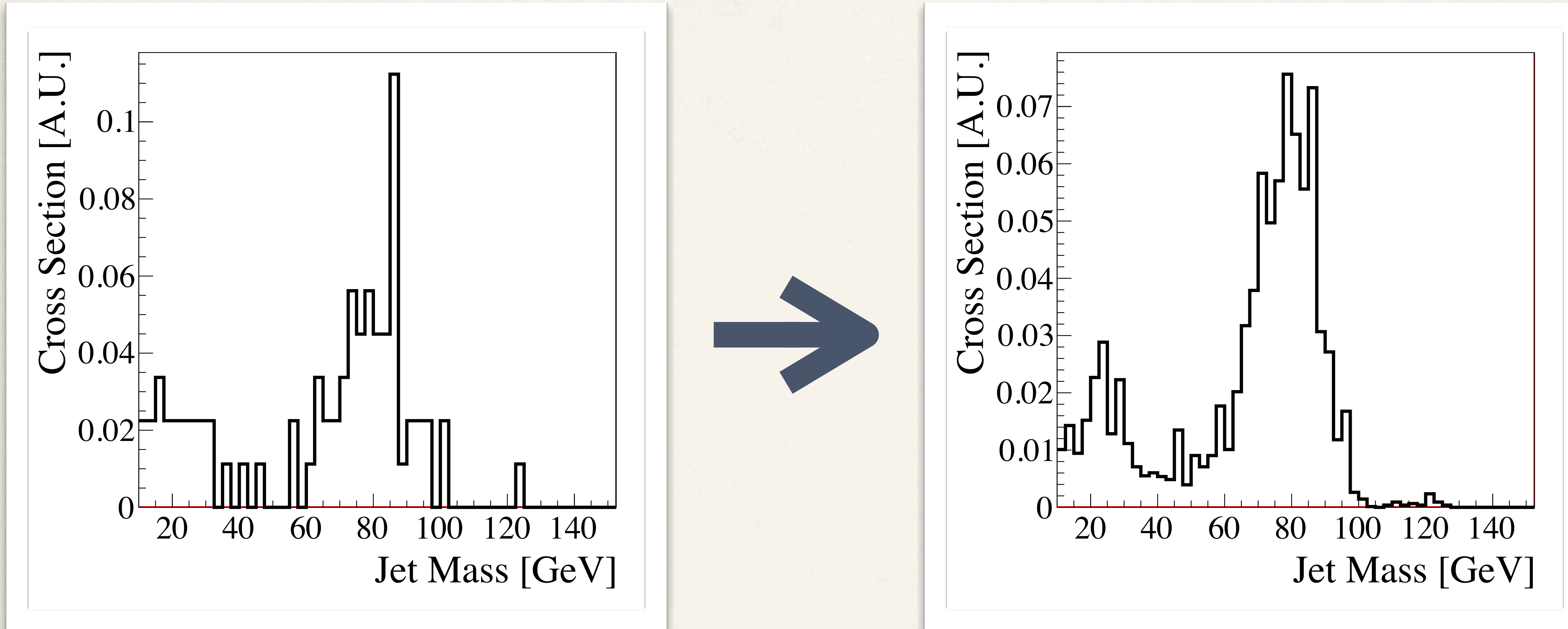


Qjets

David Krohn (Harvard)

Rutgers 4/17/12

Based on work with S. Ellis, A. Hornig, T. Roy, and M. Schwartz: arXiv:1201.1914
and work in progress with above plus D. Kahawala, A. Thalapillil, and L.-T. Wang

Outline

- Review of Jets & Jet Substructure
- Introduction to Qjets
- Example: Jet Pruning
- Future Directions
- Conclusions

Takeaway

- Many jet substructure analyses employ trees
- But, more than one tree can plausibly be associated with a jet
    - Typically, we use k_T or C/A to chose the “best” tree
- However, if we force ourselves to only consider a single tree for each jet, we make ourself more susceptible arbitrary choices of the jet algorithm
- By looking at many trees for each jet, we can decrease random fluctuations and create a more powerful analysis

Review of Jets & Jet Substructure

Types of Algorithms

- There are two main classes of jet algorithm
- Sequential recombinations
  - Combine four-momenta one by one
- Cone algorithms
  - Stamp out jets as with a cookie cutter

Focus on these

Sequential Recombination

- Define a distance measure between every pair of four-momenta in an event (jet-jet distances)

$$d_{ij}$$

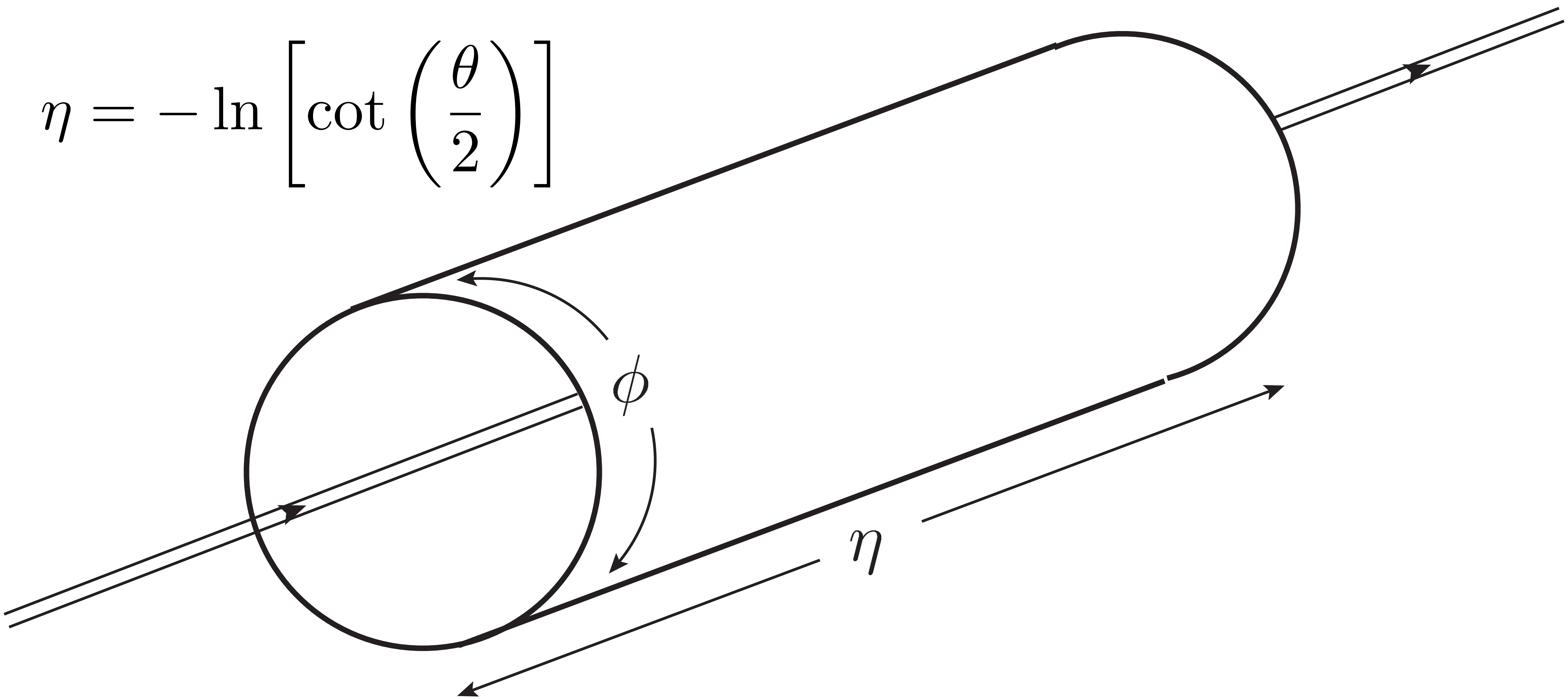
- Define a distance measure for each four-momenta individually (jet-beam distances)

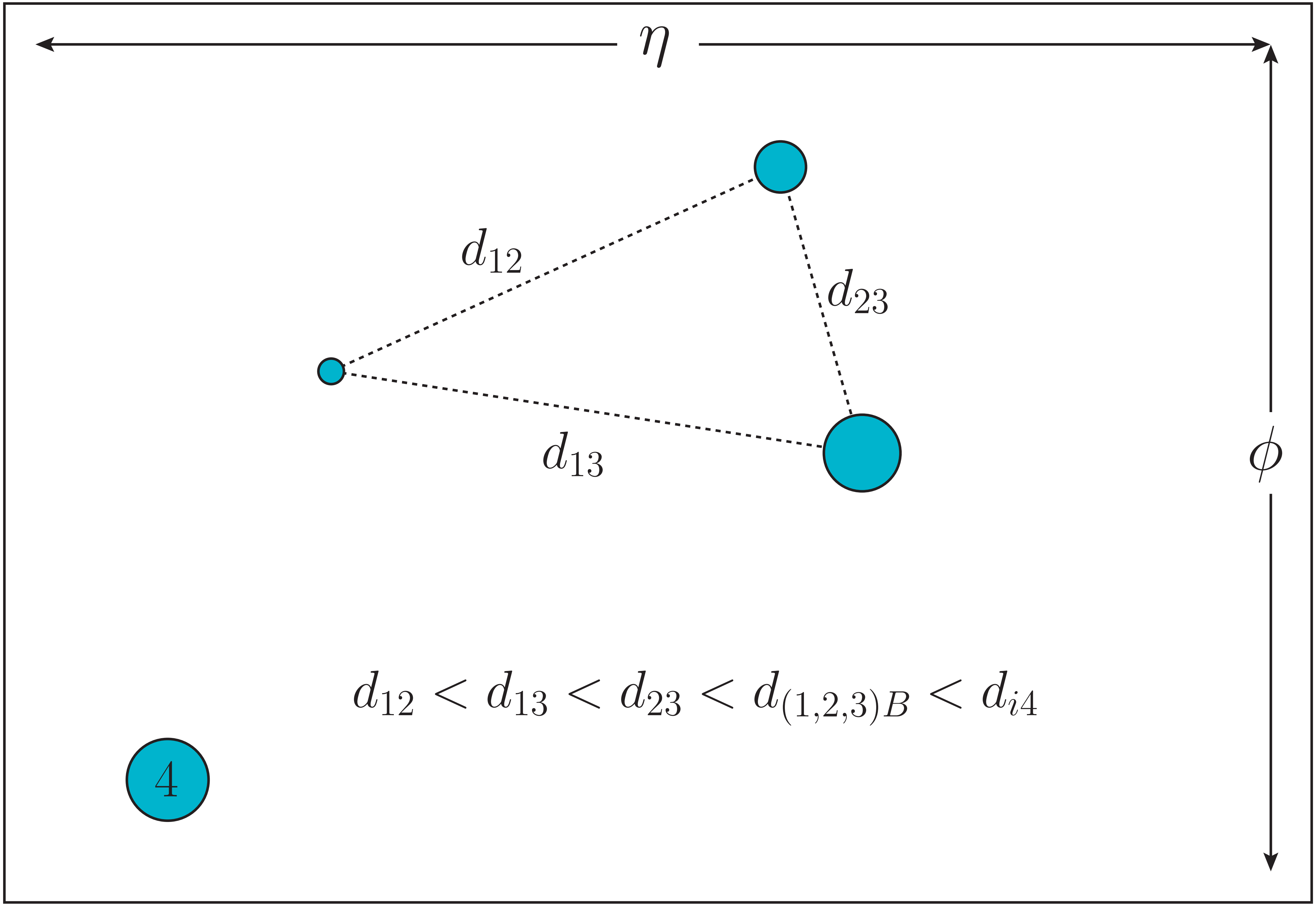
$$d_{iB}$$

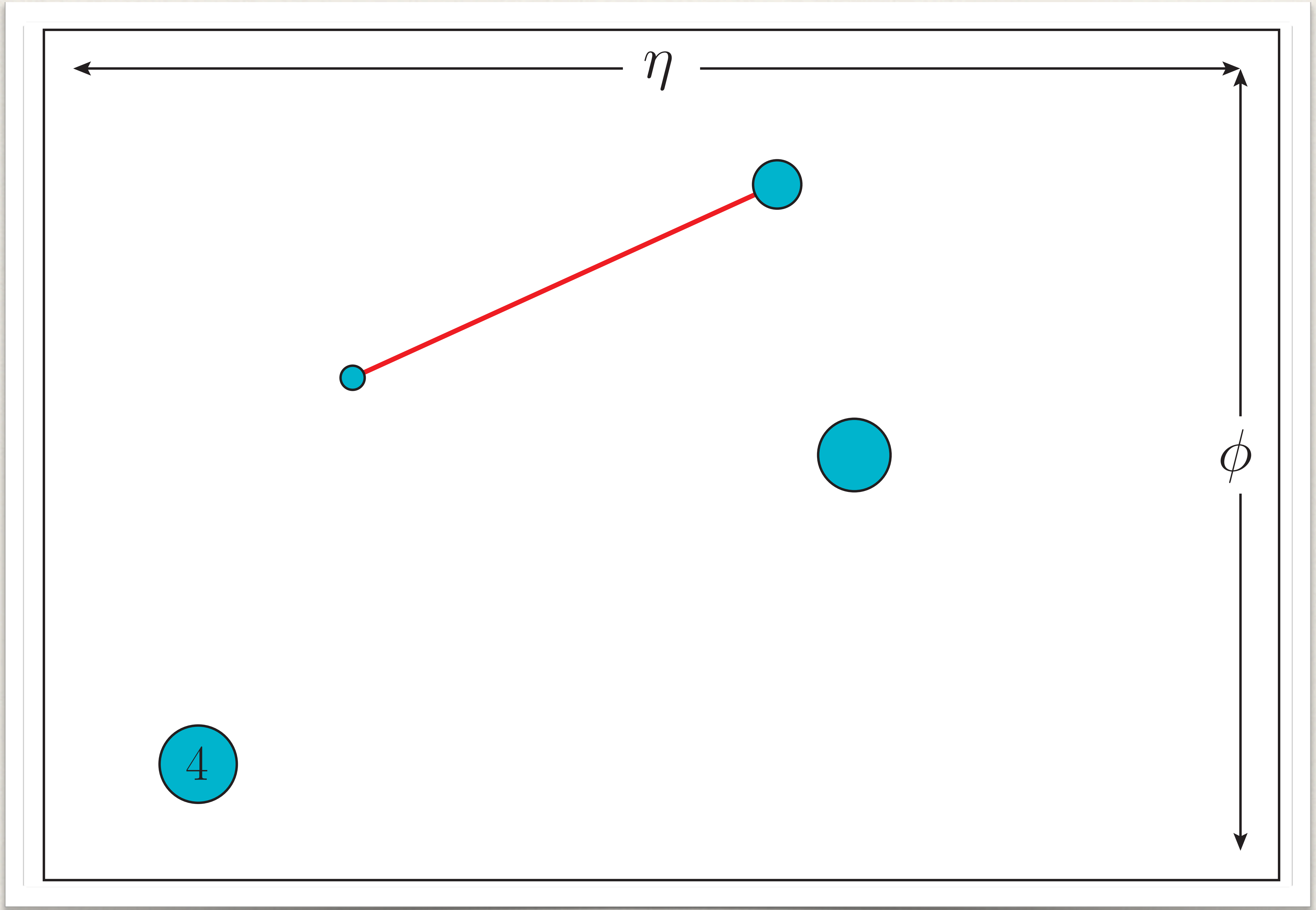
- If smallest distance at any stage in clustering is jet-jet, add together corresponding four-momenta
  - Otherwise take jet with smallest jet-beam distance and set it aside
- Repeat till all jets are set aside
- In this way, jets are constructed by pairwise recombinations - get a tree-like sequence at the end.

Coordinate System

$$\eta = -\ln\left[\cot\left(\frac{\theta}{2}\right)\right]$$




η
ϕ
d_{12}
d_{23}
d_{13}
$d_{12} < d_{13} < d_{23} < d_{(1,2,3)B} < d_{i4}$
4



η
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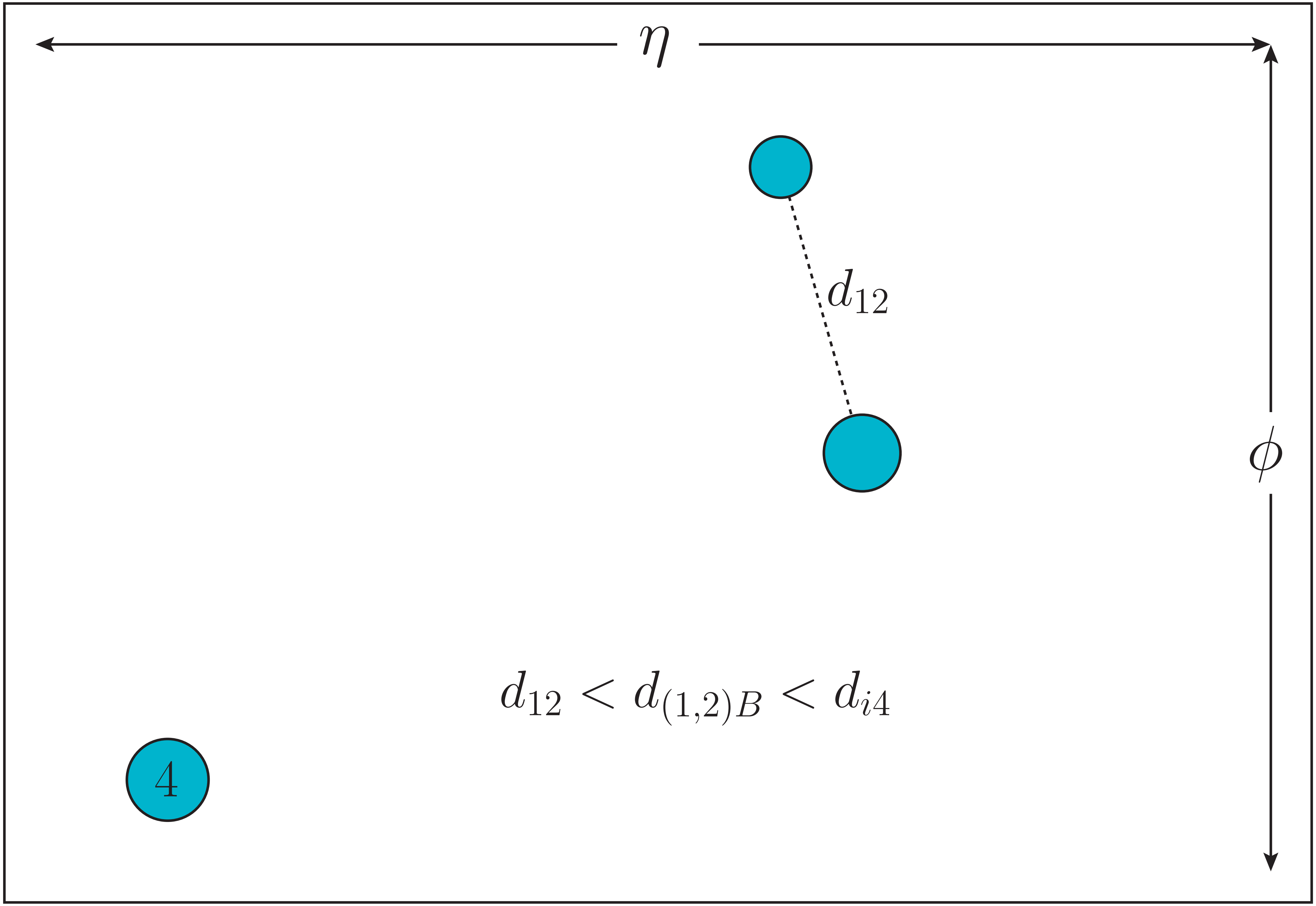
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$d_{12} < d_{(1,2)B} < d_{i4}$
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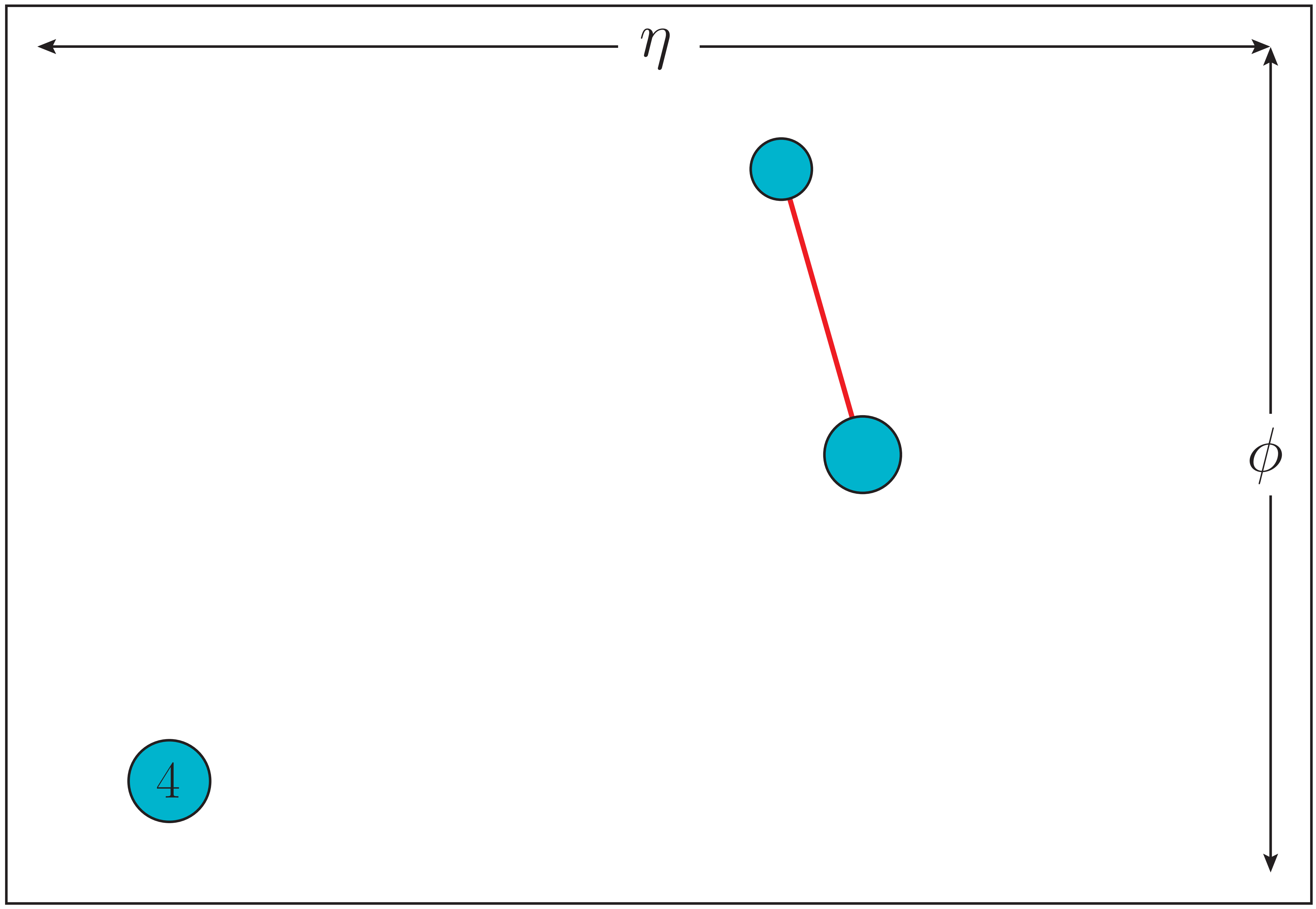

