Flux and Freund-Rubin Superpotentials in M-Theory

Neil Lambert
King’s College London

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String Theory apparently predicts a vast "Landscape" of vacua

Moduli stabilized by fluxes

Two key questions:

- Is the Standard Model there at all?
- How special or generic is it?
- i.e. How much choice did god have?
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A major program has been the statistical analysis of the Landscape [Douglas et al.].
Introduction

Here we are interested in M-theory vacua $M_4 \times X$. Statistics were recently discussed by [Acharya, Denef and Valandro]... $N=1$ Minkowski vacua $\rightarrow X$ is $G_2$ non-Abelian gauge theories from co-dimension 4 singularities $\rightarrow$ chiral Fermions from co-dimension 7 singularities... Over 20 years ago Freund and Rubin introduced a class of M-theory "compactifications"... $\rightarrow$ Spacetime is $AdS_4 \times X$... Four-form flux $G=\hat{M}dvol$... $N=1$ supersymmetry $\rightarrow X$ is weak $G_2$... $\rightarrow$ Only realistic in that $D=4$ (and even then...) $\rightarrow$ Now seen as near horizon $AdS$ duals to 3D CFT's.
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**The Good:**
- chiral Fermions and non-Abelian gauge fields from singularities
- easier to construct examples
- supersymmetric brane configurations without orientifolds
- adS duals to phenomenological models

**The Bad:**
- adS vacua - but that is generic to supersymmetric vacua
- imagine some kind of KKLT mechanism

**The Ugly:**
- KK scale = cosmological scale
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- **The Ugly:**
  - KK scale = cosmological scale
Our goals here:

- Study the effective potential for weak $G^2$ compactifications of M-theory.
- Include topological fluxes [c.f. Beasley and Witten].
- Look for mechanisms to lift the cosmological constant.
- Look for mechanisms to set KK scale greater than cosmological scale.
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▶ study the effective potential for weak $G_2$ compactifications of M-theory
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Our wish list:

▶ Look for mechanisms to lift the cosmological constant
▶ Look for mechanisms to set KK scale $>>$ cosmological scale
The KK Reduction

Bosonic sector of the low energy effective action of M-theory

\[ S = \frac{1}{2\kappa^9} \int \sqrt{-g} R - \frac{1}{2} G \wedge \ast G - \frac{1}{6} C \wedge G \wedge G \]

where

\[ G = dC \]

metric ansatz:

\[ g = V_0 \text{Vol}(X)^{-1} g_4(M) + g_7(X) \]

where

\[ \text{Vol}(X) = \int_X \sqrt{g_7} \]

Fluxes

\[ G = Mdx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 + G_X \]
The KK Reduction

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N.B. The Freund-Rubin parameter \( M \) is not in general a constant
The KK Reduction

$N = 1$ supersymmetry comes from a spinor on $X$ with

\[ \nabla_i \eta = \frac{i}{2} \lambda_7 \gamma_i \eta \]

Hence

\[ R_{ij}(X) = 6 \lambda_7^2 g_{ij}(X) \]

and

\[ d_7 \Phi = 4 \lambda_7 \ast_7 \Phi, \quad d_7 \ast_7 \Phi = 0 \]

where

\[ \Phi = \frac{i}{3!} \tilde{\eta} \gamma_{ijk} \eta dx^i \wedge dx^j \wedge dx^k \]

This is the so-called weak $G_2$ condition.
The KK Reduction

What are the light modes?
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- harmonic 3-forms $\omega_i$ give rise to massless axions $C^i$
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- harmonic 3-forms $\omega_i$ give rise to massless axions $C^i$
- massless metric deformations arise from [House and Micu]

$$\delta \Phi = s^I \phi_I \quad d \phi_I = -\lambda_7 \ast_7 \phi_I$$
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These are not in a 1-1 correspondence.

