Jets, our window on partons at the LHC

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Work (in progress) with M. Cacciari, M. Dasgupta, L. Magnea, G. Soyez

Rutgers University — New High Energy Theory Seminar
27 November 2007
Partons — quarks and gluons — are key concepts of QCD.

- It’s in terms of quark and gluon fields that we write the Lagrangian.
- Perturbative QCD *only* deals with partons.
- Concept of parton powerful even beyond perturbation theory:
  - hadron classifications
  - exotic states, e.g. colour glass condensate (high gluon densities)
Yet it is surprisingly difficult to ascribe unambiguous meaning to partons.

- Not an asymptotic state of the theory
- Because of confinement
- But also even in perturbation theory because of collinear divergences (in massless approx.)

QCD coupling has related problems (probability of emitting a gluon...
Despite this, there are two decent ways of “seeing” partons;

- Scatter some hard probe off them, e.g. a virtual photon Deep Inelastic Scattering (DIS)
- See traces of them in the final state jets

In each case ill-defined nature of a parton translates into ambiguity in the partonic interpretation of what you see.
In final state, trace of original partons is visible as collimated bunches of energetic hadrons

Picture illustrates $e^+ e^- \rightarrow Z \rightarrow q\bar{q}$

Information not just visual, but also quantitative

\[ E \sim \frac{m_Z}{2} \]
Jets are what we see.
Clearly(?) 2 jets here

How many jets do you see?
Do you really want to ask yourself this question for $10^8$ events?
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1. Introduction

Seeing v. defining jets

Jets are what we see.
Clearly (?) 2 jets here

How many jets do you see?
Do you really want to ask yourself this question for $10^8$ events?
A jet definition is a systematic procedure that projects away the multiparticle dynamics, so as to leave a simple picture of what happened in an event:

Jets are as close as we can get to a physical single hard quark or gluon: with good definitions their properties (multiplicity, energies, [flavour]) are

- finite at any order of perturbation theory
- insensitive to the parton → hadron transition

NB: finiteness ↔ set of jets depends on jet def.
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1. Introduction

1. Seeing Partons

Why does it work?

Proper jet definition gives results that are approximately invariant with respect to:

- soft and collinear branching
  - So divergent real and virtual contributions cancel
  - IR & Collinear safety
- local reshuffling of momenta

Hadronisation
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Hadronisation
Heavy objects: multi-jet final-states

- $10^7$ $t\bar{t}$ pairs for $10$ fb$^{-1}$ (1 year, low-lumi)
- Vast # of QCD multijet events

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<th># jets</th>
<th># events for 10 fb$^{-1}$</th>
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<tr>
<td>3</td>
<td>$9 \cdot 10^8$</td>
</tr>
<tr>
<td>4</td>
<td>$7 \cdot 10^7$</td>
</tr>
<tr>
<td>5</td>
<td>$6 \cdot 10^6$</td>
</tr>
<tr>
<td>6</td>
<td>$3 \cdot 10^5$</td>
</tr>
<tr>
<td>7</td>
<td>$2 \cdot 10^4$</td>
</tr>
<tr>
<td>8</td>
<td>$2 \cdot 10^3$</td>
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Tree level
\[ p_t(jet) > 60 \text{ GeV}, \theta_{ij} > 30 \text{ deg}, |y_{ij}| < 3 \]

Draggiotis, Kleiss & Papadopoulos '02

All-hadronic
(BR~46%, huge bckg)

picture: Juste LP ’05
Tree-level calculations with many partons / W / Z / H / etc.

- Alpgen
- Madgraph
- Sherpa
- Helas/Helac
- [Twistor-derived rules]

Monte Carlo event generators

- Pythia (f77), Pythia8 (C++)
- Herwig (f77), Herwig++ (C++)
- Ariadne
- Sherpa
- With NLO matching: MC@NLO, POWHEG, (Vincia, GeNeVA, . . . )

Each tool associated with 3–15 people: total of $\sim 50$
1. Introduction

2. Jets at LHC

Experimenters’ priorities

1. $pp \rightarrow WW + \text{jet}$  Les Houches

2. $pp \rightarrow H + 2 \text{jets}$
   ▶ Background to VBF Higgs production

3. $pp \rightarrow t\bar{t}b\bar{b}$

4. $pp \rightarrow t\bar{t} + 2 \text{jets}$
   ▶ Background to $t\bar{t}H$

5. $pp \rightarrow WW b\bar{b}$

6. $pp \rightarrow VV + 2 \text{jets}$
   ▶ Background to $W\ W \rightarrow H \rightarrow WW$