η
ϕ
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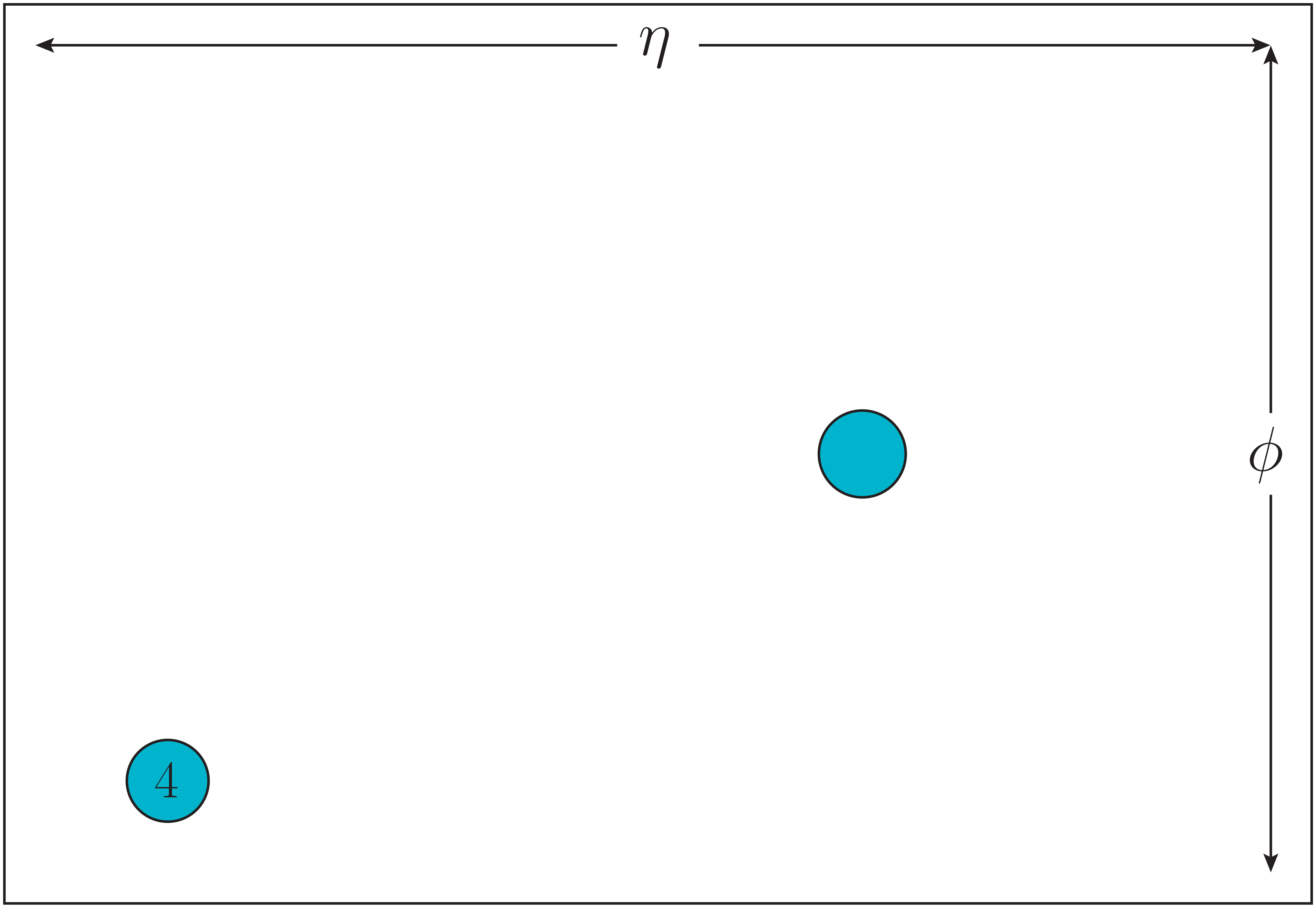


Done!

Standard Recombination Algorithms

- $\mathrm{k_T}$ algorithm

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2) \left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = p_{Ti}^2$$

- C/A algorithm

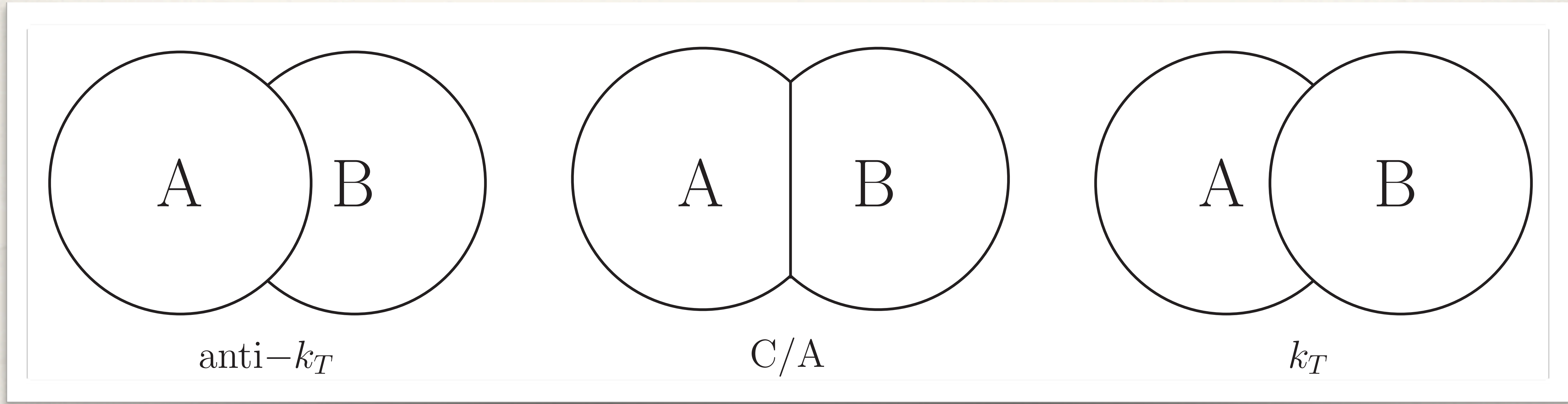
$$d_{ij} = \left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = 1$$

- anti-$\mathrm{k_T}$ algorithm

$$d_{ij} = \min(p_{Ti}^{-2}, p_{Tj}^{-2}) \left(\frac{\Delta R}{R_0}\right)^2, \; d_{iB} = p_{Ti}^{-2}$$

Approximate Jet Behavior:

$$p_{TA} > p_{TB}$$



Hard to Soft | Near to Far | Soft to Hard

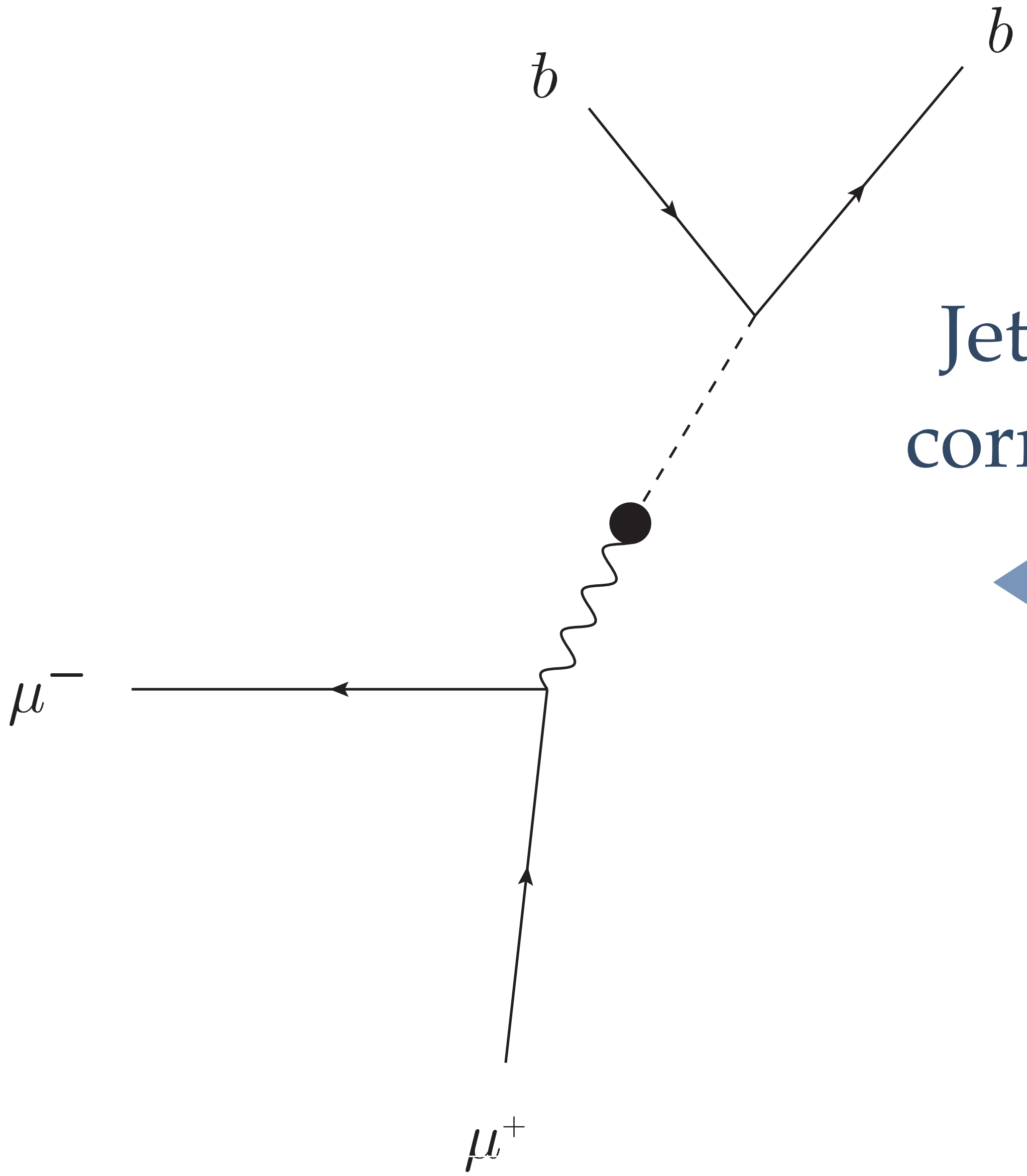
Tradeoffs

- k_T & C/A
  - Pro: Cluster near to far (both) & soft to hard (k_T). Allows us to use parton shower heuristics to understand behavior.
  - Con: Jets can have perverse shapes, weird areas
- anti-k_T
  - Pro: Jets are cone-like. Area relatively well defined.
  - Con: The ordering of the shower has little or no physical significance.

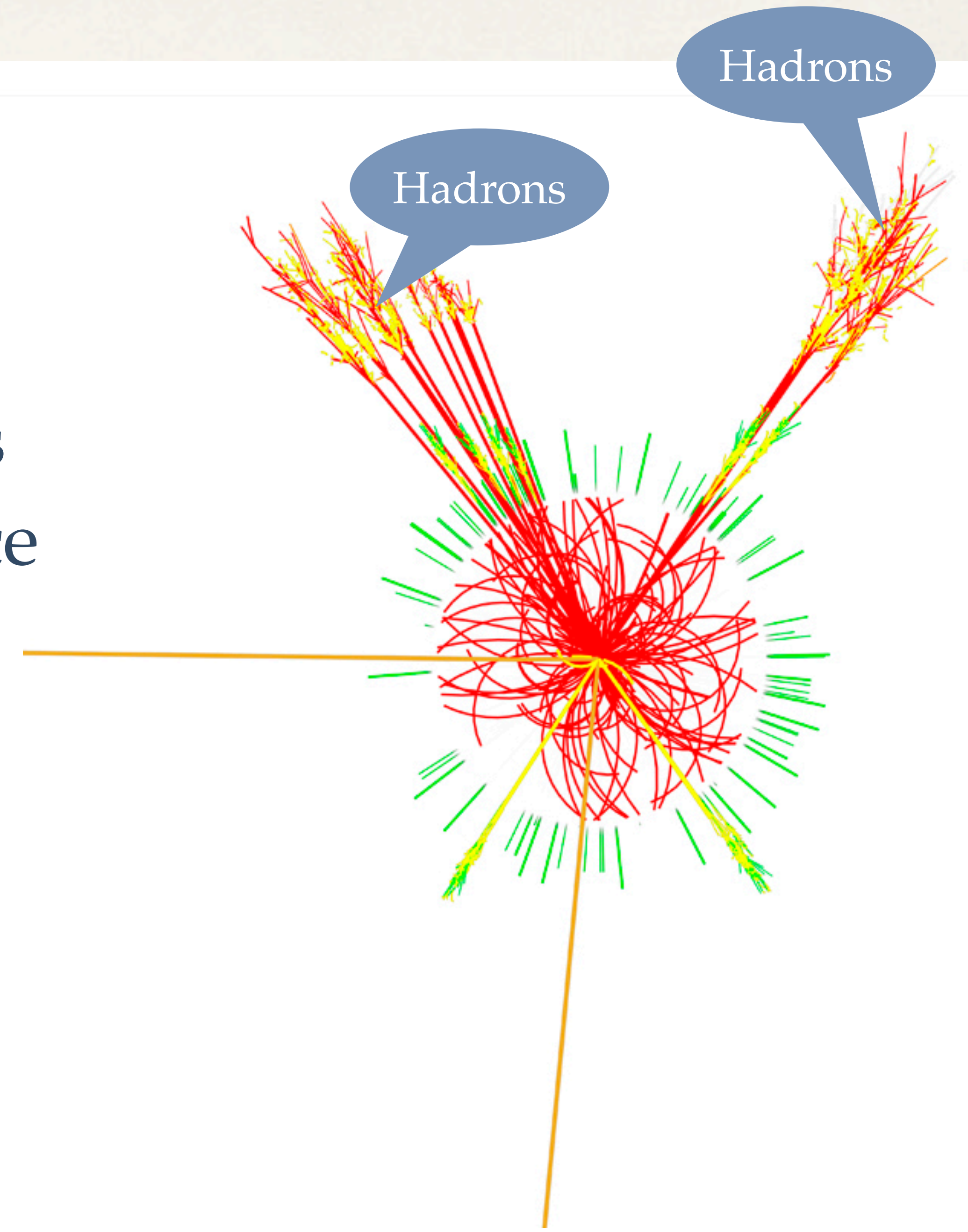
Jet-Parton Correspondence

- Jets allow us to make the connection between what we calculate (feynman diagrams) and what we measure in the detector.
- For instance, we'd expect to see two jets for each h->b bbar decay.

Jets make this correspondence



What we calculate



What we measure

Event picture from http://atlas.ch/photos/events.html

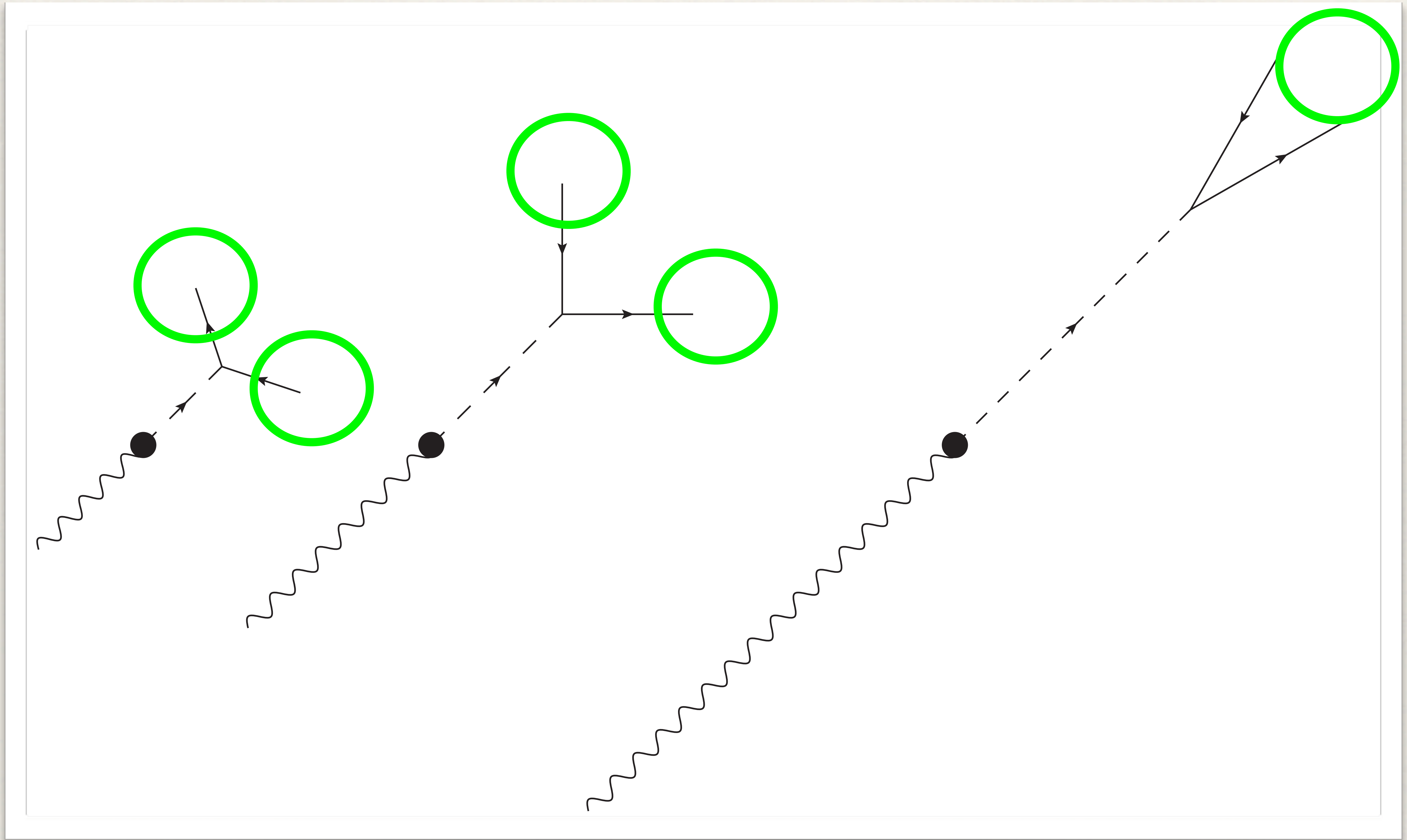
- However, this heuristic correspondence between jets and partons breaks down when things become collimated.