To obtain chiral supermultiplets we need Bosonic superpartners for these modes.
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- KK axion modes $\delta C = \tilde{C}^I \varphi_I$
  - massive - $m^2 \sim \lambda_7^2$
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- KK axion modes $\delta C = \tilde{C}^I \varphi_I$
  - massive - $m^2 \sim \lambda_7^2$
- non-weak $G_2$ metric deformations $\delta \Phi = \tilde{s}^i \omega_i$
  - tachonic $m^2 \sim -\lambda_7^2$
The KK Reduction

So we introduce two sorts of light multiplets:

1. $z^i = C^i + i\tilde{s}^i + \text{Fermions}$
2. $z^I = \tilde{C}^I + is^I + \text{Fermions}$
The KK Reduction

So we introduce two sorts of light multiplets:

- $z^i = C_i + i\tilde{s}^i + $ Fermions
- $z^l = \tilde{C}^l + i s^l + $ Fermions

Note that if we take $\lambda_7 \to 0$ then we obtain two copies of the massless supermultiplet.

- origin: $\omega_i$ and $\varphi_I$ are orthogonal modes if $\lambda_7 \neq 0$

$$\int_X \omega_i \wedge \star_7 \varphi_I \sim \lambda_7^{-1} \int \omega_i \wedge d\varphi_I = 0$$
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$$\int_X \omega_i \wedge *_7 \varphi_l \sim \lambda_7^{-1} \int \omega_i \wedge d\varphi_l = 0$$

Thus we wish to expand

- $C = C_0 + C_X + \sum C^i \omega_i + \sum_l \tilde{C}^l \varphi_l$
- $\Phi = \sum_i \tilde{s}^i \omega_i + \sum_l s^l \varphi_l$

set $\tilde{s}^i = 0$ for now
So we introduce two sorts of light multiplets:

- \( z^i = C^i + i\tilde{s}^i + \text{Fermions} \)
- \( z^l = \tilde{C}^l + i\tilde{s}^l + \text{Fermions} \)

Note that if we take \( \lambda_7 \rightarrow 0 \) then we obtain two copies of the massless supermultiplet.

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- \( \Phi = \sum_i \tilde{s}^i \omega_i + \sum_l s^l \varphi_I \)
  - set \( \tilde{s}^i = 0 \) for now
The KK Reduction

Substituting into the action gives

\[ S_{\text{eff}} = \frac{1}{\kappa_4^2} \int \sqrt{-g_4} \left( \frac{1}{2} R_4 - g_{ij} \partial_\mu z^i \partial^{\mu} \bar{z}^j - g_{IJ} \partial_\mu z^I \partial^{\mu} \bar{z}^J - V \right) + T \]

where

\[ g_{ij} = \frac{1}{4 \text{Vol}(X)} \int_X \omega_i \wedge \ast_7 \omega_j \quad \quad g_{IJ} = \frac{1}{4 \text{Vol}(X)} \int_X \varphi_I \wedge \ast_7 \varphi_J \]

\[ V = 16 \lambda_7^2 \frac{V_0}{\text{Vol}(X)} \tilde{C}^I \tilde{C}^J g_{IJ} - \frac{21 V_0 \lambda_7^2}{\text{Vol}(X)} + \frac{\text{Vol}(X)^3}{4 V_0^3} M^2 \]

\[ + \frac{1}{4} \frac{V_0}{\text{Vol}(X)^2} \int_X G_X \wedge \ast_7 G_X \]

\[ T = - \frac{1}{4 V_0} M C^i \int_X \omega_i \wedge G_X - \frac{1}{4 V_0} M \tilde{C}^I \tilde{C}^J \int \varphi_I \wedge d\varphi_J \]

\[ - \frac{1}{4 V_0} M \int_X C_X \wedge G_X \]
The KK Reduction

Note that we can’t think of either $V$ or $V - T$ as a potential.
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Note that we can’t think of either $V$ or $V - \mathcal{T}$ as a potential.

- even if $G_X = \tilde{C}' = 0$, it does not have the correct Freund-Rubin extremum
The KK Reduction

Note that we can’t think of either $V$ or $V - T$ as a potential.