7. $pp \rightarrow V + 3 \text{jets}$
   ▶ General background to new physics

8. $pp \rightarrow VVV + \text{jet}$
   ▶ Background to SUSY trilepton

Currently available

NLOJET++, MCFM, PHOX, ...
http://www.cedar.ac.uk/hepcode/

Theorist’s list (G. Heinrich)

▶ $2 \rightarrow 3$ (OK for a good student!)
   ▶ $pp \rightarrow WW + \text{jet}$
   ▶ $pp \rightarrow VVV$
   ▶ $pp \rightarrow H + 2 \text{jets}$

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NLO wishlist (2005)

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Experimenters’ priorities

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   ■ Background to VBF Higgs production

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Another 30-50 people active
Jet (definitions) provide central link between expt., “theory” and theory
What’s new for jets @ LHC?

Number of particles:

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- Range & complexity of signatures (jets, \( t\bar{t}, tj, Wj, Hj, t\bar{t}j, WWj, Wjj, SUSY, \) etc.)
- Theoretical investment
  \( \sim 100 \text{ people} \times 10 \text{ years} \)
  \( 60 – 100 \text{ million $} \)

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Approach of LHC provides motivation for taking a new, fresh, systematic look at jets.

This talk: some of the discoveries along the way

Definitions shown are those with widest exptl. impact

NB: also ARCLUS, OJF, ...
1. Introduction

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Jet Definition History

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Sequential recombination algorithms

**$k_t$ algorithm**  
Catani, Dokshizter, Olsson, Seymour, Turnock, Webber ’91–’93  
Ellis, Soper ’93

- Find smallest of all $d_{ij} = \min(k_{ti}^2, k_{tj}^2) \Delta R_{ij}^2 / R^2$ and $d_{iB} = k_i^2$
- Recombine $i, j$ (if $iB$: $i \rightarrow$ jet)
- Repeat

**NB: hadron collider variables**

- $\Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$
- Rapidity $y_i = \frac{1}{2} \ln \frac{E_i + p_{z_i}}{E_i - p_{z_i}}$
- $\Delta R_{ij}$ is boost invariant angle

$R$ sets jet opening angle
2. Safe, practical jet-finding

1. Sequential recombination

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Why $k_t$?

$k_t$ distance measures

$$d_{ij} = \min(k_{ti}^2, k_{tj}^2) \Delta R_{ij}^2, \quad d_{iB} = k_{ti}^2$$

are closely related to structure of divergences for QCD emissions

$$[dk_j]|_{M_{g\rightarrow g+g}(k_j)} \sim \frac{\alpha_s C_A}{2\pi} \frac{dk_{tj}}{\min(k_{ti}, k_{tj})} \frac{d\Delta R_{ij}}{\Delta R_{ij}}, \quad (k_{tj} \ll k_{ti}, \Delta R_{ij} \ll 1)$$

and

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$k_t$ algorithm attempts approximate inversion of branching process
‘Trivial’ computational issue:

- for $N$ particles: $N^2 \ d_{ij}$ searched through $N$ times $= N^3$
- 4000 particles (or calo cells): 1 minute
  
  NB: often study $10^7 - 10^8$ events (20-200 CPU years)

- Heavy Ions: 30000 particles: 10 hours/event

As far as possible physics choices should not be limited by computing.

Even if we’re clever about repeating the full search each time, we still have $O(N^2) \ d_{ij}$’s to establish
Can we do better than $N^2$?

There are $N(N - 1)/2$ distances $d_{ij}$ — surely we have to calculate them all in order to find smallest?

$k_t$ distance measure is partly geometrical:

- Consider smallest $d_{ij} = \min(k_{ti}^2, k_{tj}^2)R_{ij}^2$
- Suppose $k_{ti} < k_{tj}$
- Then: $R_{ij} \leq R_{i\ell}$ for any $\ell \neq j$. \[\text{[If } \exists \ell \text{ s.t. } R_{i\ell} < R_{ij} \text{ then } d_{i\ell} < d_{ij}\]

**In words:** if $i, j$ form smallest $d_{ij}$ then $j$ is geometrical nearest neighbour (GNN) of $i$.

$k_t$ distance need only be calculated between GNNs

Each point has 1 GNN $\rightarrow$ need only calculate $N$ $d_{ij}$’s
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Finding Geom Nearest Neighbours

Given a set of vertices on plane (1...10) a Voronoi diagram partitions plane into cells containing all points closest to each vertex

Dirichlet '1850, Voronoi '1908

A vertex’s nearest other vertex is always in an adjacent cell.