Kinematics of Boosted Particles

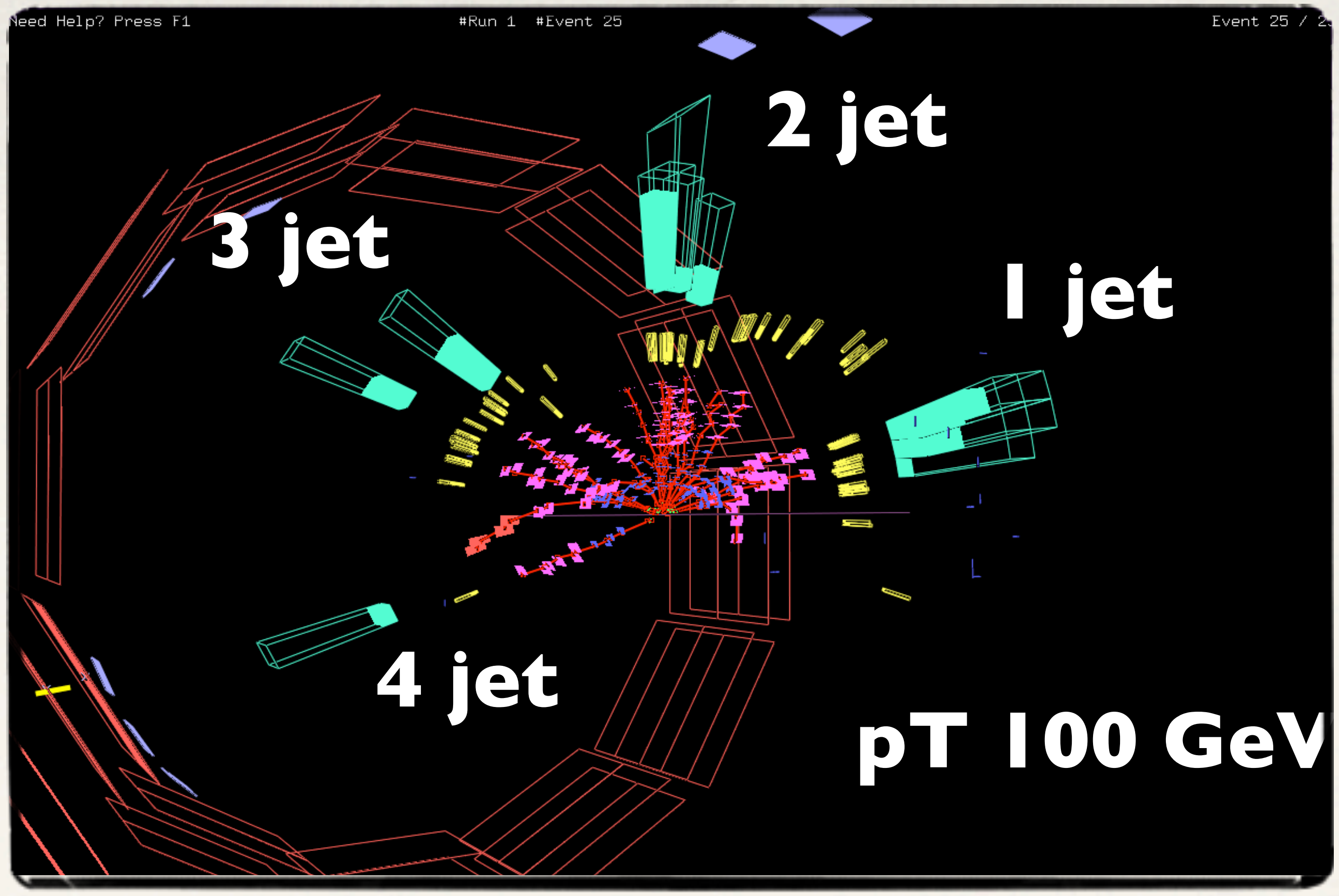
- The cone containing the decay products of a particle scales as

$$R \sim \frac{2m_X}{p_T}$$

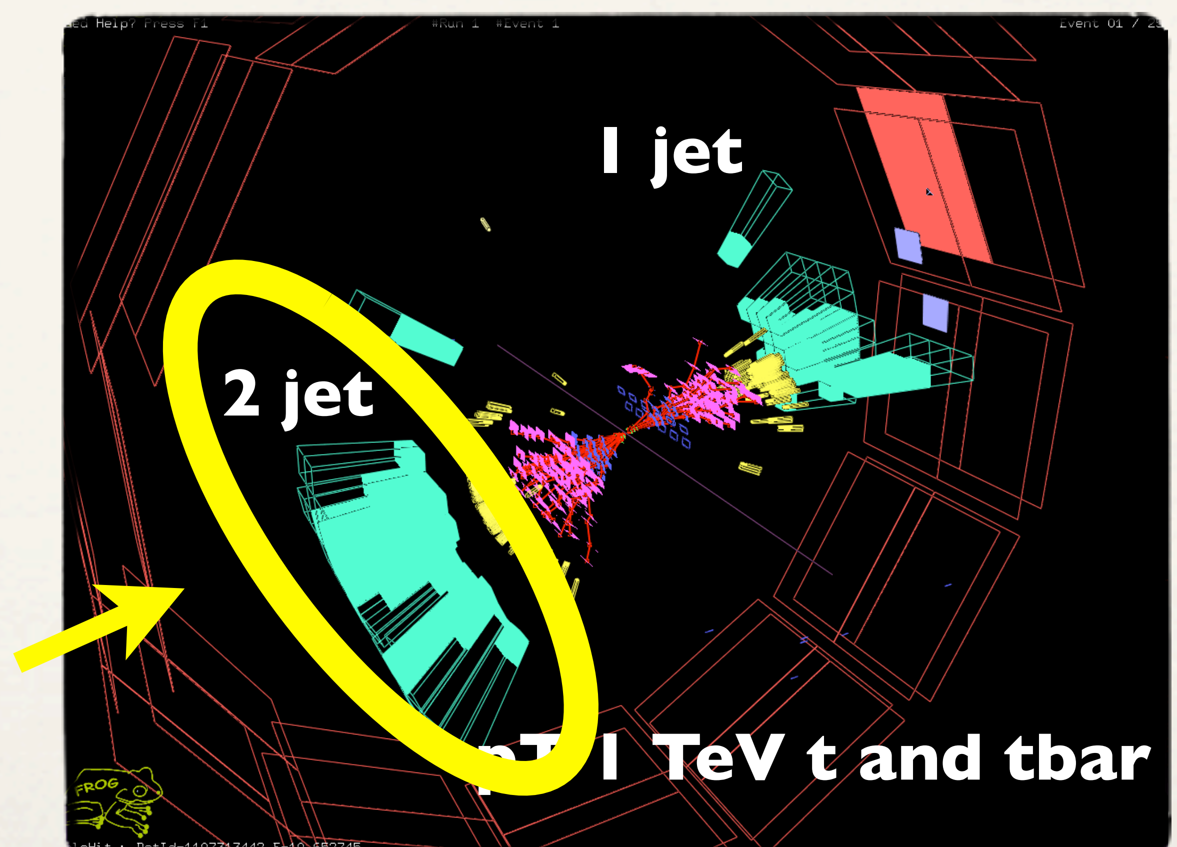
- At LHC energies, even the heaviest particles we know of (Top, W, Z, Higgs) become can become collimated.
- When this happens we say that they're "boosted".
- So we find that EW scale particles are clustered as a single jet as soon as their p_T exceeds a few hundred GeV.



Here one can see the effect - as we boost more and more (i.e. go to higher pT), the particles become more collimated.



Unboosted t-tbar pair



Boosted t-tbar pair

All three decay products of the top go into one jet

Figure source: http://www.pha.jhu.edu/groups/particle-theory/seminars/talks/F08/Yumiceva.pdf

Boosted Collider Physics

- This can be a problem!
- Most new physics models include heavy states at the TeV scale
    - If these decay down to W/Z/t, what do we do if everything's collimated?
    - Traditional answer: use the leptonic decays to avoid this mess.
- Modern answer: look inside the jet and make use of QCD to see if the jet came from a boosted heavy object.

Tools

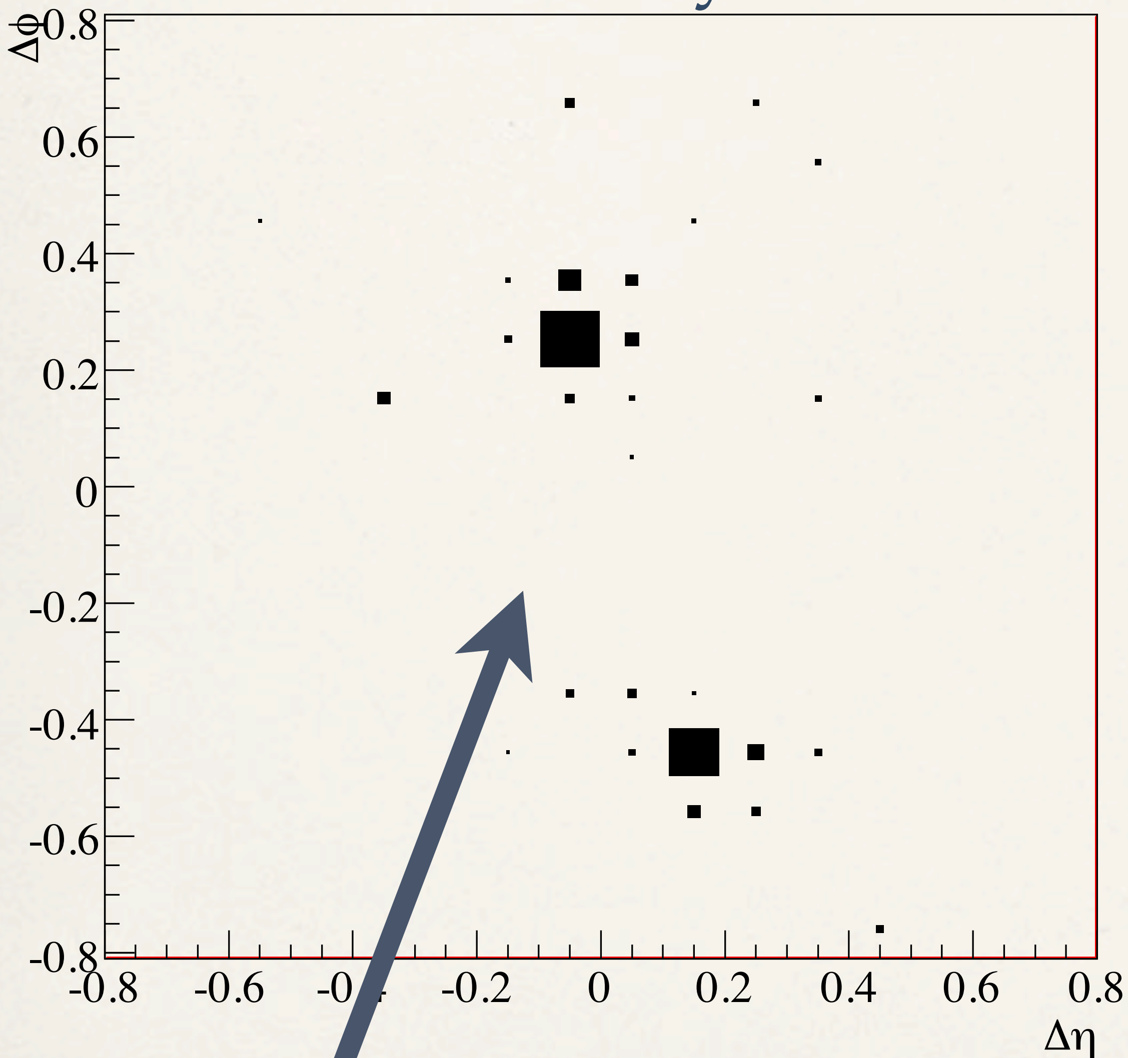
- QCD jets look really different than the jets of boosted heavy objects.
- QCD has soft/collinear singularities.
  - If we start with a high energy gluon/quark, it wants to emit soft/collinear gluons:

$$P_{q\to qg}(z) = C_F\,\frac{1+z^2}{1-z},$$
$$P_{g\to gg}(z) = C_A\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$
$$P_{g\to q\bar{q}}(z) = T_R\left[z^2 + (1-z)^2\right],$$

  - Here P(z) measures how much a particle wants to emit another with energy fraction "z" (Altarelli-Parisi splitting fcns.).

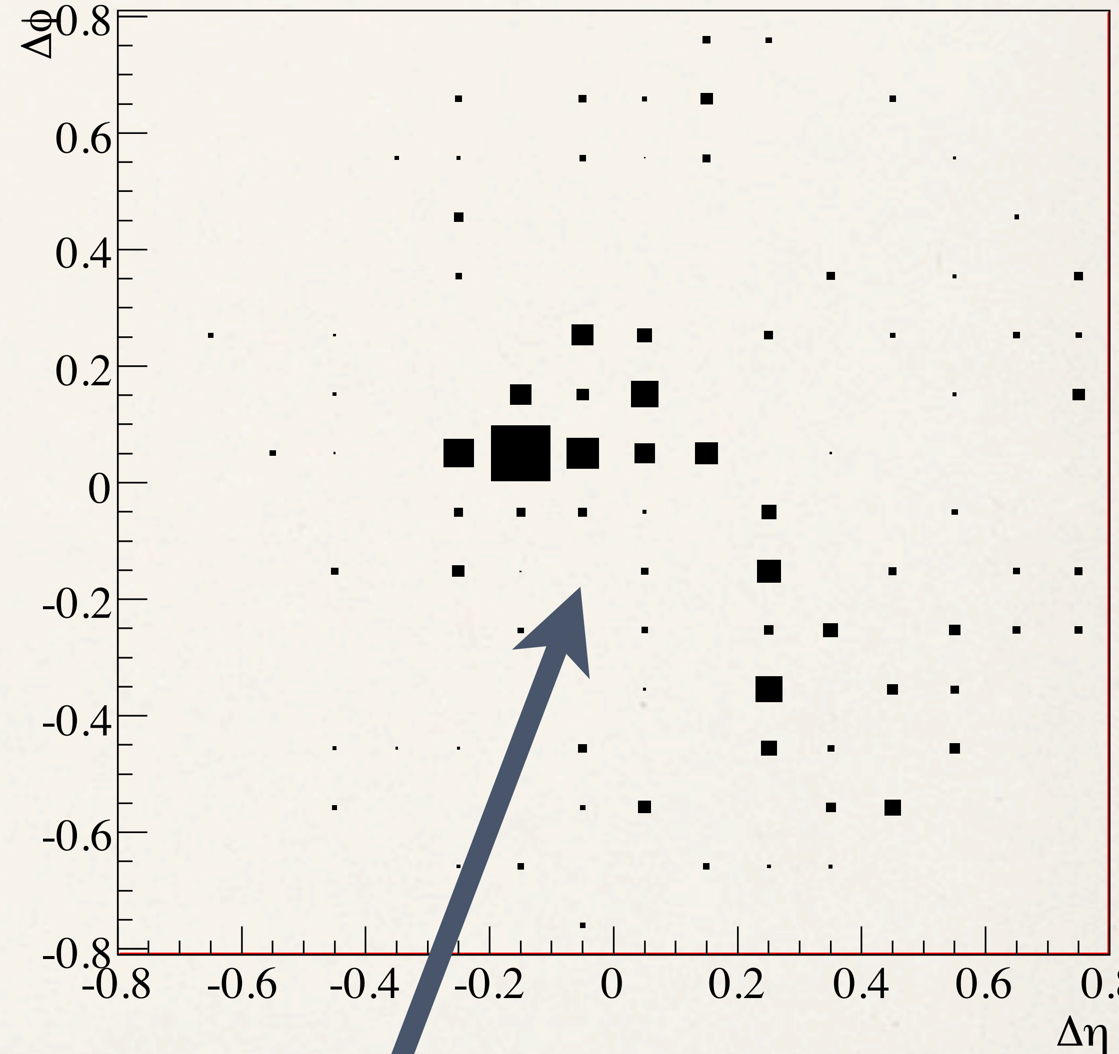
- However, a high energy heavy particle (W/Z/t/h) just decays - it has no singularity.

Boosted Heavy Particle



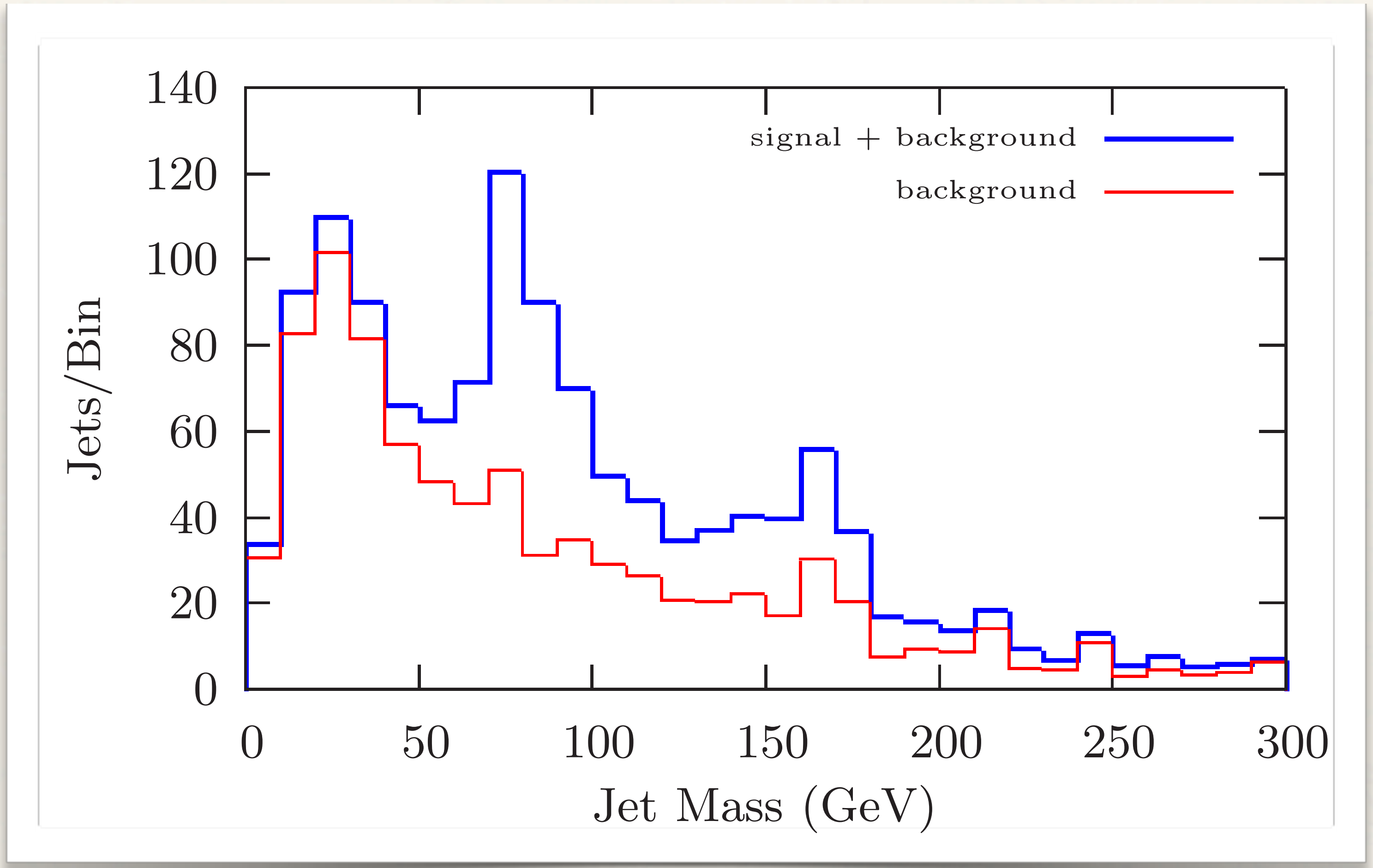
Hard splitting, energy shared equally

QCD Jet



Softer splittings. Unequal sharing of energy (note only one hard center)

- Moreover, QCD jets have a continuum mass distribution, while the jets of boosted heavy particles have a fixed mass.



- These will form our main tools.