- even if $G_X = \tilde{C}^I = 0$, it does not have the correct Freund-Rubin extremum
- $T$ is topological - doesn’t contribute to $T_{\mu\nu}$
The KK Reduction

Note that we can’t think of either $V$ or $V - \mathcal{T}$ as a potential.

- even if $G_X = \tilde{C}^I = 0$, it does not have the correct Freund-Rubin extremum
- $\mathcal{T}$ is topological - doesn’t contribute to $T_{\mu\nu}$

$M$ cannot be a constant parameter in the presence of fluxes:

$$d \star G + \frac{1}{2} G \wedge G = 0 \rightarrow d \star G_X + MG_X = 0$$

i.e. a constant $M$ implies the fluxes are topologically trivial
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In addition we see that if $M \neq 0$ then the fluxes are a source for the axions:

$$\frac{\partial \mathcal{T}}{\partial C^i} = \frac{1}{4V_0} M \int_X \omega_i \wedge G_X$$
To correctly reproduce the M-theory dynamics we can remove $M$ by using the $C$-field equation of motion:

$$\int_{X} \omega^i \wedge G_X + \frac{1}{2} \int_{X} \tilde{C} I \wedge C J \int_{X} \phi^I \wedge d \phi^J$$
The KK Reduction

To correctly reproduce the M-theory dynamics we can remove $M$ by using the $C$-field equation of motion:

- conservation of Page charge

\[
P_0 = \int_X \star G + \frac{1}{2} C \wedge G
\]

\[
= - \frac{\text{Vol}(X)^3}{V_0^2} M + \frac{1}{2} C^i \int_X \omega_i \wedge G_X + \frac{1}{2} \int_X C_X \wedge G_X
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\[
+ \frac{1}{2} \tilde{C}^I \tilde{C}^J \int_X \varphi_I \wedge d\varphi_J
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+ \frac{1}{2} \tilde{C}^l \tilde{C}^j \int_X \varphi_l \wedge d\varphi_j
\]

- use this to remove $M$ in terms of $P_0$, $\text{Vol}(X)$, $C^i$ and $\tilde{C}^l$ in the remaining equations of motion
The KK Reduction

All this leads to a system of equations for the other fields which come from the action

\[ S_{\text{eff}} = \frac{1}{\kappa_4^2} \int \sqrt{-g_4} \left( \frac{1}{2} R_4 - g_{ij} \partial_\mu z^i \partial^\mu \bar{z}^j - g_{IJ} \partial_\mu z^I \partial^\mu \bar{z}^J - U \right) \]

\[ U = \frac{16 \lambda_7^2 V_0}{\text{Vol}(X)} \tilde{C}^I \tilde{C}^J g_{IJ} - \frac{21 V_0 \lambda_7^2}{\text{Vol}(X)} \int_X G_X \wedge \ast_7 G_X \]

\[ + \frac{V_0}{4 \text{Vol}(X)^3} \left( \frac{1}{2} C^k \int_X \omega_k \wedge G_X + \frac{1}{2} \tilde{C}^I \tilde{C}^J \int_X \varphi_I \wedge d\varphi_J - \tilde{P}_0 \right)^2 \]

\[ \tilde{P}_0 = P_0 - \frac{1}{2} \int_X C_X \wedge G_X \]
The KK Reduction

Two cases to consider:

If $G_X = 0$

$\rightarrow$ extremum at $\tilde{C}_I = 0$

$\rightarrow$ effective potential for the volume is

$U = -2V_0^2V_0^2\lambda^27Vol(X) + V_0^2\tilde{P}_0^24Vol(X)^3$

$\rightarrow$ $Vol(X) \sim \lambda^{-\frac{1}{7}}P_0$

$\rightarrow$ Freund-Rubin

If $P_0$ is big enough then there is a solution with $\tilde{C}_I \neq 0$

$\rightarrow$ Englert-type solution with topologically trivial flux
The KK Reduction

Two cases to consider:

If $G_X = 0$

- extremum at $\tilde{C}' = 0$
- effective potential for the volume is

$$U = -\frac{21V_0\lambda_7^2}{\text{Vol}(X)} + \frac{V_0\tilde{P}_0^2}{4\text{Vol}(X)^3}$$

- $\text{Vol}(X) \sim \lambda_7^{-1}P_0$
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The KK Reduction

If $G_X \neq 0$
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If $G_X \neq 0$

- can set $\tilde{P}_0 = 0$ by a large gauge transformation
- minimum for $C^i$ occurs at $M = 0$
- minimum is at $\tilde{C}^I = 0$
- effective potential for the volume is

$$U = -\frac{21 V_0 \lambda_7^2}{\text{Vol}(X)} + \frac{1}{4} \frac{V_0}{\text{Vol}(X)^2} \int_X G_X \wedge *_7 G_X$$

- $\text{Vol}(X) \sim \lambda_7^{-2} \int_X G_X \wedge *_7 G_X$
The Superpotential

We want to find a superpotential (and Kahler potential) to reproduce $U$

$$U = e^K (g^{I\bar{J}} D_I W \bar{D}_J \bar{W} + g^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3 W \bar{W})$$
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There is a general form for the Kahler potential (at large volume)

$$K = -3 \ln \left( \frac{\text{Vol}(X)}{V_0} \right)$$

We assume that

$$\text{Vol}(X) = \frac{1}{7} \int_X \Phi \wedge \ast_7 \Phi$$
The Superpotential

Next we need to give the form of the superpotential.
The Superpotential

Next we need to give the form of the superpotential.

For $\lambda_7 = P_0 = 0$ Beasley and Witten use (cf. Gukov-Vafa-Witten)

$$W = \frac{1}{4V_0} \int_X \left( \frac{1}{2} C + i\Phi \right) \wedge G$$

but this is not holomorphic if $d\Phi \neq 0$.

A natural generalization is

$$W = -\frac{1}{4V_0} P_0 + \frac{1}{8V_0} \int_X \left( C + i\Phi \right) \wedge d\left( C + i\Phi \right)$$

clearly holomorphic

reduces to the above when $d\Phi = 0$ and $P_0 = 0$

must take $P_0$ to be a constant (not dependent on $C$).
The Superpotential

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Next we need to give the form of the superpotential.

For $\lambda_7 = P_0 = 0$ Beasley and Witten use (cf. Gukov-Vafa-Witten)

$$W = \frac{1}{4V_0} \int_X \left( \frac{1}{2} C + i\Phi \right) \wedge G$$

but this is not holomorphic if $d\Phi \neq 0$.

A natural generalization is

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Explicit calculation shows that

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\[ - \frac{21 \lambda^2 V_0}{\text{Vol}(X)} - \frac{16 \lambda_7^2 V_0}{\text{Vol}(X)} \left( g_{i\bar{j}} \tilde{s}^i \tilde{s}^j - (g_{i\bar{j}} \tilde{s}^i \tilde{s}^j)^2 \right) \]
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Applications

We would like to break the relation: KK scale = cosmological scale and lift to de Sitter solutions.
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To break this one needs to fine-tune the potential so that additional contributions cancel the $-21\lambda_7^2$ term
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Let us try to model supersymmetry breaking by the gauge theory sector localized at co-dimension 7 singularities of $X_7$.

$$S_{susy} = -2 \sum_A \Lambda_A \int d^4x \sqrt{-g}$$

Leads to

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Next consider including the (complex) Chern-Simons invariant [Acharya]

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Comments

What we have done:

- We constructed the effective potential and superpotential for Freund-Rubin compactifications in the presence of topological fluxes.
- Turning on fluxes drives the Freund-Rubin parameter to zero and results in a non-supersymmetric minimum.
- No supersymmetric vacua except pure Freund-Rubin or pure $G_2$ (need $SU(3)$ structure?)
- Looked at methods to lift cosmological constant and KK scale but with no success.

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- Consider cosmological constant issues in more detail
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