E.g. GNN of point 7 will be found among 1,4,2,8,3 (it turns out to be 3)

Construction of Voronoi diagram for $N$ points: $N \ln N$ time  Fortune '88

Update of 1 point in Voronoi diagram: $\ln N$ time

Devillers '99 [+ related work by other authors]

Convenient C++ package available: CGAL  http://www.cgal.org

Assemble with other comp. science methods: FastJet

Cacciari & GPS, hep-ph/0512210  
http://www.lpthe.jussieu.fr/~salam/fastjet/
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http://www.lpthe.jussieu.fr/~salam/fastjet/
2. Safe, practical jet-finding
   1. Sequential recombination

FastJet performance

NB: for $N < 10^4$, FastJet switches to a related geometrical $N^2$ alg.
Conclusion: speed issues for $k_t$ resolved
Modern cone algs have two main steps:

- Find some/all stable cones
  \[\equiv\text{cone pointing in same direction as the momentum of its contents}\]
- Resolve cases of overlapping stable cones

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**Qu:** How do you find the stable cones?

All experiments use iterative methods:

- use each particle as a starting direction for cone; use sum of contents as new starting direction; repeat.
- use additional ‘midpoint’ starting points between pairs of initial stable cones.

‘Midpoint’ algorithm
Stable cones
with midpoint: \{1,2\} & \{3\}

Jets with midpoint \( f = 0.5 \) \{1,2\} & \{3\}

Midpoint cone alg. misses some stable cones; extra soft particle \( \rightarrow \) extra starting point \( \rightarrow \) extra stable cone found

**MIDPOINT IS INFRARED UNSAFE**

Or collinear unsafe with seed threshold
2. Safe, practical jet-finding

- Cone algorithms

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**IR/Collinear unsafety is a serious problem!**

- Invalidates theorems that ensure finiteness of perturbative QCD
  - Cancellation of real & virtual divergences
- Destroys usefulness of (intuitive) partonic picture
  - You cannot think in terms of hard partons if adding a 1 GeV gluon changes 100 GeV jets
- ‘Pragmatically:’ limits accuracy to which it makes sense to calculate

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**A cone algorithm should find all stable cones**

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Guarantees IR safety of the set of stable cones

Only issue: you still need to find the stable cones in practice.

One known exact approach:

- Take each possible subset of particles and see if it forms a stable cone. Tevatron Run II workshop, '00 (for fixed-order calcs.)
- There are $2^N$ subsets for $N$ particles. Computing time $\sim N2^N$.
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Finding all distinct circular enclosures of a set of points is *geometry*:

![Diagram](image)

Any enclosure can be moved until a pair of points lies on its edge.

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A Seedless Infrared Safe Cone: SISCone

Naive implementation of this idea would run in $N^3$ time.

- $N^2$ pairs of points, pay $N$ for each pair to check stability
- $N^3$ is also time taken by midpoint codes (smaller coeff.)

With some thought, this reduces to $N^2 \ln N$ time.

- Traversal order, stability check
- checkxor
- GPS & Soyez '07

- Much faster than midpoint with no seed threshold
  - IR unsafe

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MC cross check of IR safety

- Generate event with
  \[2 < N < 10\] hard particles, find jets

- Add \[1 < N_{\text{soft}} < 5\] soft particles, find jets again
  [repeatedly]

- If the jets are different, algorithm is IR unsafe.

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Be careful with split–merge too.
2. Safe, practical jet-finding

- Cone algorithms

- MC cross check of IR safety

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A full set of algs

Complementary set of IR/Collinear safe jet algs → flexibility in studying complex events.

Consider families of jet algs: e.g. sequential recombination with

\[ d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \Delta R_{ij}^2 / R^2 \]

<table>
<thead>
<tr>
<th>Alg. name</th>
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<th>time</th>
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<tr>
<td>( p = 1 )</td>
<td></td>
<td></td>
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<tr>
<td>( k_t )</td>
<td>Dynamic Nearest Neighbour</td>
<td>( N \ln N ) exp.</td>
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<tr>
<td>CDOSTW ’91-93; ES ’93</td>
<td>CGAL (Devillers et al)</td>
<td></td>
</tr>
<tr>
<td>( p = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cambridge/Aachen</td>
<td>Dynamic Closest Pair</td>
<td>( N \ln N )</td>
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<tr>
<td>Dok, Leder, Moretti, Webber ’97</td>
<td>T. Chan ’02</td>
<td></td>
</tr>
<tr>
<td>Wengler, Wobisch ’98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p = -1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>anti-( k_t ) (cone-like)</td>
<td>Dynamic Nearest Neighbour</td>
<td>( N^{3/2} )</td>
</tr>
<tr>
<td>Cacciari, GPS, Soyez, in prep.</td>
<td>CGAL (worst case)</td>
<td></td>
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<tr>
<td>cone</td>
<td>All circular enclosures</td>
<td>( N^2 \ln N ) exp.</td>
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<tr>
<td>SISCone</td>
<td>previously unconsidered</td>
<td></td>
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<td>GPS Soyez ’07 + Tevatron run II ’00</td>
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All accessible in FastJet

FastJet in software of all (4) LHC collaborations
Once you have a decent set of jet algs, *start asking questions about them.*

- They share a common parameter $R$ (angular reach). How do results depend on $R$?
- In what way do the various algorithms differ?
- How are they to be best used in the challenging LHC environment?