1. Jet radiation distribution

2. Jet mass

Figure source: Using jet mass to discover vector quarks at the LHC, W. Skiba, D. Tucker-Smith, [hep-ph/0701247] Phys.Rev. D75 (2007) 115010

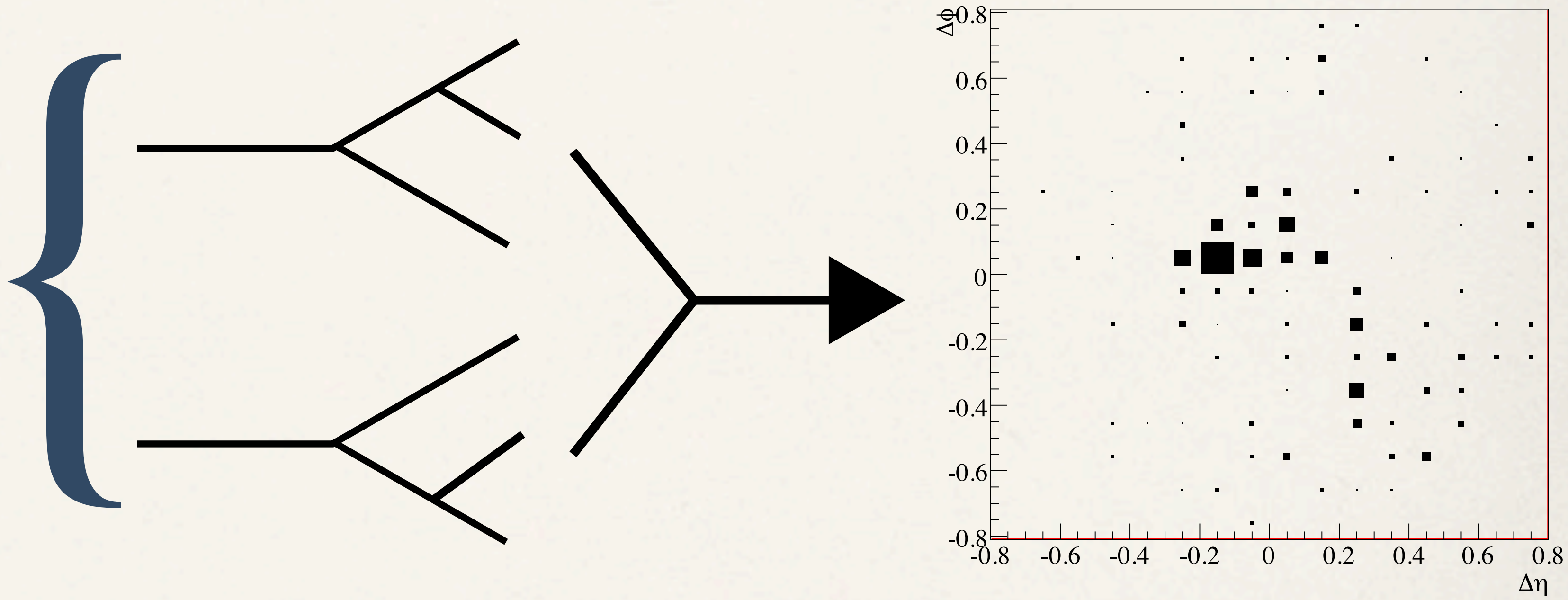
Qjets

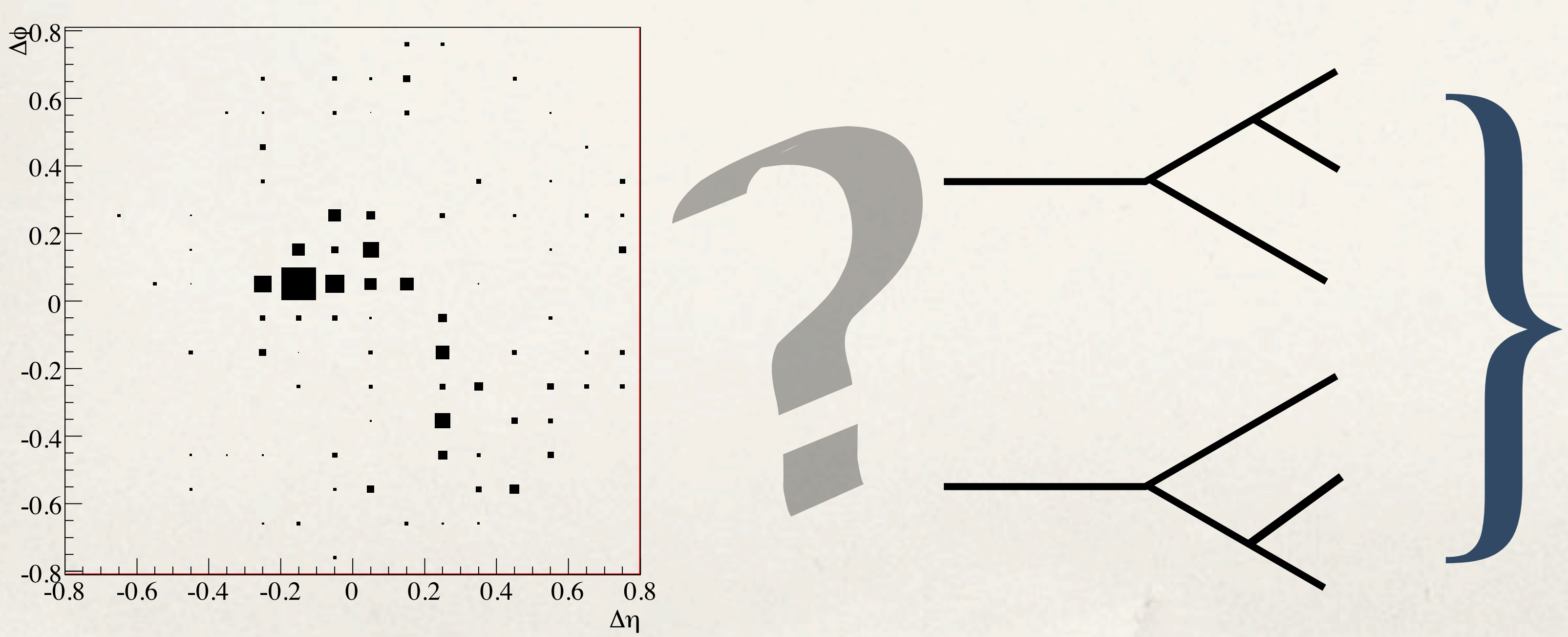
Two Basic Approaches to Substructure

1. Consider only the two-dimensional distribution of energy in a jet
   - Examples: Trimming & Filtering, N-Subjettiness, Jet substructure w/o trees
2. Try to associate a tree structure with a jet
   - Allows one to use heuristic pictures of parton shower & decay chains.
   - Examples: Pruning, energy sharing variables, mass drop
   - However, the current procedure for constructing a tree is not ideal.

Mapping Jets to Trees

The energy distribution for a particular tree is unambiguous

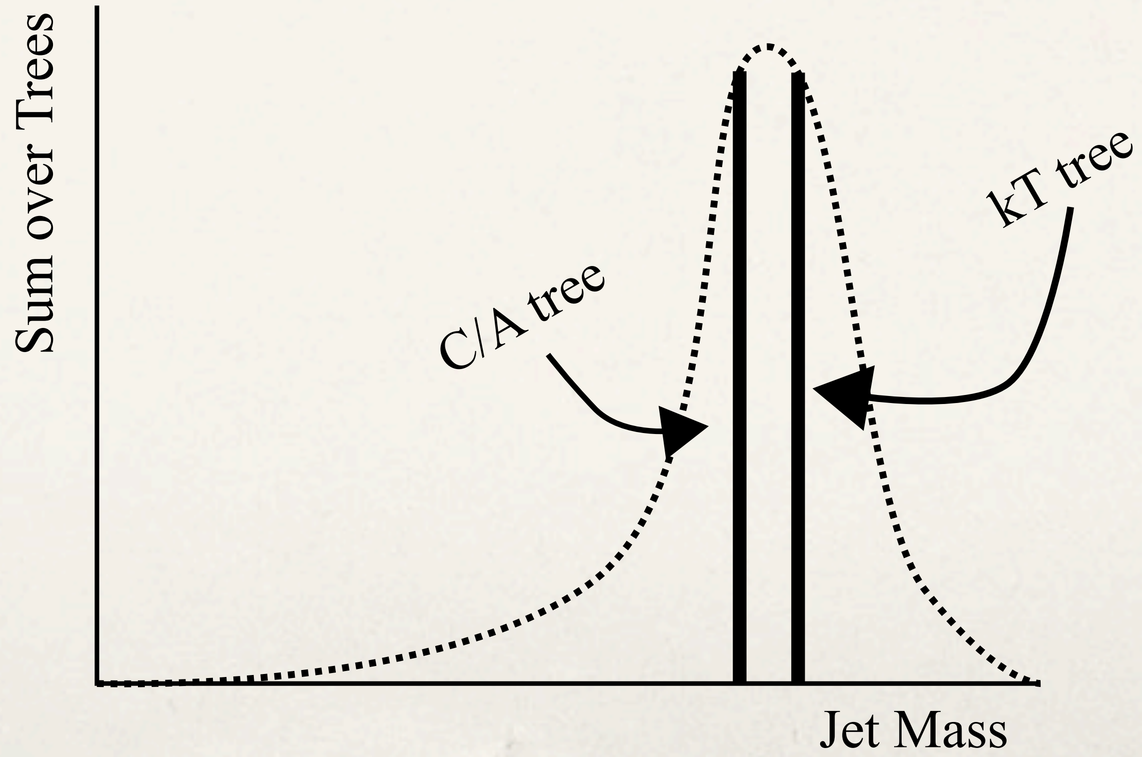




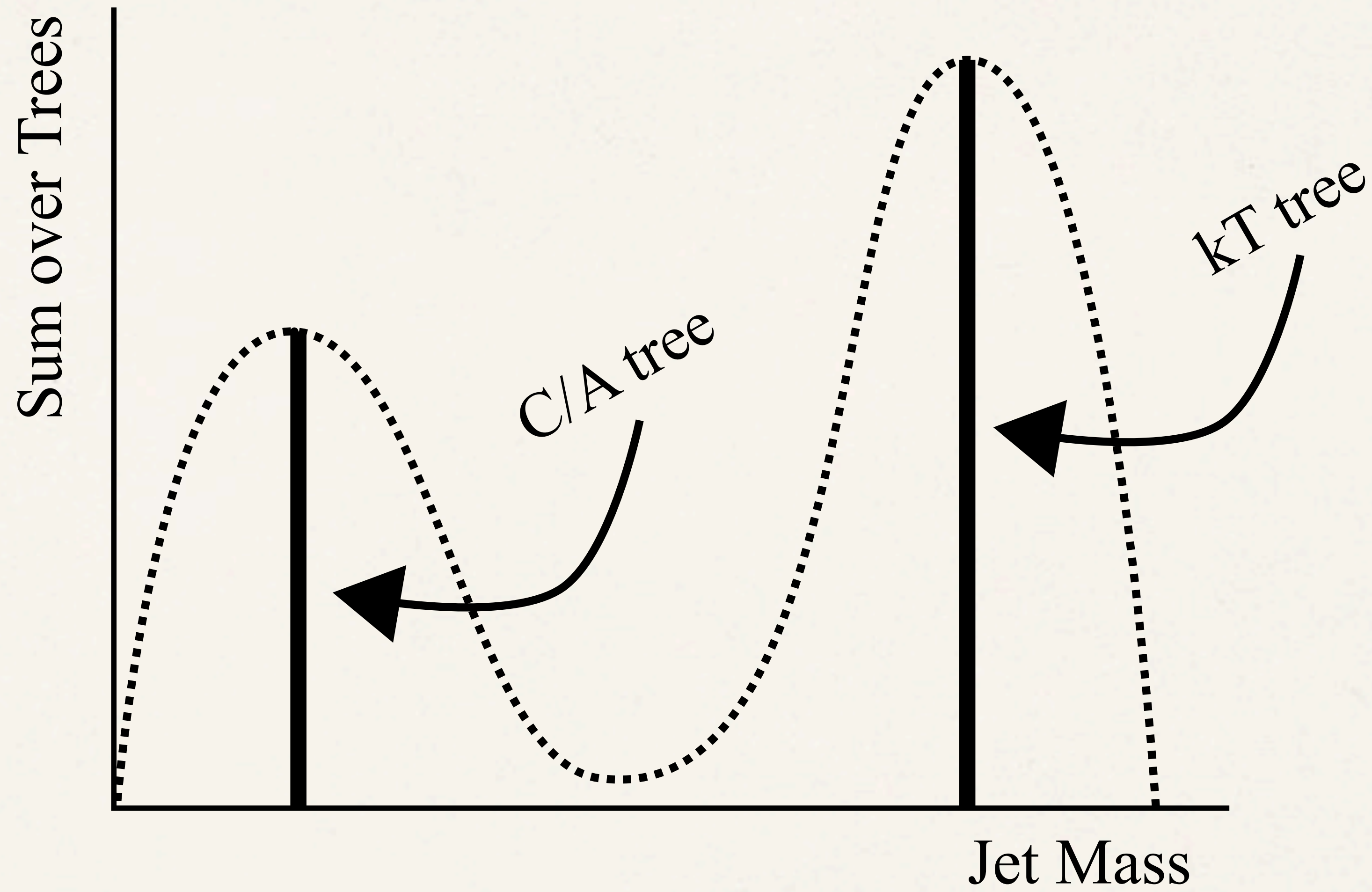
But, more than one tree can correspond to the same energy distribution

Unnecessary Choices

- How do we assign a particular tree to an energy distribution?
- Standard answer: Use a well motivated algorithm like C/A or kT
- Ideally, since both are well motivated algorithms they'll give the same answer:



- However, sometimes the answers are very different.

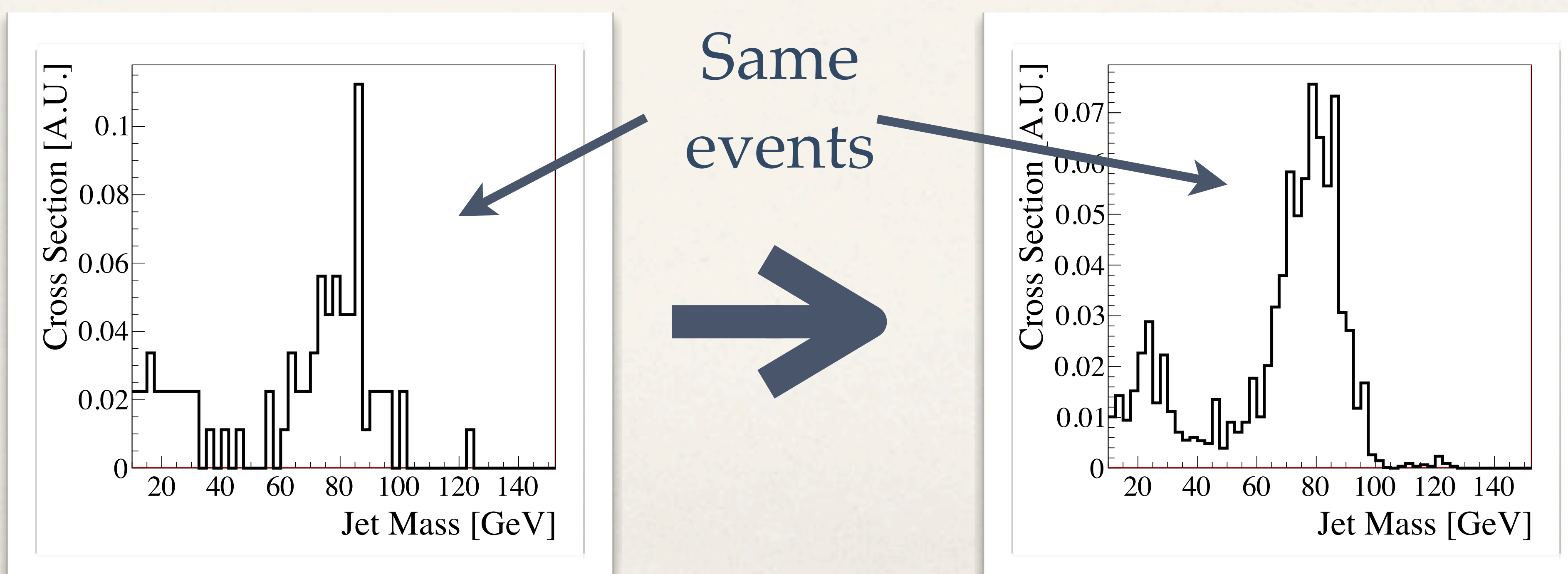


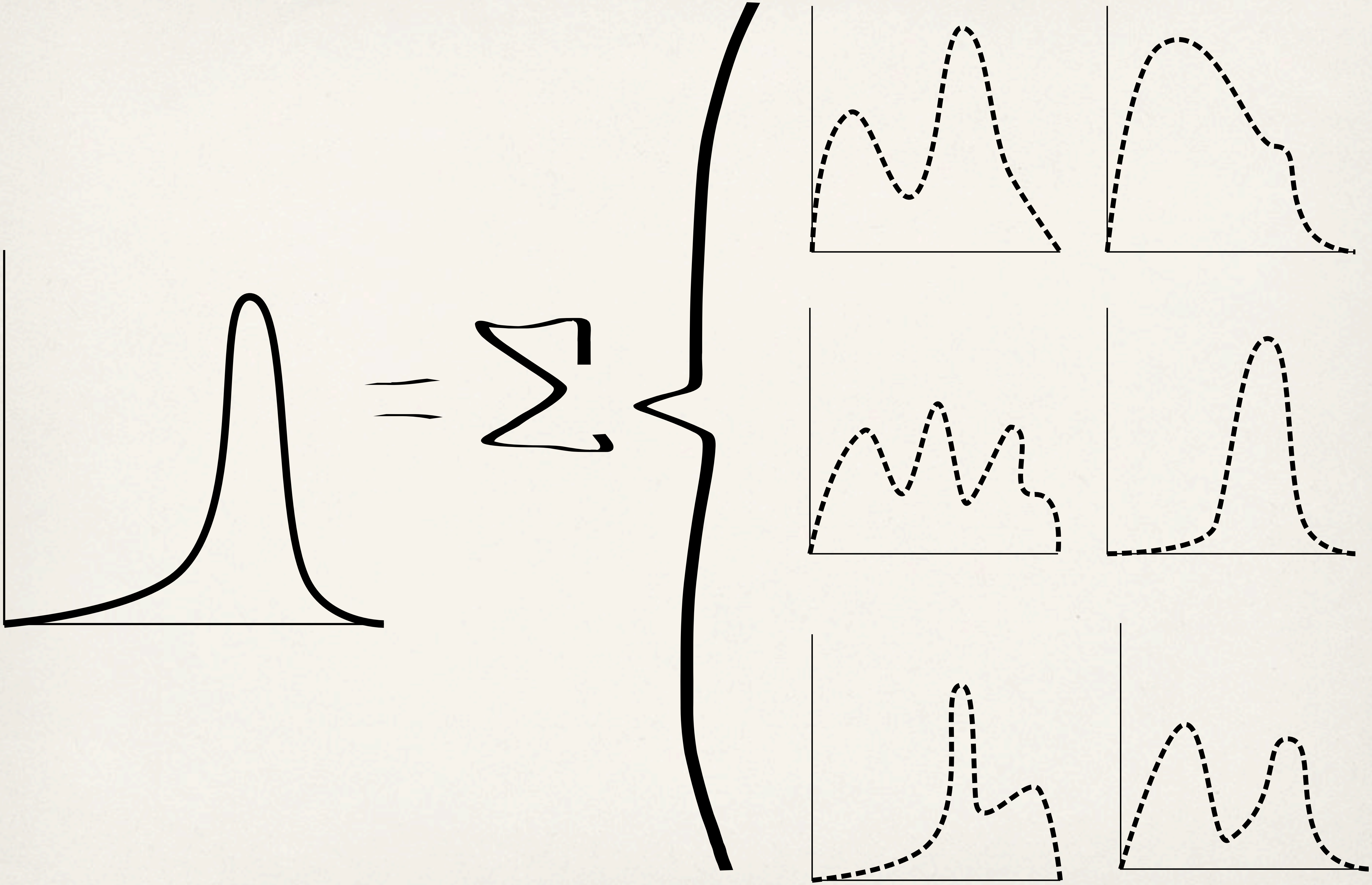
- Considering only the kT or C/A tree introduces an element of randomness into this process, resulting in unnecessary fluctuations in the final state observable.

- Intuitively it makes sense that defining an observable in a way which reflects the ambiguity of this clustering should yield better results.

Solution: Sum over Trees

- We propose that rather than assigning a single number to each event, instead each event should contribute a distribution obtained by summing the observable over many trees.
- When we sum these together, the result is much more stable than the histogram we would have had if we just considered one number per event.





Weights

- The only question is: when we add together the result obtained from different trees, how should we weight each tree's contribution?
- Surely they should not all count equally. If they did, then why would we use kT or C/A to find our trees in the first place?
- In theory, one could weight each tree by the product of splitting functions and Sudakovs one would obtain from a parton shower.
    - Work in progress.

