Supercurrents

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Motivation/Outline

- Embed the supercurrent in a supermultiplet – not always well defined

- A new supermultiplet

- Field theory applications

- Supergravity applications

- String construction applications

The conserved currents

- Energy momentum tensor $T_{\mu\nu}$
  Ambiguity (improvement):

$$T'_{\mu\nu} = T_{\mu\nu} + \left(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right) t$$

- Supersymmetry current $S_{\mu\alpha}$
  Ambiguity (improvement):

$$S'_{\mu\alpha} = S_{\mu\alpha} + (\sigma_{\mu\nu})_\alpha^{\ \beta}\,\partial^\nu s_\beta$$

Improvement terms do not affect the conserved charges and the current conservation.

The Ferrara-Zumino multiplet

The most widely known multiplet which includes $T_{\mu\nu}$ and $S_{\mu\alpha}$ is the FZ-multiplet $\mathcal{J}_{\alpha\dot{\alpha}}$

$$\overline{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X \ .$$

X is chiral

$$X = x + \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \overline{S}^{\dot{\alpha}}_\mu + \theta^2 (T^\mu_\mu + i\partial \cdot j) \ .$$

$j_\mu = \mathcal{J}_{\alpha\dot{\alpha}}\Big|_{\theta=\overline{\theta}=0}$ is an R-current.

It includes 12 fermionic operators $S_{\mu\alpha}$ and 12 bosonic operators: $T_{\mu\nu}$ $(10 - 4 = 6)$, j_μ (4) and x (2).

The FZ-multiplet in superconformal theories

Superconformal theories are characterized by $X = 0$, i.e.

$$\overline{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = 0 \ .$$

The R-current is conserved, $T_{\mu\nu}$ is traceless and $S_{\mu\alpha}$ has only spin $\frac{3}{2}$.

This multiplet includes 8 fermionic operators $S_{\mu\alpha}$ (with $\sigma^{\mu\alpha}_{\dot{\alpha}} S_{\mu\alpha} = 0$) and 8 bosonic operators: $T_{\mu\nu}$ $(10-4-1=5)$, j_μ $(4-1=3)$.

Ambiguity – improvement

The defining relation

$$\overline{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

is invariant under

$$\mathcal{J}'_{\alpha\dot{\alpha}} = \mathcal{J}_{\alpha\dot{\alpha}} - i\partial_{\alpha\dot{\alpha}} \left(\Omega - \overline{\Omega}\right)$$
$$X' = X + \frac{1}{2}\overline{D}^2\overline{\Omega}$$

with any chiral Ω.

$T_{\mu\nu}$ and $S_{\mu\alpha}$ change by improvement terms.

Example 1: Wess-Zumino models

In a theory based on a Kähler potential K and a superpotential W

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2g_{i\bar{i}}(D_\alpha \Phi^i)(\overline{D}_{\dot{\alpha}}\overline{\Phi}^{\bar{i}}) - \frac{2}{3}[D_\alpha, \overline{D}_{\dot{\alpha}}]K$$

$$X = 4W - \frac{1}{3}\overline{D}^2 K \ .$$

These are not invariant under Kähler transformations – they change by improvement terms.

Hence, if the Kähler class is nontrivial, $\mathcal{J}$ is not globally well defined – not a good operator.

Example 2: Fayet-Iliopoulos terms

An FI-term shifts $K \to K + \xi V$. As in the previous example

$$\mathcal{J}_{\alpha\dot{\alpha}} = \ldots - \frac{2}{3}\xi[D_\alpha, \overline{D}_{\dot{\alpha}}]V$$
$$X = \ldots - \frac{1}{3}\xi\overline{D}^2 V \ .$$

These are not gauge invariant – they change by improvement terms.

Now $\mathcal{J}$ is not gauge invariant – not a good operator.

The R-multiplet

If the theory has a $U(1)_R$ symmetry, we can embed $T_{\mu\nu}$, $S_{\mu\alpha}$ and the conserved R-current in $\mathcal{R}_{\alpha\dot{\alpha}}$ which satisfies

$$\overline{D}^{\dot{\alpha}}\mathcal{R}_{\alpha\dot{\alpha}} = \chi_\alpha$$

where χ_α is chiral and

$$\overline{D}_{\dot{\alpha}}\overline{\chi}^{\dot{\alpha}} = D^\alpha\chi_\alpha \ .$$

These $T_{\mu\nu}$ and $S_{\mu\alpha}$ differ from those in the FZ-multiplet by improvement terms.

This multiplet is Kähler invariant and gauge invariant even with FI-terms.

