Signatures of Naturalness

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Outline

• Motivation
  • Model Independent approach to Naturalness

• Minimal Naturalness at current experiments

• Indirect evidence of naturalness : top divergence
  • Little Higgs Theories
  • SUSY theories
Quadratic Divergences

- Masses of scalars sensitive to the cutoff scale
- Mass splitting between charged and neutral pions well explained by an EM quadratic divergence
- Without fine tuning bare mass and quantum corrections, expect scalars to have masses of order the cutoff.
- Higgs at 125 GeV so cutoff/new physics around the corner
- So why haven’t we seen it?
What are we looking for?

- Supersymmetry
- Little Higgs
- Extra Dimensions/CFTs
- Fourth Generation
- Leptoquarks
- ...

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What are we looking for?

- Supersymmetry
- Little Higgs
- Extra Dimensions/CFTs
- Fourth Generation
- Leptoquarks
- ...
Simplified Models

- Supersymmetry
  - Light Stops - Cancels the top quadratic divergence to the Higgs mass
  - Light Higgsinos - Z mass not Higgs Mass
  - Light Gluinos - So that the Stops are not too light

\[ \tilde{g} \]

\[ \tilde{t}_L \quad \tilde{t}_R \]

\[ \tilde{b}_L \quad \tilde{H} \]
Simplified Models

- Little Higgs/Extra Dimensions/CFTs
  - Light fermionic top partners
  - single or pair produced
  - 3 different decay channels

\[ \mathcal{L} \ni THQ_3 \]
Signals

- The signals are due to the additional structure of the solutions!
- Stops solve naturalness with $\mathcal{L} \supset \phi_t \phi_t^\dagger H H^\dagger$
  - but decay via $\mathcal{L} \supset \phi_t \tilde{H} t$
- Fermionic partners use $\mathcal{L} \supset T T^c H H^\dagger$
  - but decay via $\mathcal{L} \supset T H Q_3$
Question

• If all of these signatures are not related to Higgs naturalness, then what are the model independent signatures of a Natural Higgs?

• Leads to Minimal Naturalness.
Minimal Naturalness

- Minimal Naturalness
  - Add a new particle to the SM
  - Impose that it cancels a quadratic divergence
  - No additional interactions

- All signatures are directly tied to the cancelation of quadratic divergences
  - Signatures of the model vanish in the limit that the new particle does not contribute to the Higgs quadratic divergence
Minimal Naturalness

- Model Independent Naturalness
  - Naturalness requires cancelation of quadratic divergences
  - The UV symmetry can add any number of additional interactions
  - A true model independent approach would consider all models where quadratic divergences are canceled with every possible additional interaction
  - A more tractable approach: consider all these additional terms vanishing
Outline

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  - SUSY theories
Minimal Naturalness

- Yukawa terms
  \[ \mathcal{L} \supset \lambda H \bar{\psi}_1 \psi_2 \]

- Quartic terms
  \[ \mathcal{L} = \lambda H H^\dagger \Phi \Phi^\dagger \]
Yukawa Interactions

\[ \mathcal{L} \supset \lambda H \bar{\psi}_1 \psi_2 \]

- Either \( \psi_1 \) or \( \psi_2 \) is charged under SU(2)
- Pair or associated production via gauge bosons
- Decays through W/Z/H
Yukawa Interactions

\[ \mathcal{L} \supset \lambda H \bar{\psi}_1 \psi_2 \]

- Case 1: Both are new particles
  - Electroweakino phenomenology
  - They might be in different representations than electroweakinos.
  - Decays that end in MET/R-hadrons/CHAMPs with different sized production cross sections
Yukawa Interactions

\[ \mathcal{L} \supset \lambda H \bar{\psi}_1 \psi_2 \]

- **Case 2:** \( \psi_2 \) is a SM particle
  - \( \psi_1 \) must have the same quantum numbers as a SM field
  - A 4th generation model whose interactions cancel a quadratic divergence
  - Single production through yukawa interaction and pair production via gauge interactions

Table 1: Possible signatures associated with minimal models which use the interaction term in Eq. 2.1 to
gauge and/or Higgs bosons and either missing

<table>
<thead>
<tr>
<th>Case</th>
<th>( \psi_2 )</th>
<th>Production Channel</th>
<th>Signatures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>has to be charged electroweakly and ( \psi_1 ) is heavier than its Standard Model partner</td>
<td>Yes single production</td>
<td>( 2 V/H + R-hadrons/CHAMPS/ )</td>
</tr>
<tr>
<td>2</td>
<td>has to be charged and ( \psi_1 ) is a new particle, ( \psi_2 ) is a SM particle</td>
<td>Yes pair production</td>
<td>( 2 V/H + R-hadrons/CHAMPS/ )</td>
</tr>
<tr>
<td>3</td>
<td>has to be charged and ( \psi_2 ) is a new particle, ( \psi_1 ) is a non-SM particle</td>
<td>No single production</td>
<td>( 2 V/H + \bar{L} \bar{T} )</td>
</tr>
<tr>
<td>4</td>
<td>has to be charged and ( \psi_1 ) is a non-SM particle, ( \psi_2 ) is a SM particle</td>
<td>No pair production</td>
<td>( 2 V/H + \bar{L} \bar{T} )</td>
</tr>
<tr>
<td>5</td>
<td>has to be charged and ( \psi_1 ) is a non-SM particle, ( \psi_2 ) is a non-SM particle</td>
<td>No single production</td>
<td>( 2 V/H + \bar{L} \bar{T} )</td>
</tr>
<tr>
<td>6</td>
<td>has to be charged and ( \psi_1 ) is a non-SM particle, ( \psi_2 ) is a non-SM particle</td>
<td>No pair production</td>
<td>( 2 V/H + \bar{L} \bar{T} )</td>
</tr>
</tbody>
</table>