Implementation

- Instead, we find a simpler Monte-Carlo procedure works quite well.
  - As in a sequential recombination algorithm, assign every pair of proto-jets a distance measure d_{ij}.
  - However, unlike a normal sequential algorithm (where the pair with the smallest measure is selected clustered), here we suggest that a given pair be randomly selected for merging with probability

$$\Omega_{ij} \equiv \frac{1}{\Omega} \exp\left(-\alpha \frac{d_{ij}}{d_{ij}^{\min}}\right), \quad \alpha = \text{rigidity parameter}$$

  - Thus, paths which deviate from the CA or kT behavior are less likely to occur
  - Repeat many (~100) times, till the distribution stabilizes

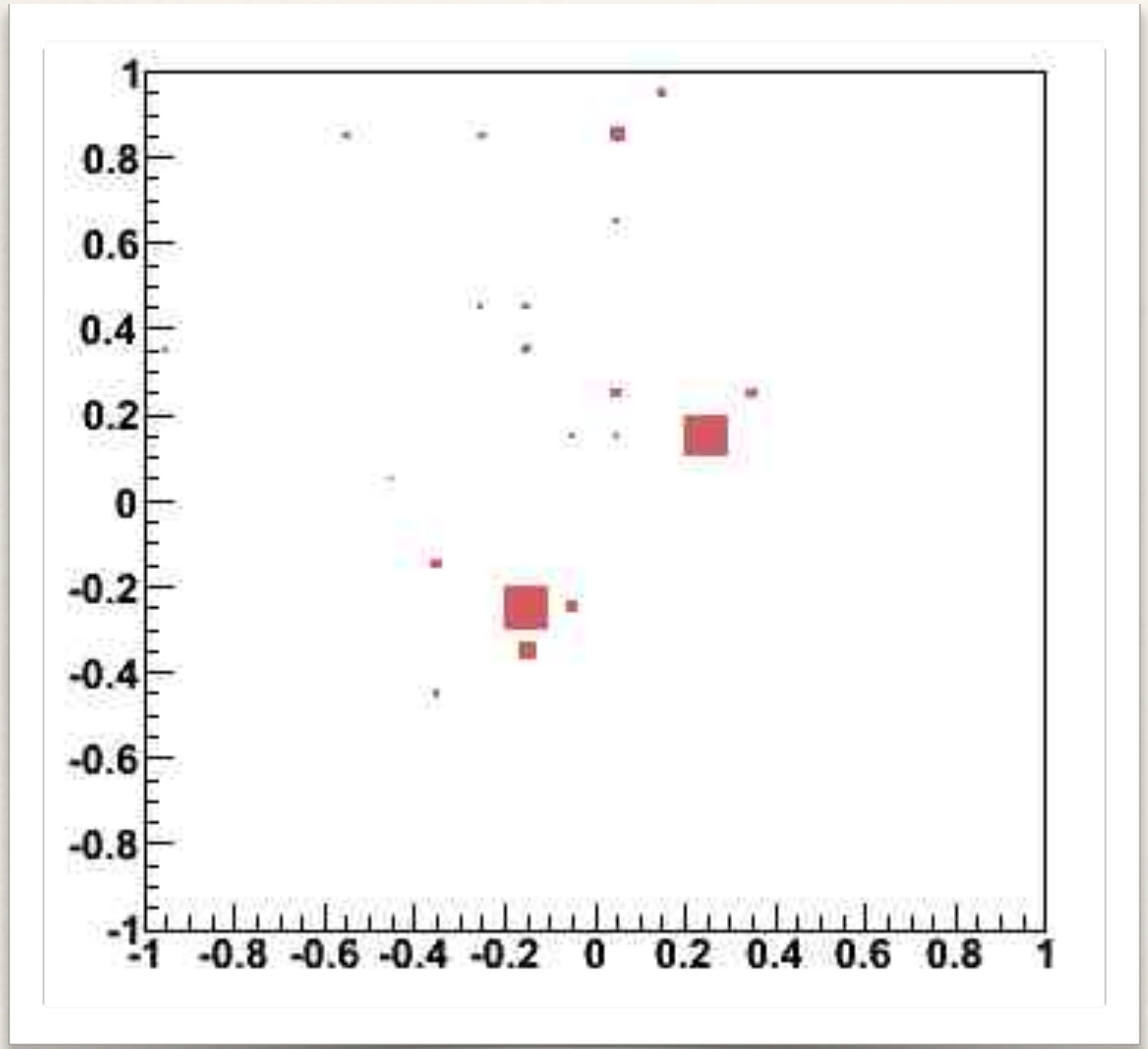
- The result is that you get many trees
- The probability of finding a given tree decreases as it becomes less k_T or C/A like
- Available as a Fastjet plugin:

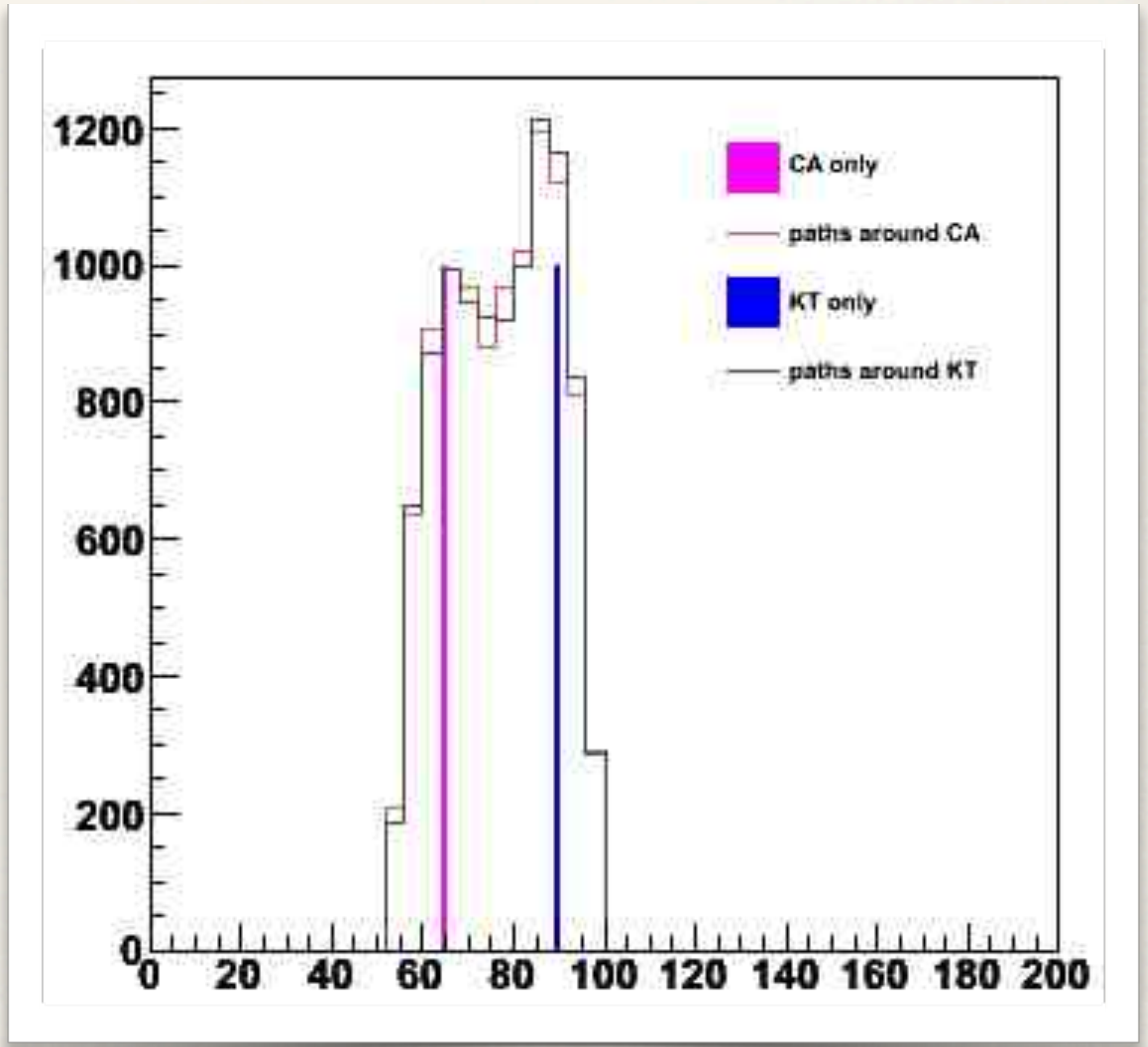
  http://jets.physics.harvard.edu/Qjets

IR/Collinear Safety

- As long as the rigidity variable (alpha) is non-zero, then infinitely soft or collinear particles will not change the observable at hand.
- How will this affect real analytical calculations?
  - Still unknown
  - Perhaps there is a better, more theory-friendly weight?

Example: Boosted W-Jets with Pruning





Pruning

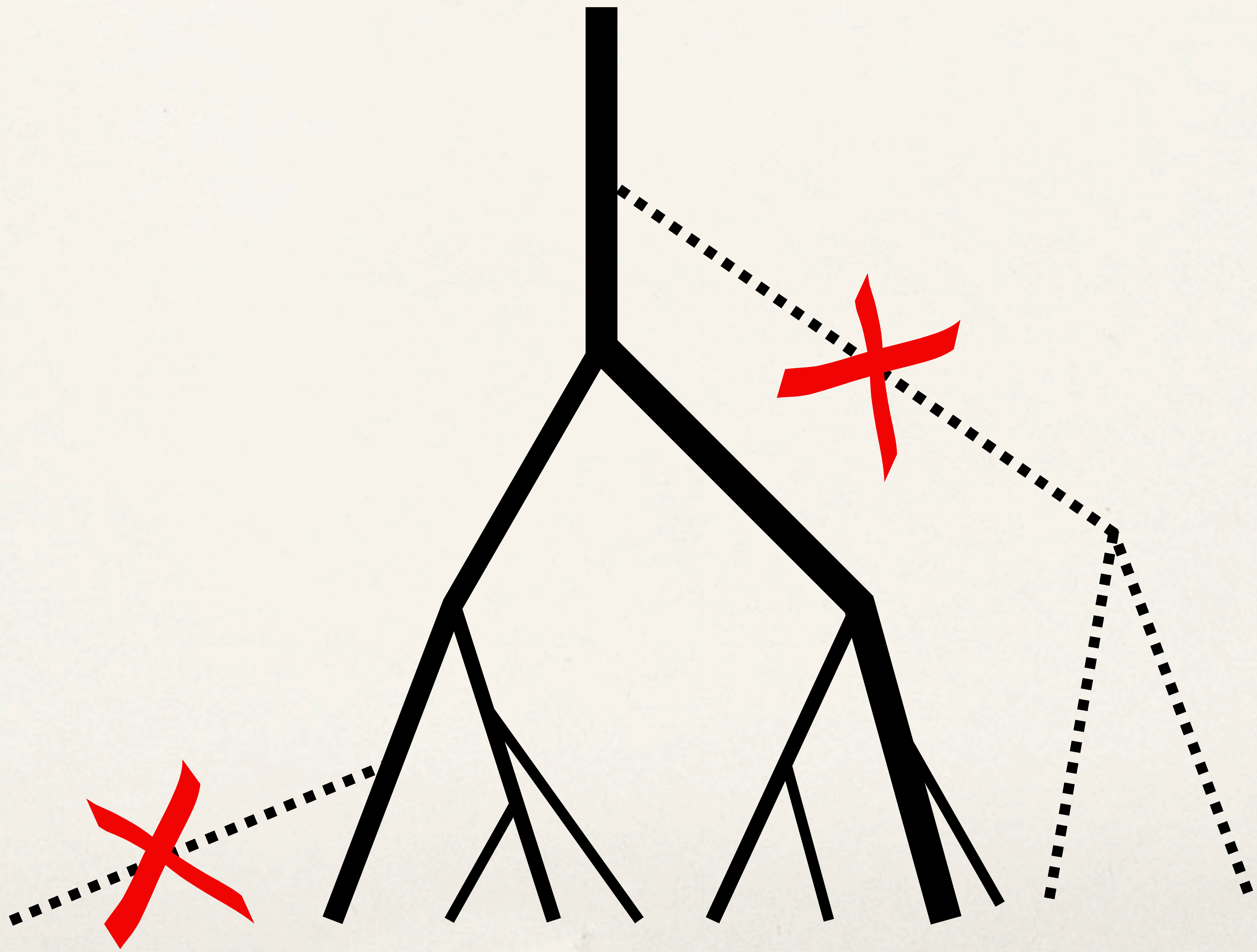
- Pruning was introduced to look for boosted heavy objects (e.g., tops, higgses, W's, etc) by cleaning up their mass.
- Intuition: QCD has soft/collinear singularities. Wide-angle emissions should come from hard decays.
  - Remove all parts of the jet which are *both* soft and wide angle.
- Two main advantages:
  - Boosted objects see their mass reconstruction improved
  - Massive QCD jets (a large background) see their mass substantially decreased -> lower backgrounds

Pruning (Ellis, Vermilion, Walsh - 0903.5081, 0912.0033)

Pruning in Practice

- To run pruning:
  - Take the constituents of an ordinary jet (formed using any algorithm).
  - Recluster them using a modified version of C/A and k_T
  - When C/A or k_T says that a pair of subjets should be recombined, ask: are the two subjets separated by more than a fixed amount (d_{cut}) and is one much softer than the other ($p_{T1}/p_{T2} < z_{cut}$)?
  - If so, set aside the softer particle and don't merge it with the main jet.

A Pruned Tree



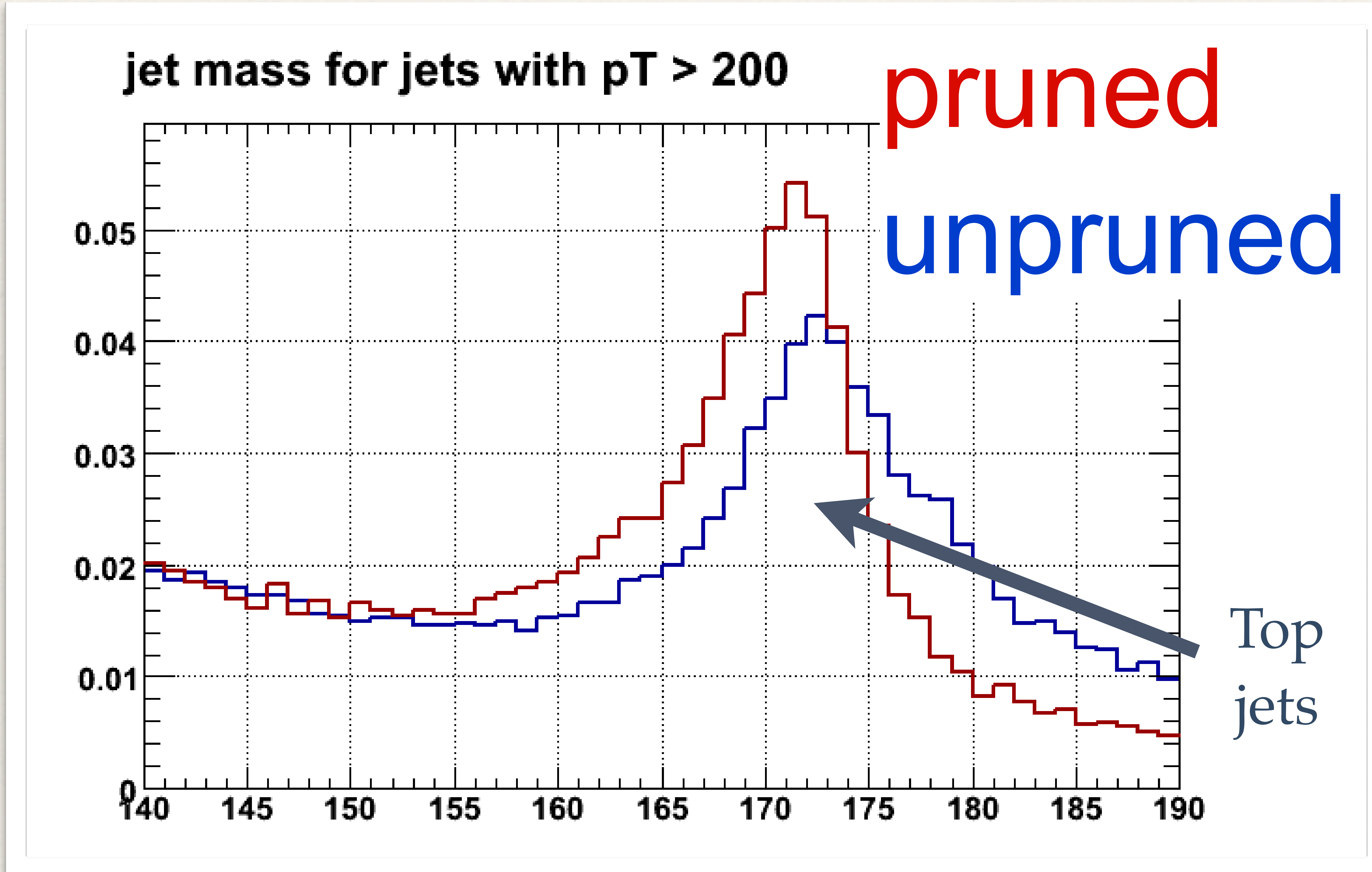


Figure source: http://www.phys.washington.edu/users/ellis/USATLAS.pdf

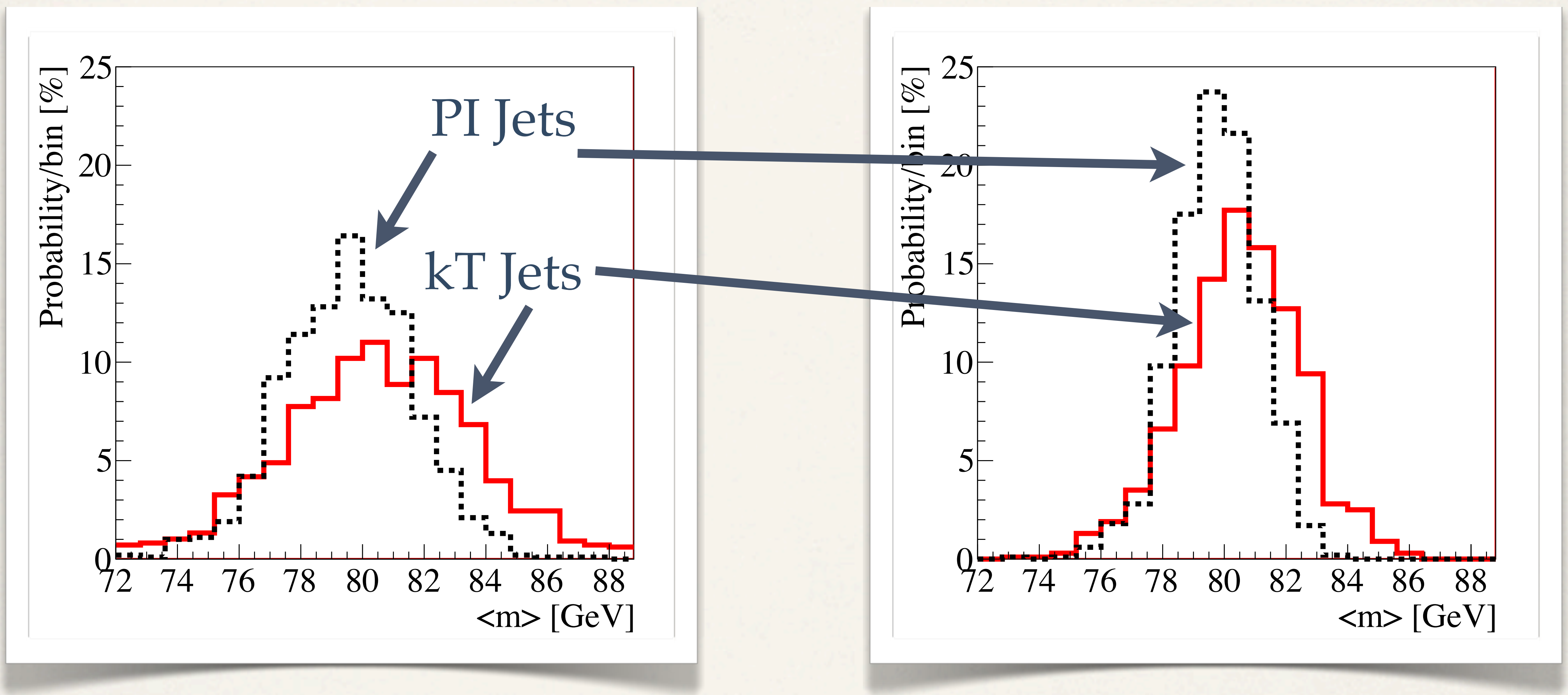
- Let’s see what happens when we modify pruning so that it runs over trees generated via the Qjet procedure.