Generic theories

Generic theories do not have a continuous R-symmetry.

Do they have a well defined (gauge invariant, globally well defined) energy momentum tensor and supersymmetry current?

Are they in a good supermultiplet?

Goal

Look for a globally well defined and gauge invariant multiplet which exists even when the target space has a non-trivial Kähler class or when FI-terms are present.

The S-multiplet

We combine the ideas in the FZ-multiplet and the R-multiplet and embed $T_{\mu\nu}$ and $S_{\mu\alpha}$ in $\mathcal{S}_{\alpha\dot{\alpha}}$ which satisfies

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = D_\alpha X + \chi_\alpha$$

where χ_α and X are chiral and

$$\overline{D}_{\dot{\alpha}}\overline{\chi}^{\dot{\alpha}} = D^\alpha \chi_\alpha \ .$$

These $T_{\mu\nu}$ and $S_{\mu\alpha}$ differ from those in the FZ-multiplet by improvement terms.

We will see that this multiplet is always Kähler invariant and gauge invariant!

The components of the S-multiplet

The S-multiplet is a larger multiplet.

It includes $16 + 16$ operators.

In addition to the $12 + 12$ operators of the FZ multiplet it includes $4 + 4$ operators from χ_α.

The 4 additional bosons are a closed 2-form (3 operators) and a scalar.

Examples

In Wess-Zumino models

$$\mathcal{S}_{\alpha\dot{\alpha}} = 2g_{i\bar{i}}(D_\alpha\Phi^i)(\overline{D}_{\dot{\alpha}}\overline{\Phi}^{\bar{i}})$$
$$X = 4W$$
$$\chi_\alpha = \overline{D}^2 D_\alpha K \ .$$

With nonzero FI-terms

$$\chi_\alpha = -4\xi W_\alpha \ .$$

We see that $\mathcal{S}_{\alpha\dot{\alpha}}$ is gauge invariant and Kähler invariant and hence it is globally well defined.

Ambiguity – improvement

The defining relation

$$\overline{D}^{\dot\alpha}\mathcal{S}_{\alpha\dot\alpha} = D_\alpha X + \chi_\alpha$$

is invariant under

$$\begin{aligned}
\mathcal{S}'_{\alpha\dot\alpha} &= \mathcal{S}_{\alpha\dot\alpha} + [D_\alpha, \overline{D}_{\dot\alpha}]U \\
X' &= X + \frac{1}{2}\overline{D}^2 U \\
\chi'_\alpha &= \chi_\alpha + \frac{3}{2}\overline{D}^2 D_\alpha U
\end{aligned}$$

for any real U.

It changes the components by improvement terms.

Consequences of the improvement

If we can solve

$$\chi_\alpha = \overline{D}^2 D_\alpha U$$

with a globally well defined and gauge invariant U, we can set $\chi_\alpha = 0$, and find the FZ-multiplet.

If we can solve

$$X = \overline{D}^2 U$$

with a well defined U, we can set $X = 0$, and find the R-multiplet – the theory is R-invariant.

Field theory applications

Consider a supersymmetric field theory in which the FZ-multiplet exists in the UV description.

For example, consider a gauge theory with canonical kinetic terms and without FI-terms.

A nonrenormalization theorem: since the FZ-multiplet exists in the UV, it exists at all length scales. Hence the IR effective theory should also have an FZ-multiplet.

The low energy theory

- It does not have FI-terms either for elementary or emergent gauge fields.

- Its target space (and moduli space of vacua) has an exact Kähler form.

- The topology of the moduli space of vacua can be different than in the UV, but its Kähler class remains trivial (e.g. SQCD with $N_f = N_c$).

Previous related results

Similar nonrenormalization theorems from different points of view were given before:

- about FI terms by [Shifman, Vainshtein, Dine, and Weinberg]

- about the topology by [Witten]

Coupling to supergravity

We limit ourselves to field theories with no dimensionful parameters of order the Planck scale (e.g. no mass, or FI-term $\sim M_P$) and consider the large M_P limit.

In this limit we can focus on linearized supergravity – small coupling of matter to the graviton and the gravitino.

SUGRA from the FZ-multiplet

The most common presentation of supergravity is based on the FZ-multiplet [Wess, Zumino, Stelle, West, Ferrara, van Nieuwenhuizen].

At the linearized level we couple the gravity multiplet $H_{\alpha\dot{\alpha}}$ to the FZ-current

$$\int d^4\theta H_{\alpha\dot{\alpha}} \mathcal{J}^{\alpha\dot{\alpha}}$$

This is the old minimal multiplet of supergravity.

