\mu \frac{d}{d\mu} S(\ell^+, \ell^-, \mu) = \int d\ell'^+ d\ell'^- \gamma_S(\ell^+-\ell'^+, \ell^- - \ell'^-) S(\ell'^+, \ell'^-, \mu)

S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \int d\ell'^+ d\ell'^- \ U_S(\ell^+-\ell'^+, \ell^- - \ell'^-, \mu, \mu_m) S_{\text{hemi}}(\ell'^+, \ell'^-, \mu_m)

U_{J_n}(s-s', \mu, \mu_m) = \frac{e^{L_1}}{\Gamma(-\omega_1)} \frac{e^{\gamma_E}}{\Gamma(-\omega_1)} \frac{\theta(s-s')}{(s-s')^{1+\omega_1}}

\omega_1(\mu, \mu_m) = -\frac{4C_F}{\beta_0} \ln \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_m)} \right]

\omega_1 + \omega_2 = 0

U_S(\ell^+, \ell^-, \mu, \mu_0) = \frac{e^{2L_2}}{\Gamma(-\omega_2)^2} \frac{(\mu_m e^{\gamma_E})^{2\omega_2}}{\Gamma(-\omega_2)^2} \left[ \frac{\theta(\ell^+)}{(\ell^+)^{1+\omega_2}}+\frac{\theta(\ell^-)}{(\ell^-)^{1+\omega_2}} \right]

\omega_2(\mu, \mu_m) = \frac{4C_F}{\beta_0} \ln \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_m)} \right]
SCET Log resummation

Matching Functions

$H(Q, \mu_h)$

$C(m, \mu_m)$

$S(\Delta, \mu_\Delta)$

$B_{\pm}(\Gamma, \mu_\Gamma)$

Top-down running

$U_H$

$U_C$

$U_{B_+}$

Bottom-up running

$U_{J_+}$

$U_{B_+}$

Scales

$Q$

$SCET$

$m$

$\Delta$

$\Gamma \sim s/m$

$\Lambda_{QCD}$

consistency:

$$U_{HQ}(\mu, \mu_m) \delta(s - Q\ell'^+) \delta(\bar{s} - Q\ell'^-)$$

$$= \int d\ell^+ d\ell^- U_{J_n}(s - Q\ell^+, \mu, \mu_m) U_{J_n}(\bar{s} - Q\ell^-, \mu, \mu_m) U_{S}(\ell^+ - \ell'^+, \ell^- - \ell'^-, \mu, \mu_m)$$

$$\omega_1 + \omega_2 = 0$$

cancellation between soft & collinear factors
HQET Log resummation

Matching Functions

\begin{align*}
H(Q, \mu_h) & \quad \text{Top-down running} \\
C(m, \mu_m) & \quad \text{Bottom-up running} \\
S(\Delta, \mu_\Delta) & \\
B_\pm(\Gamma, \mu_\Gamma)
\end{align*}

Scales

\begin{align*}
Q & \\
m & \\
\Delta & \\
\Gamma & \sim s/m \\
\Lambda_{\text{QCD}}
\end{align*}

**top-down:**

\[
\mu \frac{d}{d\mu} H_m(m, \frac{Q}{m}, \mu) = \gamma_{Hm}(\frac{Q}{m}, \mu) H_m(m, \frac{Q}{m}, \mu) \quad H_m(\mu) = U_{Hm}(\mu, \mu_m) H_m(\mu_m)
\]

**bottom-up:**

\[
B_\pm(\hat{s}, \mu) = \int d\hat{s}' U_{B\pm}(\hat{s} - \hat{s}', \mu, \mu_\Gamma) B_\pm(\hat{s}', \mu_\Gamma)
\]

\[
S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \int d\ell'^+ d\ell'^- U_S(\ell'^+ - \ell'^+, \ell^- - \ell'^-, \mu, \mu_m) S_{\text{hemi}}(\ell'^+, \ell'^-, \mu_m)
\]

Similar to SCET

\[
\omega_1 + \omega_2 = 0
\]
HQET Log resummation

Matching Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Top-down running</th>
<th>Bottom-up running</th>
<th>Scales</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(Q, \mu_h)$</td>
<td>$U_H$</td>
<td>$U_{J+}$</td>
<td>$Q$</td>
</tr>
<tr>
<td>$C(m, \mu_m)$</td>
<td>$U_C$</td>
<td>$U_{J-}$</td>
<td>$m$</td>
</tr>
<tr>
<td>$S(\Delta, \mu_\Delta)$</td>
<td>$U_{B_+}$</td>
<td>$U_{B_-}$</td>
<td>$\Delta$</td>
</tr>
<tr>
<td>$B_{\pm}(\Gamma, \mu_\Gamma)$</td>
<td>$U_{B_-}$</td>
<td>$U_{S}$</td>
<td>$\Gamma \sim s/m$</td>
</tr>
</tbody>
</table>

consistency:

$$U_{H_m}(\mu, \mu_\Delta) \delta \left( \hat{s} - \frac{Q\ell'^+}{m} \right) \delta \left( \hat{s} - \frac{Q\ell'^-}{m} \right)$$

$$= \int d\ell^+ d\ell^- U_{B_+} \left( \hat{s} - \frac{Q\ell^+}{m}, \mu, \mu_\Delta \right) U_{B_-} \left( \hat{s} - \frac{Q\ell^-}{m}, \mu, \mu_\Delta \right) U_S(\ell^+ - \ell'^+, \ell^- - \ell'^-, \mu, \mu_\Delta)$$

$$\omega_1 + \omega_2 = 0$$

cancellation between soft & collinear factors again

an observable that did not account for the soft radiation would not have this property.
**BHQET Jet Function** \( B_\pm(\hat{s}, \mu) \)

**LL running** in our case large logs do not effect the normalization

\[
B_\pm(\hat{s}, \mu) = \int d\hat{s}' \ U_{B_\pm}(\hat{s} - \hat{s}', \mu, \mu_\Gamma) \ B_\pm(\hat{s}', \mu_\Gamma)
\]

\[
U_{B_\pm}(s - s', \mu, \mu_i) = \frac{e^{L_2(\mu, \mu_i)} m \mu_i e^{\gamma_E} \omega_1}{\Gamma(-\omega_1)} \left[ \frac{\theta(s - s')}{(s - s')^{1+\omega_1}} \right]_+, \quad \omega_1(\mu, \mu_i) = -\frac{4C_F}{\beta_0} \ln \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_i)} \right]
\]

LL Running from 1.5 to 3.0 GeV

Tree level Breit Wigner

\[
\mu = \Gamma
\]

\[
\frac{M_i^2 - m^2}{m} \quad \text{(GeV)}
\]
Lessons, Implications, and Conclusion

- Factorization allows us to keep track of how the observable effects corrections from other categories (hadronization, final state radiation, etc.)
- In our analysis the inclusive nature of the hemisphere mass definition reduces the uncertainty from hadronization. The jet functions sum over hadronic states up to $m \Gamma$ and are perturb. The soft functions is universal. If we switch observables (eg. like thrust) we can in some cases relate the soft functions.
- Gluon radiation between the decay products is power suppressed
- Summation of Large Logs, control of final state radiation
- Definition of a short-distance mass scheme for jets
- Results are observable dependent and will be different for the LHC. The corr. analysis may help reduce uncertainties.
The END