Example 1/3: Mass Measurement

- As an example, let's take a sample of boosted W jets (pt>500), clean them up via jet pruning, and ask for the average jet mass.
- The uncertainty in this measurement goes down by ~1/3 when the technique described is applied.
- Need roughly half the luminosity to make a measurement of the same precision

$$\delta\langle m\rangle \propto 1/\sqrt{N}$$

α	$\frac{\delta\langle m\rangle\vert_{\mathrm{cl}}}{\delta\langle m\rangle\vert_{\mathrm{Q}}}$
0	1.32
0.01	1.31
0.1	1.25
1.0	1.10
100	1.03


PI Jets
kT Jets
Probability/bin [%]
25
20
15
10
5
0
72 74 76 78 80 82 84 86 88
<m> [GeV]
Probability/bin [%]
25
20
15
10
5
0
72 74 76 78 80 82 84 86 88
<m> [GeV]

<N>=10

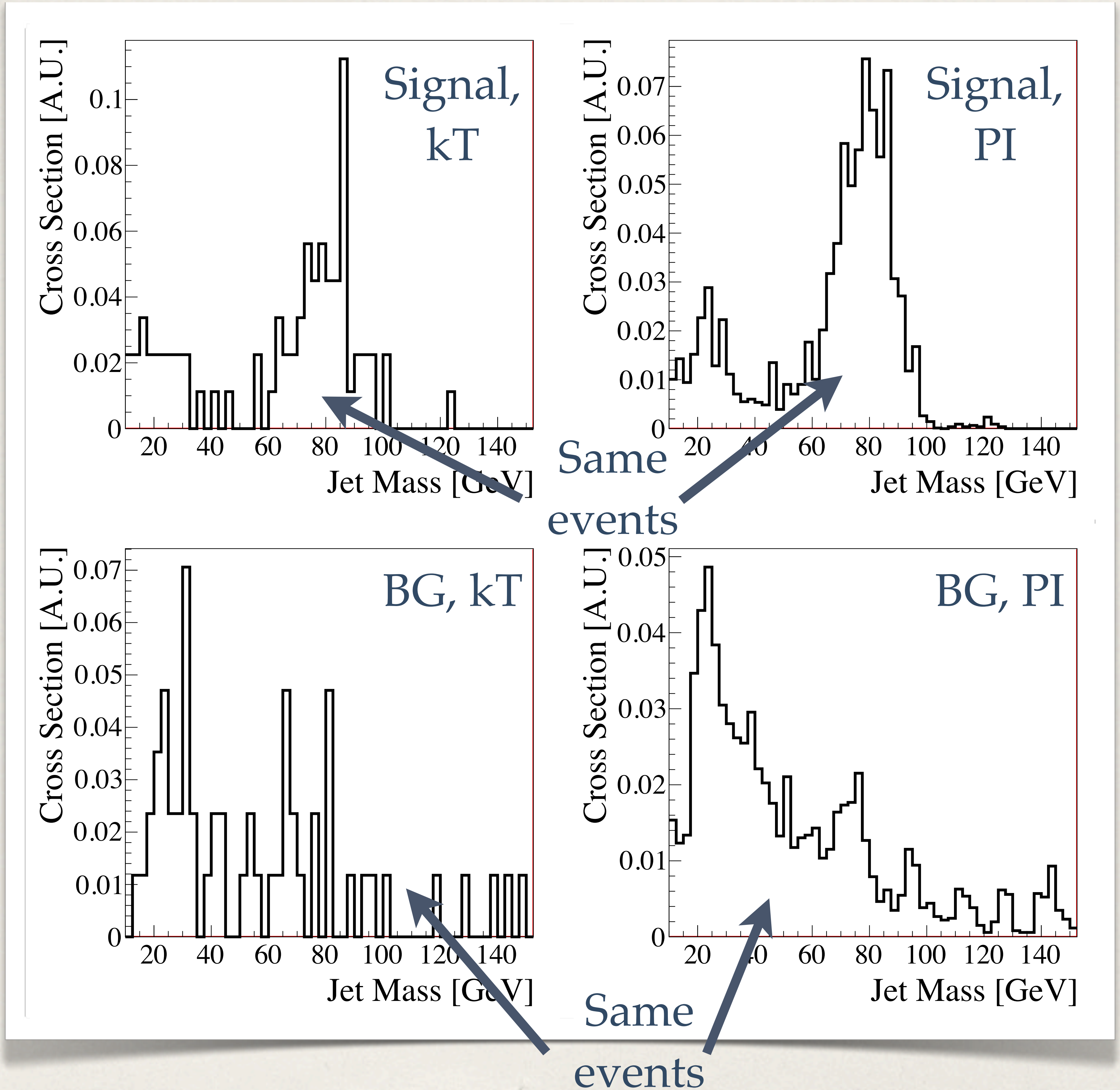
<N>=20

Example 2/3: Signal Discovery &Exclusion

- Signal = boosted W-jets, pT > 500
- BG = light QCD jets, pT > 500
- Measure the signal size in a bin (here 70-90 GeV) and compare it to the size of the BG fluctuations (Poisson stats included)
- Need only ~70% the luminosity to have the same significance

$$S/\delta B \propto \sqrt{N}$$

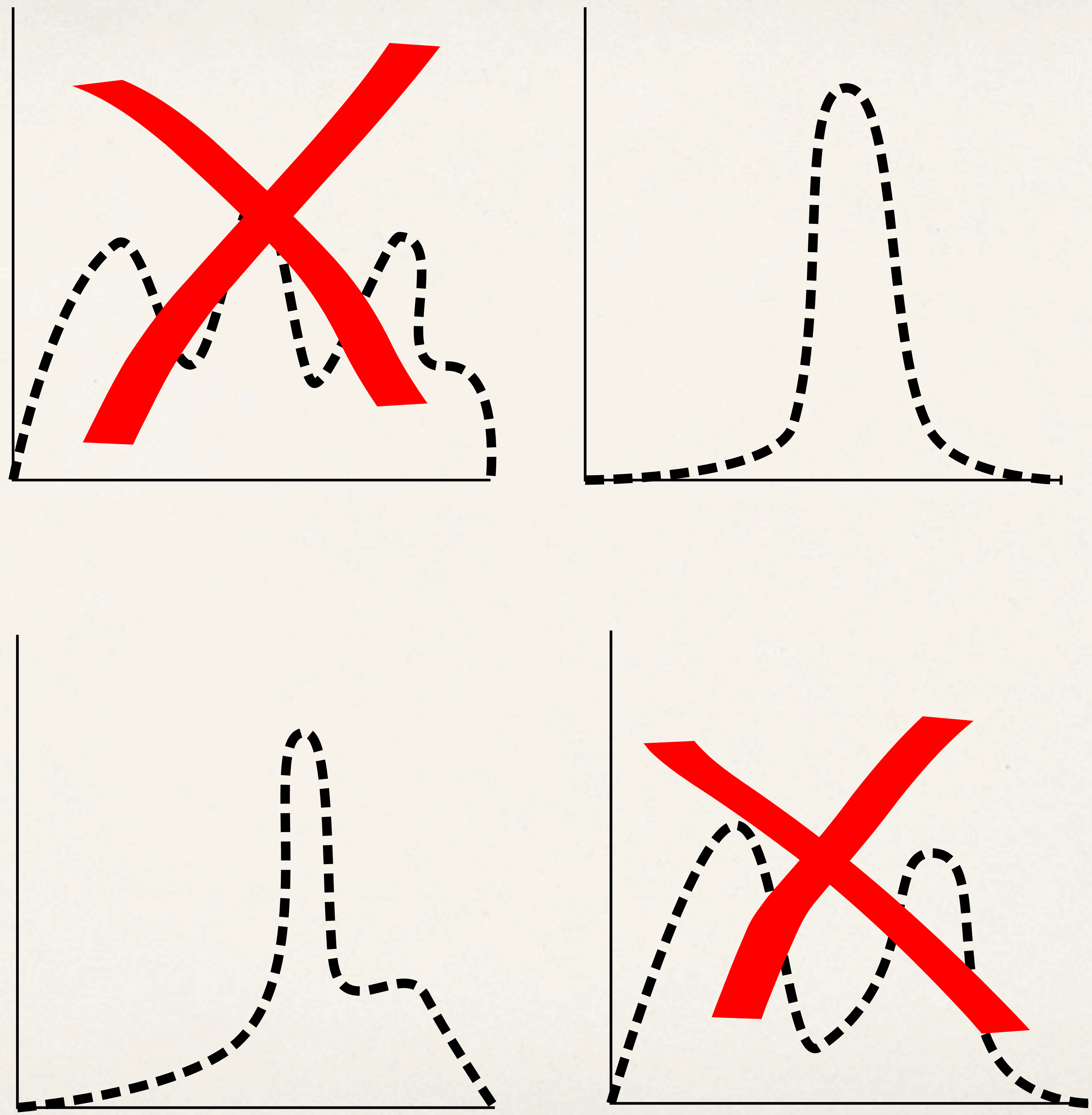
α	$\frac{\langle S\rangle/\delta B\vert_{\mathrm{Q}}}{\langle S\rangle/\delta B\vert_{\mathrm{cl}}}$
0.0	1.07
0.01	1.13
0.1	1.18
1.0	1.14
100	1.06



Signal, kT
Signal, PI
BG, kT
BG, PI
Cross Section [A.U.]
Jet Mass [GeV]
Same events
Same events

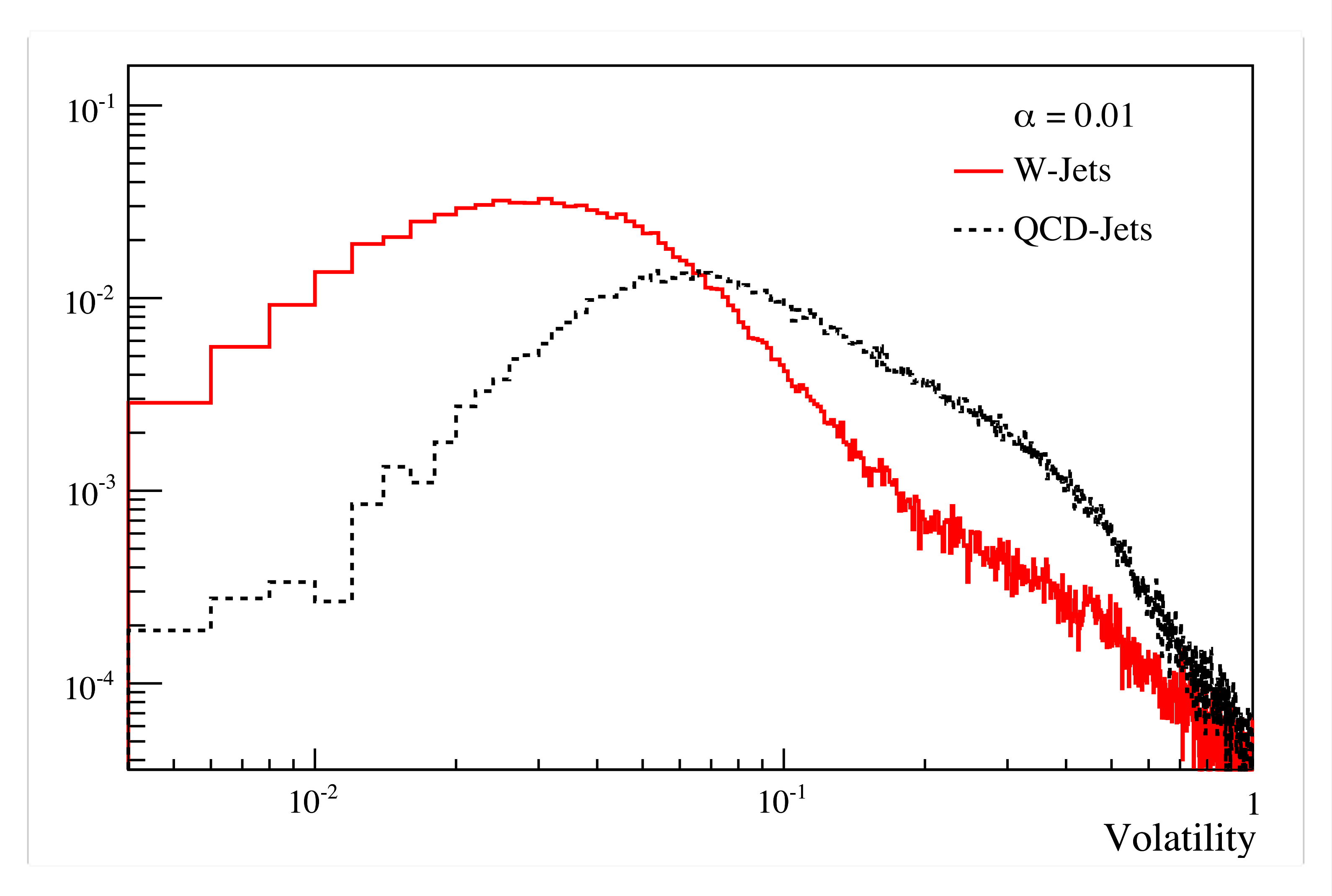
Example 3/3: Signal vs. Background Discriminant

- When there's a "right answer" for a jet's mass, most of the trees tend to center around that value.
  - There's a "right answer" for the pruned mass of a boosted particle's jet, but not for a background QCD jet
- The width of a mass distribution serves as a good signal to background discriminant!



Width to Mass Distribution

- volatility = width of pruned mass distribution

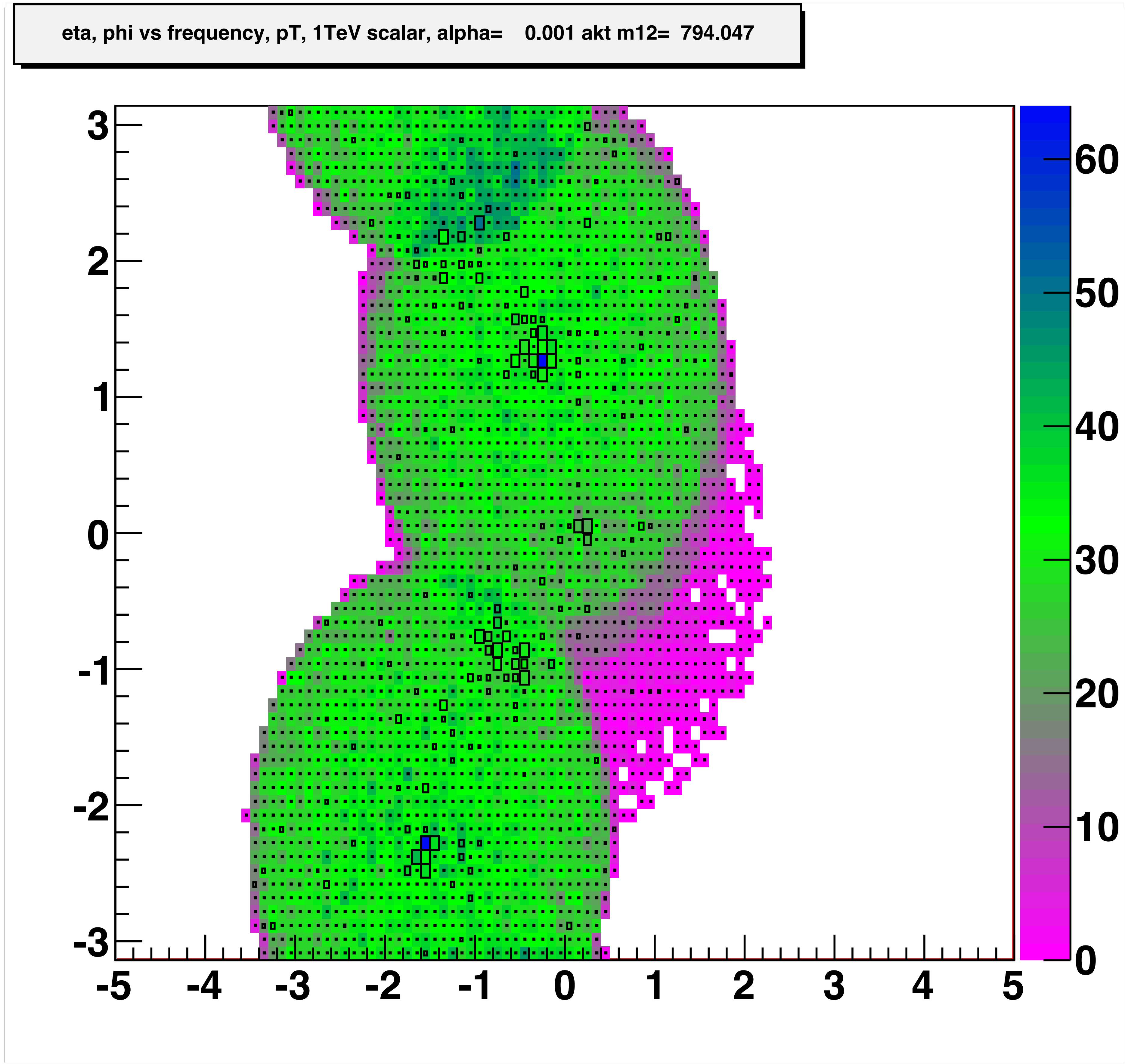


Future Directions

- Perhaps we should consider "summing" over multiple parameters, not just trees.
  - Jet radii, trimming parameters, etc.
- We've only looked at considering multiple tree structures for the radiation inside a jet.
  - Can this procedure be extended to an entire event?
  - Could this help with precision quantities like y_{23}?