It cannot be used when $\mathcal{J}_{\alpha\dot{\alpha}}$ does not exist.

SUGRA from the R-multiplet

If the rigid theory has a $U(1)_R$ symmetry we can couple the gravity multiplet $H_{\alpha\dot{\alpha}}$ to the R-current.

This leads to the new minimal multiplet [Akulov, Volkov, Soroka, Sohnius, West]

General considerations of gravity theory exclude the existence of global continuous symmetries and hence we will not pursue it.

SUGRA from the S-multiplet

Recall the S-multiplet which every SUSY field theory has

$$\overline{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = D_\alpha X + \chi_\alpha$$

If we can solve $\chi_\alpha = \overline{D}^2 D_\alpha U$ with a well defined U, the FZ-multiplet exists and we can use standard SUGRA.

Alternatively, we couple

$$\int d^4\theta H_{\alpha\dot{\alpha}}\mathcal{S}^{\alpha\dot{\alpha}} \ .$$

SUGRA from the S-multiplet – the spectrum

The S-multiplet is larger than the FZ-multiplet. Correspondingly, the off-shell gravity multiplet includes $16 + 16$ fields.

On shell spectrum: graviton, gravitino, complex scalar and Weyl fermion.

The 4 additional fields are similar to the dilaton, two-form field (dual to a scalar) and dilatino of string theory.

This theory is related to 16+16 SUGRA [Girardi, Grimm, Muller and Wess].

SUGRA from the S-multiplet – an alternate description

This theory can be describe as ordinary SUGRA (based on FZ-multiplet) where the matter theory includes an additional chiral superfield Φ.

The ill defined U in $\chi_\alpha = \overline{D}^2 D_\alpha U$ always appears in the combination

$$\widehat{U} = U + \Phi + \overline{\Phi}$$

SUGRA from the S-multiplet – solving the problems of the FZ-multiplet

The theory depends on

$$\widehat{U} = U + \Phi + \overline{\Phi} \ .$$

When U is not gauge invariant or not Kähler invariant:

$$U \to U + \Lambda + \overline{\Lambda}$$

the field Φ is also not invariant:

$$\Phi \to \Phi - \Lambda$$

such that $\widehat{U}$ is invariant.

Example 1: FI-terms

$$\widehat{U} = U + \Phi + \overline{\Phi}$$

When there are FI-terms, $U \sim \xi V$ is not gauge invariant. Φ makes the FI-term "field dependent."

It has the effect of Higgsing the gauge symmetry.

This is common in string theory when a theory on a brane has FI-terms, a closed string modulus Φ appears in $\widehat{U}$.

Example 2: nontrivial Kähler class

$$\widehat{U} = U + \Phi + \overline{\Phi}$$

When U is not globally well defined, Φ extends the target space and makes its Kähler class trivial. $\widehat{U}$ is well defined.

This is common in string theory. When a brane moves on a space with certain 2-cycles the Kähler form of its target space is not exact. A closed string mode Φ couples as in $\widehat{U}$ and removes the problem.

Constraints on string models

Many string constructions use D3-branes.

In the field theory limit (large extra dimensions limit) the theory on these D3-branes often has FI-terms and moduli with nontrivial target space.

When these theories are coupled to gravity (i.e. the additional dimensions are compactified) a massless superfield like our Φ must be present.

Typically this is the dilaton/radion multiplet.

Constraints on string models – moduli stabilization

Often we would like to stabilize the modulus Φ in a supersymmetric fashion leaving a low energy theory with FI-terms or nontrivial topology.

This is impossible.

Many (not all!) published models, e.g. some of those based on sequestering, flux vacua, KKLT, F-theory, etc. need to be revisited.

Conclusions

- The energy momentum tensor and the supersymmetry current should be embedded in a supermultiplet.

- The most common supermultiplet is the Ferrara-Zumino multiplet.

- When there are nonzero FI-terms or the Kähler form of the target space is not exact the FZ-multiplet is ill defined.

Conclusions, cont.

- A larger multiplet, the S-multiplet is always well defined.

- When a UV theory has a good FZ-multiplet, so should the IR theory. It cannot have FI-terms and the Kähler form of its target space must be exact.

Conclusions, cont.

- When the FZ-multiplet exists we can easily couple it to supergravity.

- When the FZ-multiplet does not exist we must couple the S-multiplet to supergravity.

- The resulting theory is ordinary supergravity with an additional chiral superfield.

Conclusions, cont.

- Interpreting this chiral superfield as part of the matter, it fixes the problems with the FZ-multiplet.

- This leads to constraints on many string constructions.