Try to answer questions with Monte Carlo? Gives little understanding of underlying principles.

⇒ *Supplement with analytical approximations.*
3. Understanding jet algs

Various contributions

- Gluon emission, $\mathcal{O}(\alpha_s)$
- Conversion of quarks, gluons $\rightarrow \pi^\pm$, etc.
- Hadronisation
- Underlying event
- Pileup
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Diagram:
- Proton
- Anti-proton
- Various partons and quark interactions
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Proton

$\sigma$

Anti-proton
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- Gluon emission, $\mathcal{O}(\alpha_s)$
- Conversion of quarks, gluons $\rightarrow \pi^\pm$, etc.
  
  **Hadronisation**

- Underlying event
- Pileup
Start with *quark* with transverse momentum $p_t$

$$\langle \delta p_t \rangle_{PT} \simeq \frac{1}{\sigma_0} \int d\Phi |M^2| \alpha_s(k_{t,rel})(p_{t,jet} - p_t)$$

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Dokshitzer & Webber; Korchemsky & Sterman
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E.g.:

$$\frac{2}{\pi} \delta \alpha_s(k_{t,\text{rel}}) = \Lambda \delta(k_{t,\text{rel}} - \Lambda)$$

$$\Lambda = \int dk_{t,\text{rel}} \delta \alpha_s(k_{t,\text{rel}}), \text{ should be}$$

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Tested for $\sim 10$ observables in $e^+e^-$ and DIS.

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Deductions from Korchemsky & Sterman '94
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3. Understanding jet algs

1. R-dependence

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3. Understanding jet algs

1. $R$-dependence

Test NP results v. MC

qq → qq, Tevatron

$\langle \delta p_t \rangle_{\text{hadr}}$ [GeV]

$R$

MC hadr. agrees with calc.
- to varying degrees for range of algs
- also in larger gluonic channels

MC UE $\gg$ naive expectation
- models tuned on same data behave differently
- UE is huge at LHC
- largely indep. of scattering channel

Scale for (non-perturbative!)
UE is $\sim 10$ GeV
3. Understanding jet algs

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Jet, our window on partons (p. 36)

3. Understanding jet algs

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Test NP results v. MC

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\[ \langle \delta p_t \rangle_{hadr} \]

UE in \( qq \rightarrow qq, \) LHC

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- Pythia tune A
- Herwig+Jimmy

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### Dependence of jet $\langle \Delta p_t \rangle$ on

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To get best experimental resolutions, minimise contributions from all 3 components.

Here: sum of squared means

Better still: calculate fluctuations

NB: this is rough picture, but can still be used to understand general principles.
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3. Understanding jet algs

Optimal $R$ vs $p_t$, proc., collider

Basic messages

- higher $p_t \rightarrow$ larger $R$
  Most say opposite

- larger $R$ for gluons than quarks
  Gluon jets wider

- smaller $R$ at LHC than Tevatron
  UE larger
This last part of talk was an overview of *1 of several* recent jet topics

**Others include**

- **Subtraction of pileup**
  - Cacciari & GPS ’07

- **Jet areas ↔ sensitivity to UE/pileup**
  - Cacciari, GPS & Soyez prelim

- **“Optimising $R$” — cross checking with MC**
  - Cacciari, Rojo, GPS & Soyez, for Les Houches

- **Jet flavour — e.g. reducing $b$-jet theory uncertainties from $40 – 60\%$ to $10 – 20\%$.**
  - Banfi, GPS & Zanderighi ’06, ’07
Jets, our window on partons (p. 40)

4. Conclusions

Conclusions / Outlook

- Jets are the closest we can get to seeing and giving meaning to partons
- Play a pivotal role in experimental analyses, comparisons to QCD calculations
- Significant progress in past 2 years towards making them consistent (IR/Collinear safe) and practical
- The physics of how jets behave in a hadron-collider environment is a rich subject — much to be understood, and potential for significant impact in how jets are used at LHC

Link with computational geometry