Qanti-kT

- Work in progress (w/ D. Kahawala, M. Schwartz)
- Take anti-kT and perturb around it as with Qjets
- Final state is now different
  - Different jet four-momenta
  - Different jet multiplicities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3
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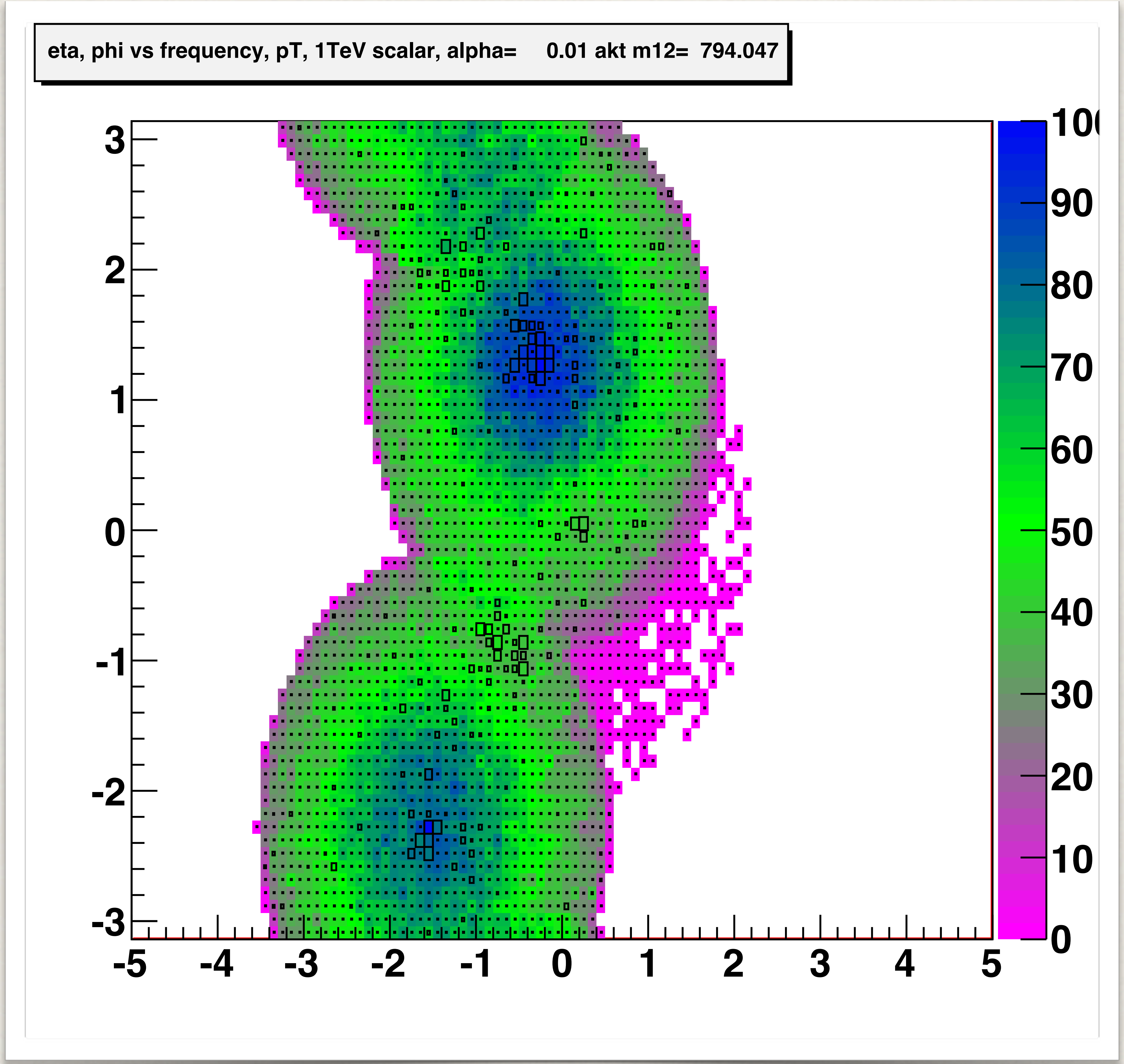
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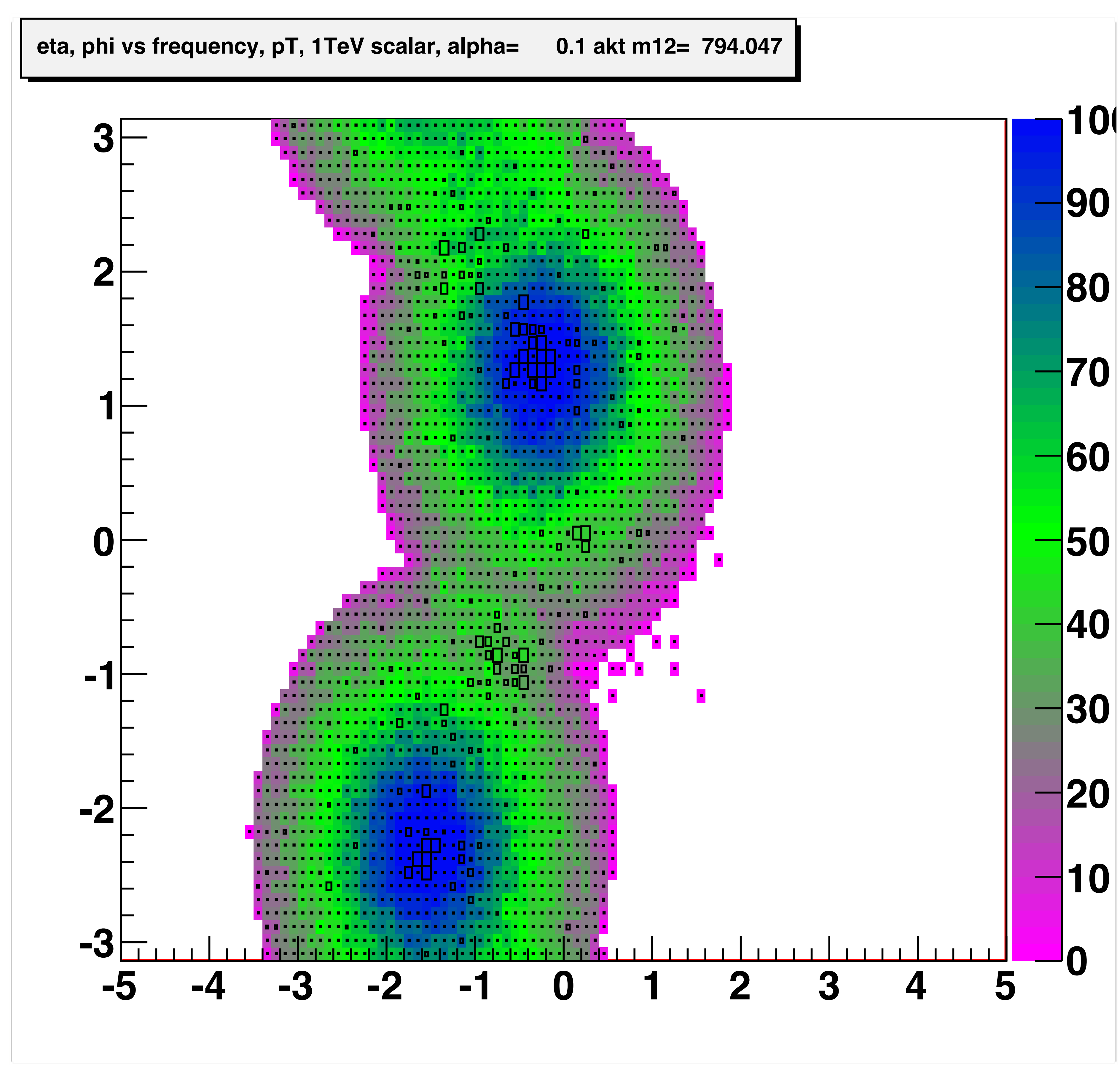
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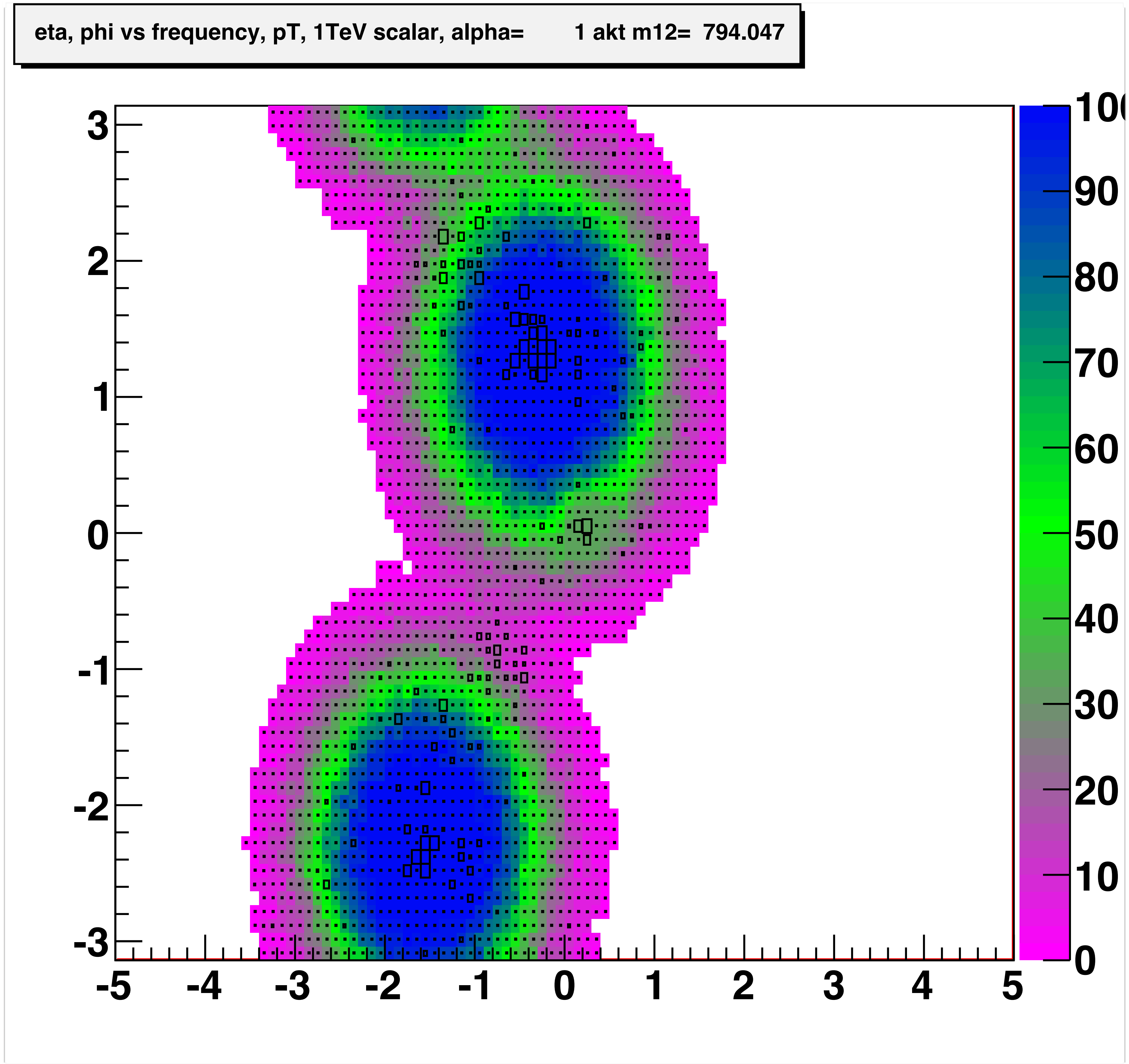
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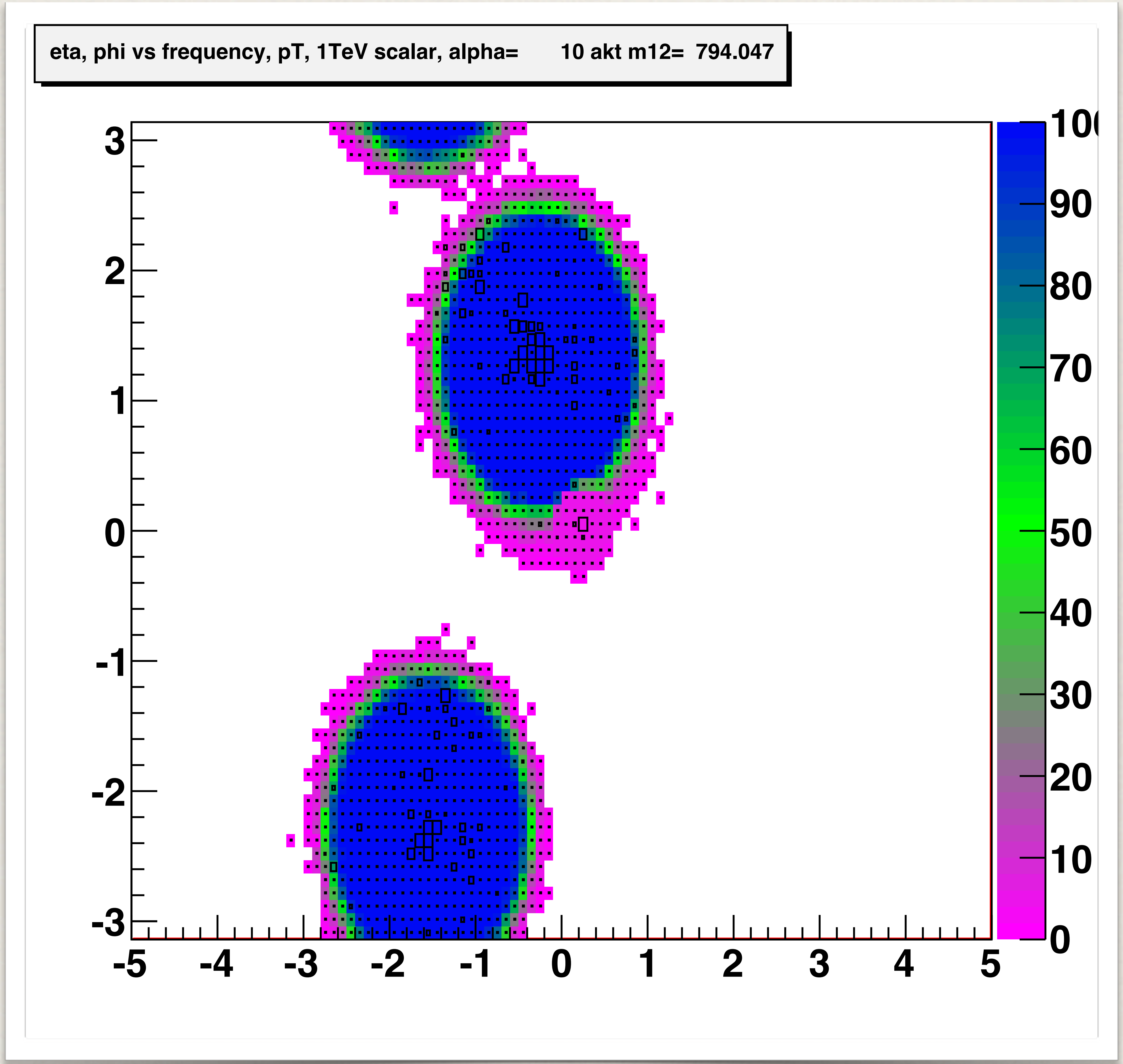
eta, phi vs frequency, pT, 1TeV scalar, alpha= 10 akt m12= 794.047
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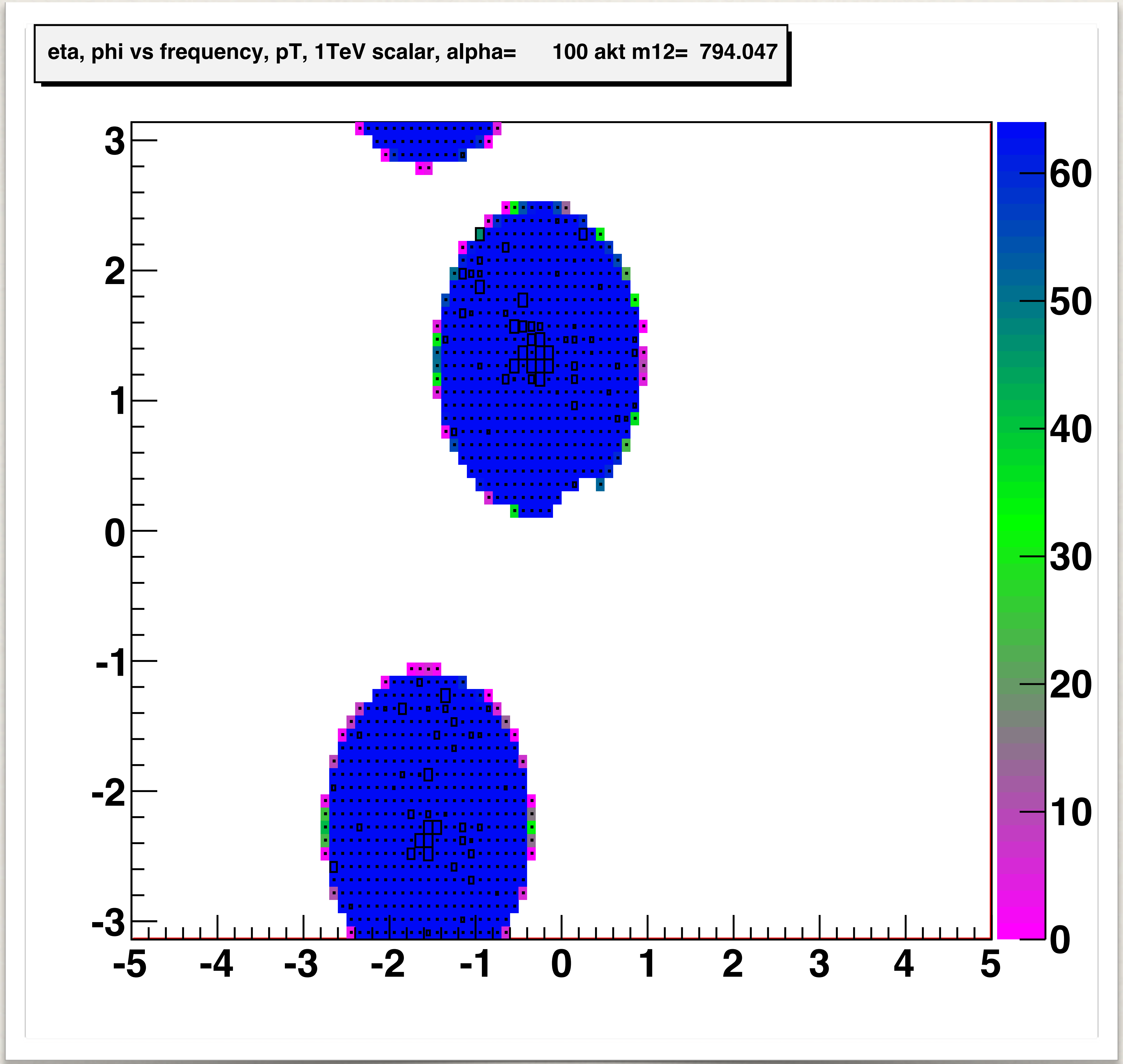
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-1
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50
40
30
20
10
0

Significant Improvement in Stability

- S/delta(B) is much larger than with traditional anti-kT.
  - Still have more optimizations to play with
  - Larger improvements as jet multiplicity increased
- Can make discoveries/exclusions much sooner!

Conclusion

- When we use C/A or k_T to associate a tree with a jet this is really just our "best guess" for the showering history.
- Sometimes these two algorithms return very different answers for the event at hand.
  - By choosing, e.g. the k_T answer over the C/A one, we introduce randomness into the picture, and the statistics are degraded.
- We propose that all trees be considered, each with a set weight, and a distribution obtained for each event (rather than a single number).
  - The results obtained from this are much less susceptible to unwanted fluctuations: equivalent to a ~2x increase in luminosity.

Backup

Classical Stats: Poisson + Binomial

$$P_N(n) \equiv \frac{e^{-N} N^n}{n!} \qquad B_\epsilon(n;r) \equiv {}_nC_r \epsilon^r (1-\epsilon)^{n-r}$$

$$F_{\epsilon,N}(r) \equiv \sum_{n=r}^{\infty} F_{\epsilon,N}(r|n) = \frac{e^{-N\epsilon} N^r \epsilon^r}{r!} \equiv P_{N\epsilon}(r)$$

$$\sigma_{\rm cl} \equiv \langle r \rangle = \sum_{r=0}^{\infty} r F_{\epsilon_{\rm cl},N}(r) = N\epsilon_{\rm cl}$$

$$\delta\sigma_{\rm cl}^2 \equiv \langle (r - \langle r \rangle)^2 \rangle = \sum_{r=0}^{\infty} (r - N\epsilon_{\rm cl})^2 F_{\epsilon_{\rm cl},N}(r) = N\epsilon_{\rm cl}$$

$$\boxed{\frac{\delta\sigma_{\rm cl}}{\sigma_{\rm cl}} = \frac{1}{\sqrt{N\epsilon_{\rm cl}}}}$$

Qstats: Poisson + Continuous

$$f_n(x) = \left(\prod_{i=1}^{n}\int_0^1 \mathrm{d}x_i f_1(x_i)\right)\delta\left(x - \frac{x_1 + \cdots + x_n}{n}\right)$$

$$\int_0^1 \mathrm{d}y\, y f_n(y) = \int_0^1 \mathrm{d}y\, y f_1(y) \equiv \epsilon_Q \qquad \sigma_n^2 \equiv \int_0^1 \mathrm{d}y\,(y-\epsilon_Q)^2 f_n(y) = \frac{1}{n}\sigma_1^2$$

$$\sigma_Q \equiv \langle yn\rangle = \sum_{n=0}^{\infty} P_N(n)\int_0^1 \mathrm{d}y\, yn f_n(y) = \epsilon_Q N$$

$$\delta\sigma_Q^2 \equiv \langle (yn - \langle yn\rangle)^2\rangle = \sum_{n=0}^{\infty} P_N(n)\int_0^1 (y\,n - \epsilon_Q N)^2 f_n(y) = (\epsilon_Q^2 + \sigma_1^2)N$$

$$\boxed{\frac{\delta\sigma_Q}{\sigma_Q} = \sqrt{\frac{1 + (\sigma_1/\epsilon_Q)^2}{N}}}$$

Properties of Jets

- What properties do we want our jets to have? Jets should be, at least,

  1. *Boost invariant*

     Easily done - cluster using rapidity / phi coordinate system:

$$\eta = -\ln\left[\cot\left(\frac{\theta}{2}\right)\right]$$

  2. *IR/Collinear safe*

IR & Collinear Safety

- Want to make jets in a way that is insensitive to soft and collinear radiation (IR & Collinear Safe)
- Necessary if we're going to employ higher order corrections.
  - If jet algorithm is not IRC safe then cancellations between real and virtual diagrams will not take place

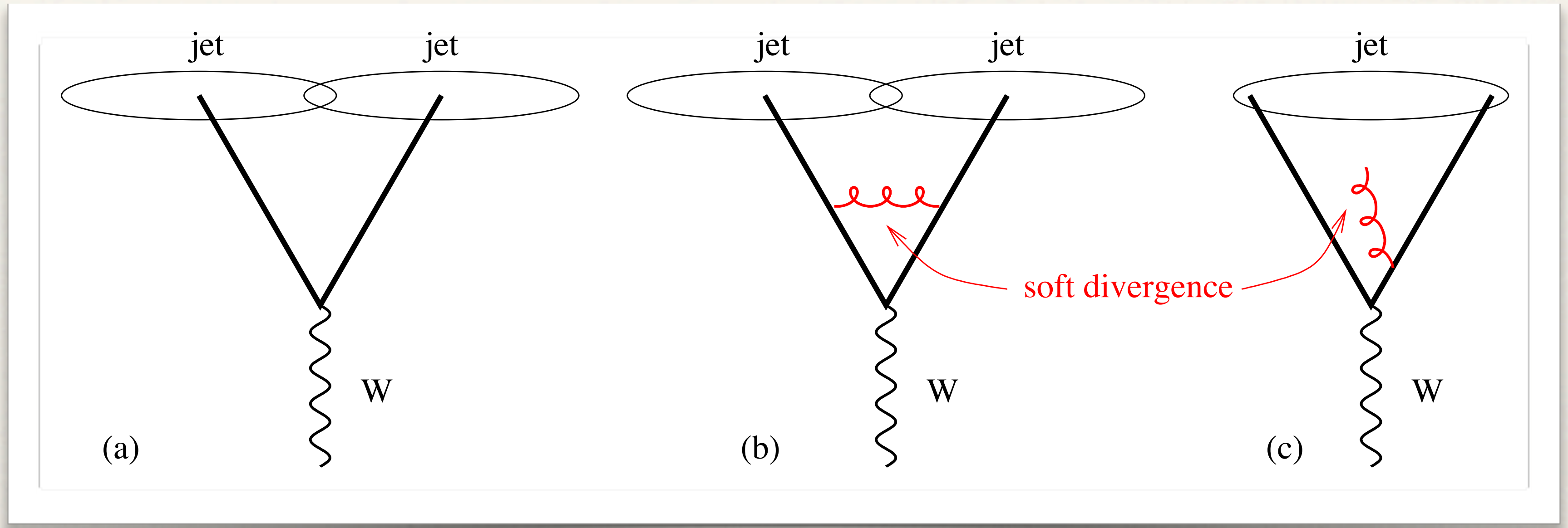
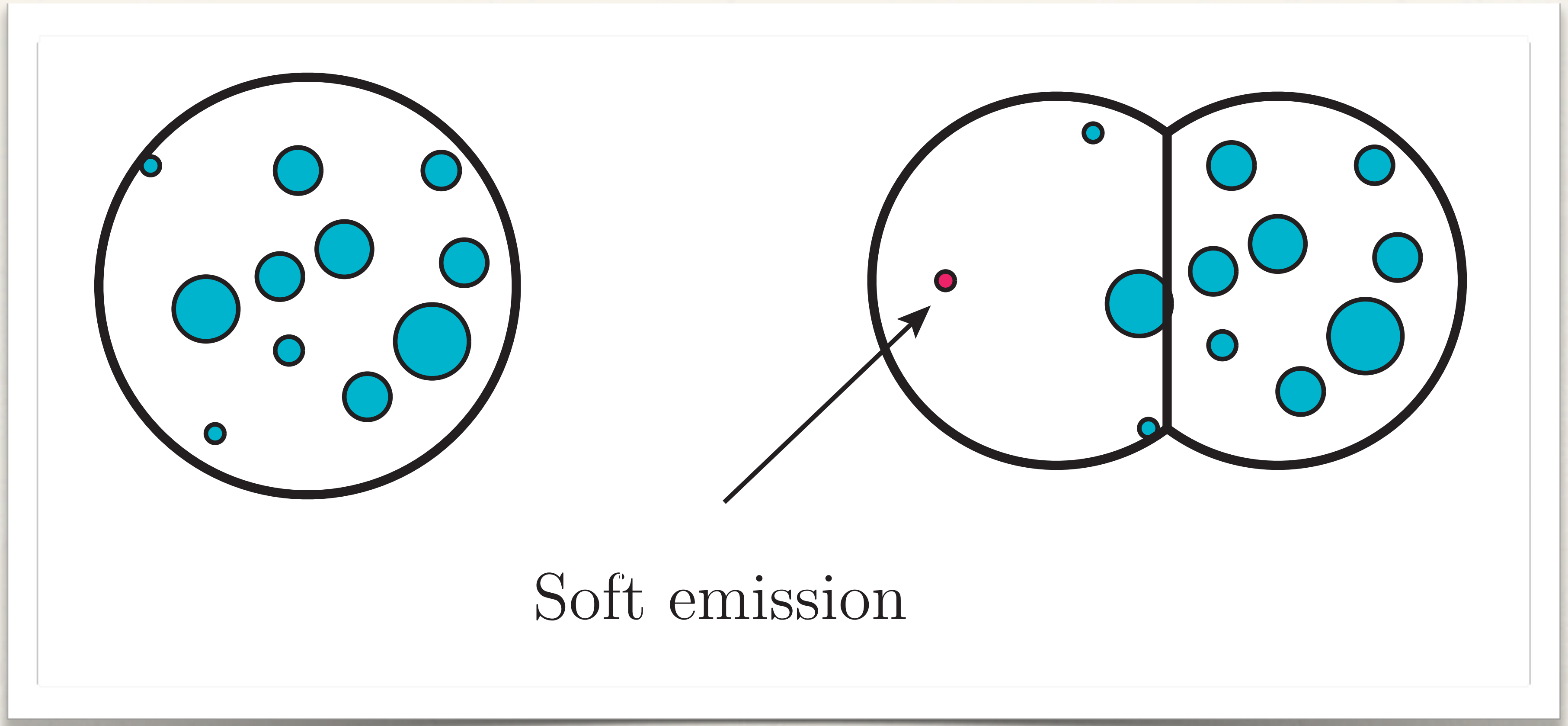


Figure source: Towards Jetography, G. P. Salam, [arXiv:0906.1833] Eur. Phys. J. C

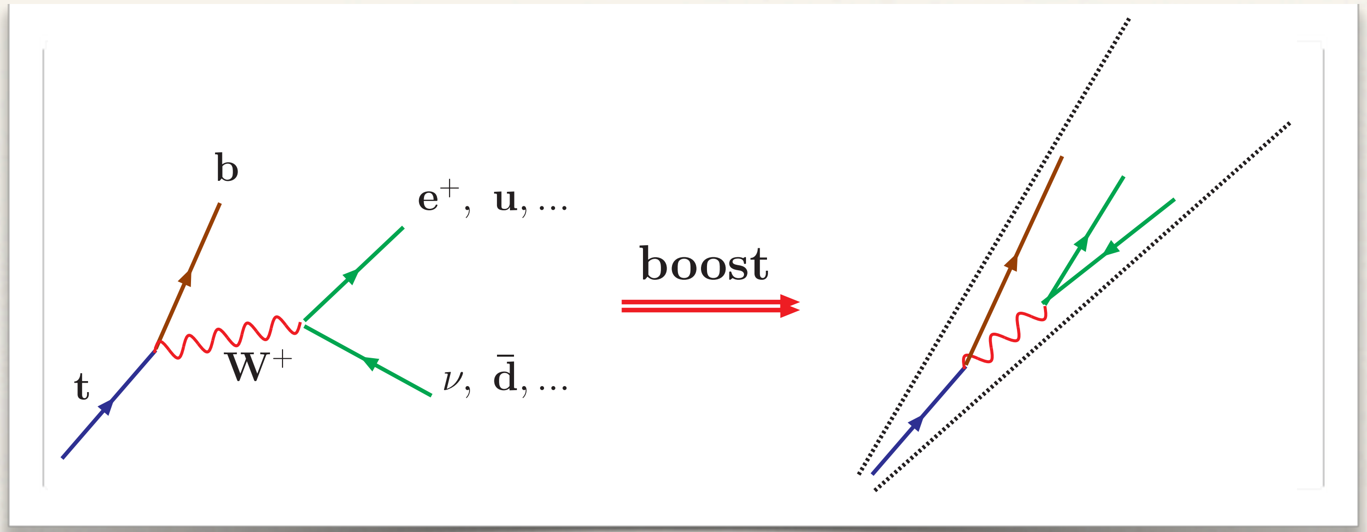
Example of an Unsafe Algorithm

- If we use calorimeter cells as seeds then even an infinitely soft emission can change the clustering behavior in a significant way:



Boosted Top

- Most models of new physics use the top quark in a special way.
- Identifying energetic tops from new physics processes will be crucial in understanding BSM phenomena at the LHC.
- If there are heavy states, the top will often be boosted



Much work on Boosted Tops

- Many approaches

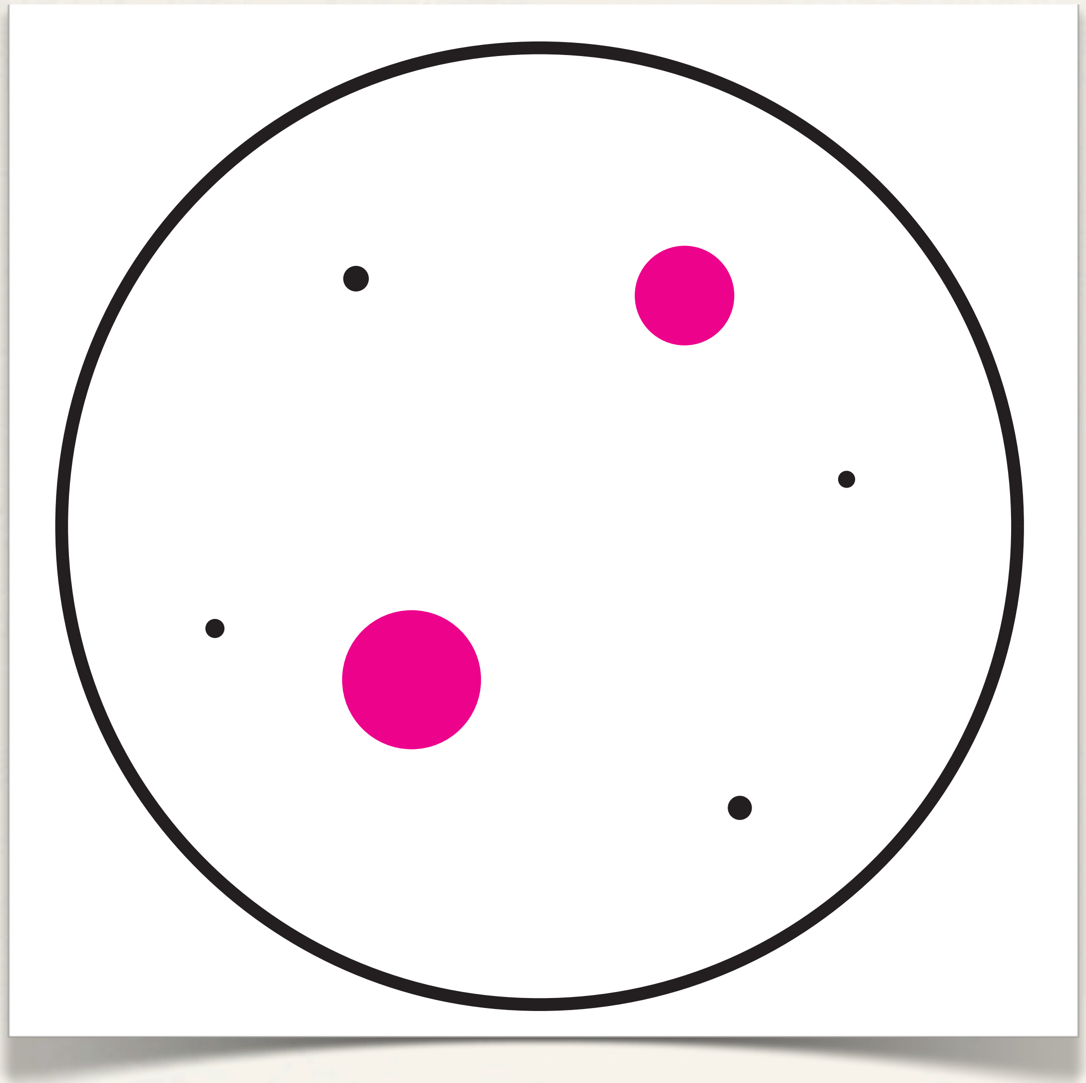
  1. Use jet shapes, analogous to event shapes (e.g. thrust & sphericity), to quantify how top-likeness of a jet.

Measure the radiation pattern.

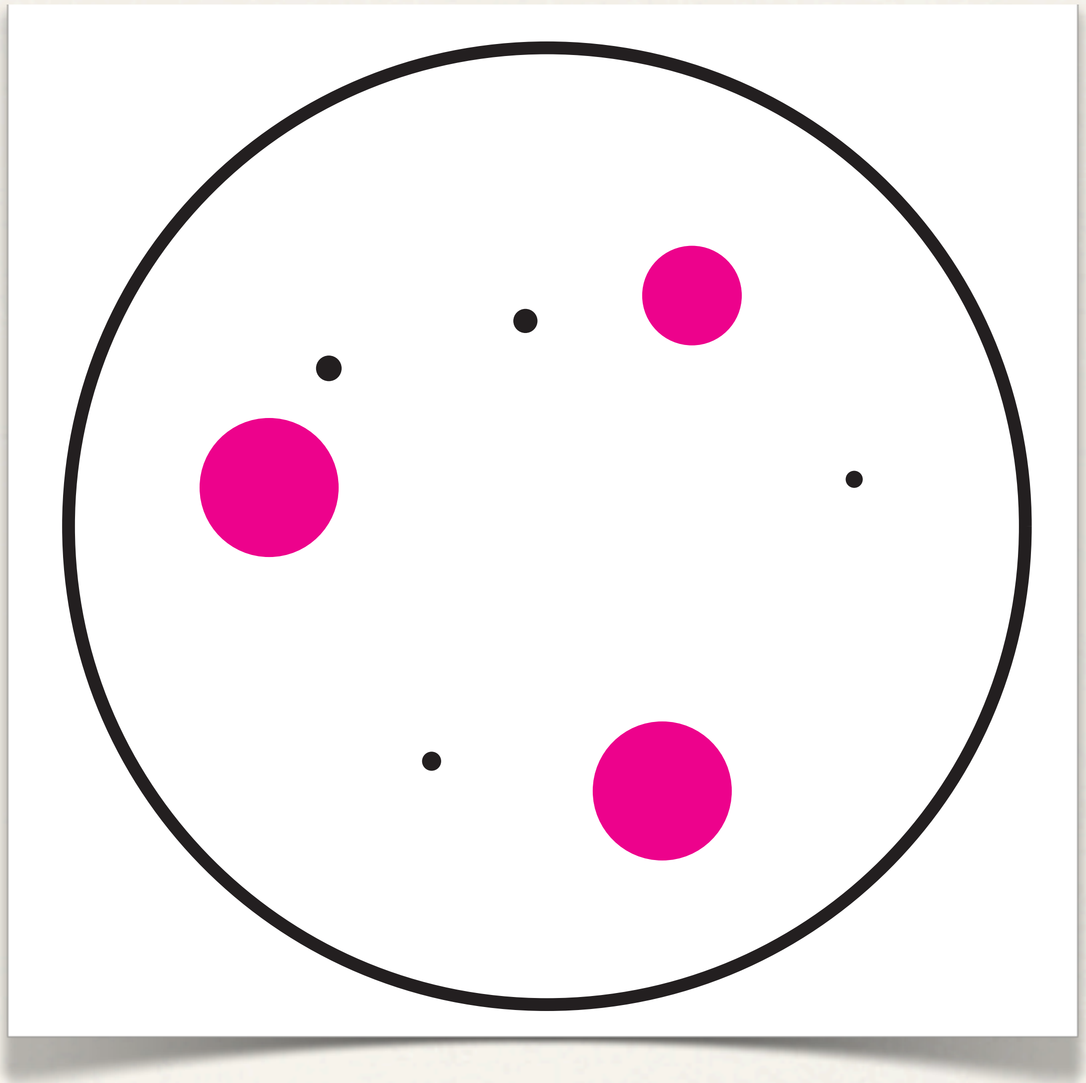
L. G. Almeida et al., *Substructure of high-pT Jets at the LHC*, Phys. Rev. D79 (2009) 074017, [0807.0234].

L. G. Almeida, S. J. Lee, G. Perez, I. Sung, and J. Virzi, *Top Jets at the LHC*, Phys. Rev. D79 (2009) 074012, [0810.0934].

QCD-like



Top-like



$$\mathrm{Pf} = \frac{4\lambda_1\lambda_2}{(\lambda_1+\lambda_2)^2} \qquad I_w^{kl} = \sum_i E_i \frac{p_{i,k}}{E_i}\frac{p_{i,l}}{E_i}$$

L. G. Almeida et al., Substructure of high-pT Jets at the LHC, Phys. Rev. D79 (2009) 074017, [0807.0234].
J. Thaler and L.-T. Wang, Strategies to Identify Boosted Tops, JHEP 07 (2008) 092, [0806.0023]

- Other approaches

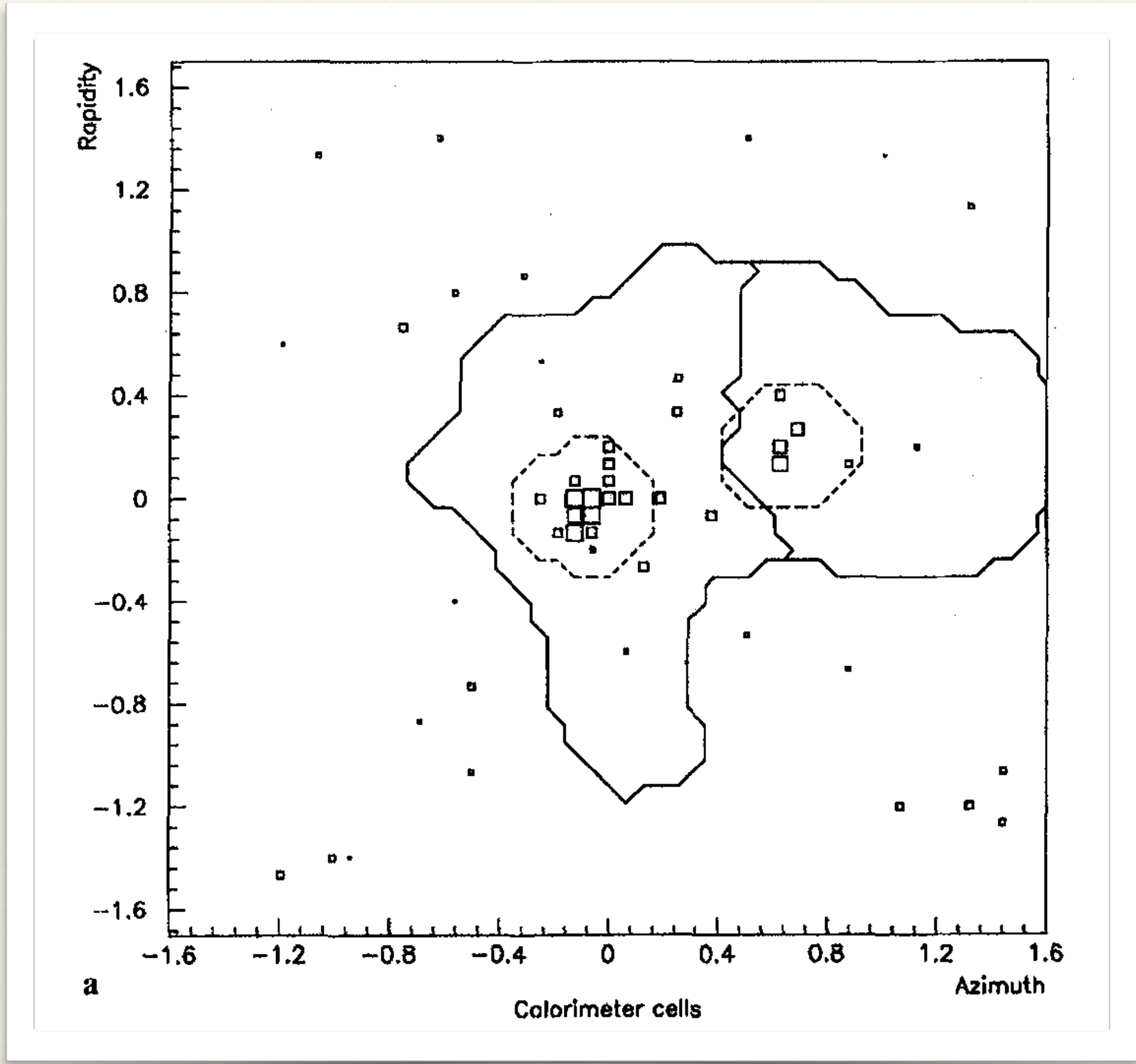
  2. Try to find subjets inside each top jet and impose kinematical constraints (using helicity structure, etc)

     Tailor made analysis

  3. See if first splitting in jet was QCD-like (soft emission) or top-like (hard emission)

D. E. Kaplan, K. Rehermann, M. D. Schwartz, and B. Tweedie, Top Tagging: A Method for Identifying Boosted Hadronically Decaying Top Quarks, Phys. Rev. Lett. 101 (2008) 142001, [0806.0848].

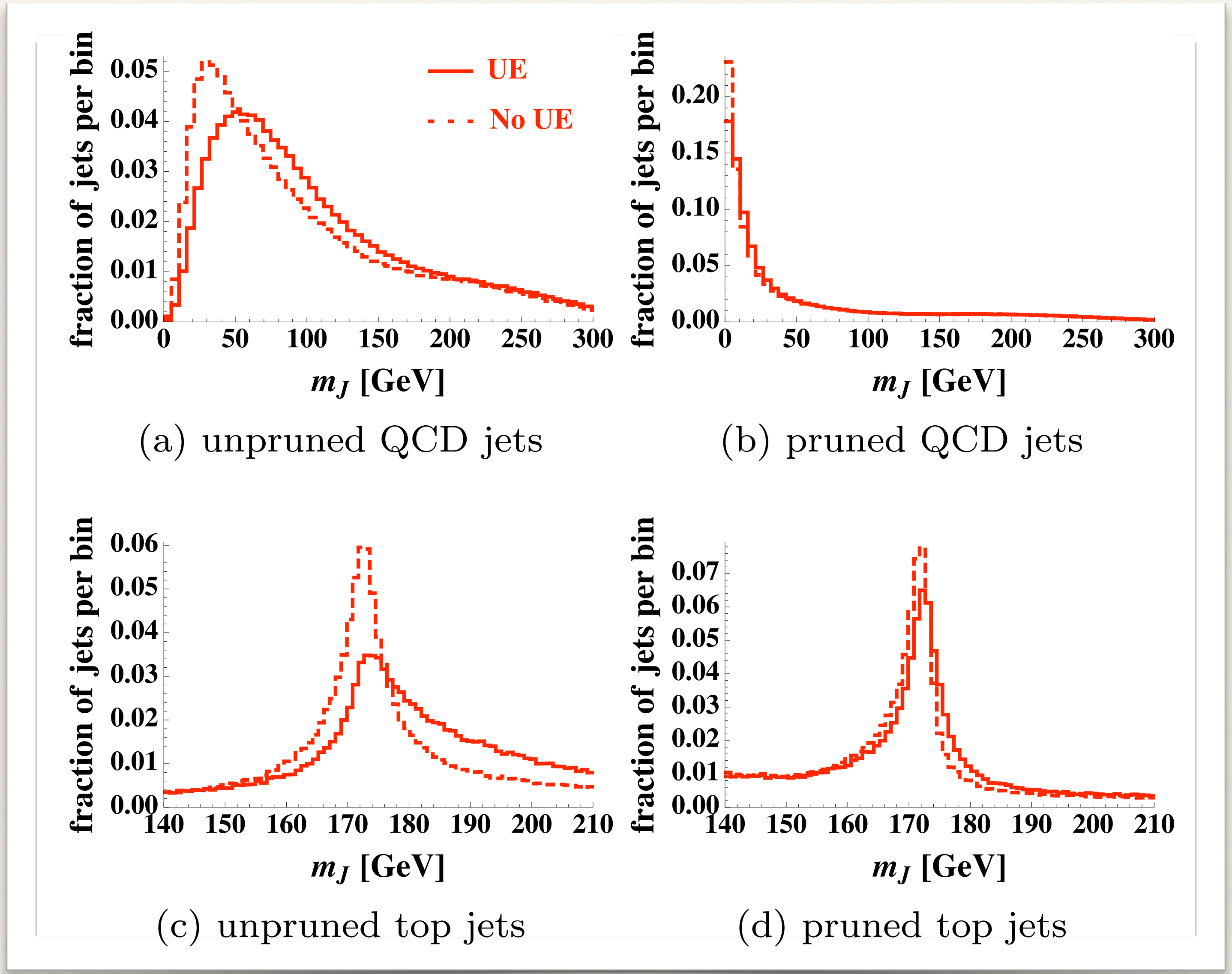
J. Thaler and L.-T. Wang, Strategies to Identify Boosted Tops, JHEP 07 (2008) 092, [0806.0023].



Try to split a jet by running an algorithm backward

Searches for New Particles Using Cone and Cluster Jet Algorithms: a Comparative Study, M. H. Seymour, Z.Phys. C62 (1994) 127-138

4. Take jet, work hard to clean it up, see if has a mass near the top's.



S. D. Ellis, C. K. Vermilion, and J. R. Walsh, Recombination Algorithms and Jet Substructure: Pruning as a Tool for Heavy Particle Searches, [arXiv:0912.0033] .

S. D. Ellis, C. K. Vermilion, and J. R. Walsh, Techniques for improved heavy particle searches with jet substructure, [arXiv:0903.5081] Phys.Rev. D80 (2009) 051501.