We consider minimal extensions of the SM where there is only the term in the Lagrangian. In order for this coupling to contribute to the Higgs quadratic divergence at 1-loop, we need to cancel Higgs quadratic divergences. In both cases particles are produced through gluons/gauge bosons.
Quartic Interactions

\[ \mathcal{L} \supset \lambda \psi^\dagger \psi HH^\dagger \]

\[ = \lambda \psi^\dagger \psi \frac{h^2}{2} + \lambda v \psi^\dagger \psi h + \frac{\nu^2}{2} \psi^\dagger \psi \]

- **Case 1:** \( m_\psi < \frac{m_h}{2} \)
  - New decays of the Higgs. Charged and Colored \( \psi \) ruled out by experiments so only invisible decays of the Higgs
  - If top/gauge quadratic divergences are canceled, decay width orders of magnitude larger than the decay width into bottoms
  - Other quadratic divergences yield decay widths too small to be observed
Quartic Interactions

\[ \mathcal{L} \supset \lambda \psi^\dagger \psi H H^\dagger \]

\[ = \lambda \psi^\dagger \psi \frac{h^2}{2} + \lambda v \psi^\dagger \psi h + \lambda \frac{v^2}{2} \psi^\dagger \psi \]

• Case 2: \( \psi \) has SM charge
  • Modified couplings to the gauge bosons at 1-loop

\[ \text{Diagram: } H \rightarrow \gamma \gamma \gamma \]
Quartic Interactions

- **Case 2: \( \psi \) has SM charge**
  - e.g. a singlet fermion with electric charge 1 canceling the top divergence

\[
\frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{\text{SM}}} = \left| 1 - \frac{1}{6.49} Q^2 \frac{4}{3} \left( \frac{\log m_\psi}{\log v} \right) \left( 1 + \frac{7m_h^2}{120m_\psi^2} \right) \right|^2
\]

- Mass term from \( \mathcal{L} \supset -m\psi^\dagger\psi + \frac{3y_t^2}{2m} \psi^\dagger\psi H H^\dagger \)
Quartic Interactions

- Case 2: $\psi$ has SM charge

![Fractional Change to the Higgs diphoton decay width](chart.png)
Quartic Interactions

\[ \mathcal{L} = \lambda H H^\dagger \Phi \Phi^\dagger \]
\[ = \lambda \nu v \Phi h \phi + \frac{\lambda}{2} v \Phi \phi h h + \frac{\lambda}{2} v \phi \phi h + \ldots \]

- Case 3: \( \langle \Phi \rangle \neq 0 \)
  - If \( \Phi \) has the quantum numbers of a Higgs, then this is a two higgs doublet model satisfying the Veltman conditions
  - Mass mixing with the Higgs!
    \[
    \begin{pmatrix}
      h_m \\
      \phi_m
    \end{pmatrix} =
    \begin{pmatrix}
      \cos \alpha & \sin \alpha \\
      -\sin \alpha & \cos \alpha
    \end{pmatrix}
    \begin{pmatrix}
      h \\
      \phi
    \end{pmatrix}
    \]
  - This suppresses all of the Higgs couplings by the same amount
Quartic Interactions

\[ \mathcal{L} = \lambda H H^\dagger \Phi \Phi^\dagger \]
\[ = \lambda \nu \nu \Phi h \phi + \frac{\lambda}{2} \nu \Phi \phi h h + \frac{\lambda}{2} \nu \phi \phi h + \ldots \]

• Case 3: \( \langle \Phi \rangle \neq 0 \)
  • Current ATLAS bounds place \( \cos \alpha \geq 0.93 \)
  • ATLAS-CONF-2013-034
Quartic Interactions

• Case 3: $\langle \Phi \rangle \neq 0$
Quartic Interactions

\[ \mathcal{L} = \lambda H H^\dagger \Phi \Phi^\dagger \]

\[ = \lambda v v_\Phi h \phi + \frac{\lambda}{2} v_\Phi \phi h h + \frac{\lambda}{2} v \phi \phi h + \ldots \]

• **Case 3:** \( \langle \Phi \rangle \neq 0 \)
  
  • Two decay channels from the quartic interaction
  
  • Mixing with the Higgs is important for \( m_\Phi < 2m_h \)
  
  • New SM-Higgs like particle with suppressed couplings
  
  • Second term important for \( m_\Phi > 2m_h \)
  
  • New scalar that decays to WW/ZZ/hh with a ratio of 2:1:1 in the large mass limit from the Goldstone boson equivalence theorem
Quartic Interactions

\[ \mathcal{L} = \lambda H H^\dagger \Phi \Phi^\dagger \]
\[ = \lambda v v_\Phi h \phi + \frac{\lambda}{2} v_\Phi \phi hh + \frac{\lambda}{2} v \phi \phi h + \ldots \]

• Case 3: \( \langle \Phi \rangle \neq 0 \)
  - Precision Higgs physics gives bounds of \( \sin^2 \alpha \leq 14\% \).
  - Bounds on Heavy Higgses are generally not competitive with current precision Higgs physics
Quartic Interactions

\[ \mathcal{L} \supset \lambda \psi^\dagger \psi H H^\dagger \]

\[ = \lambda \psi^\dagger \psi \frac{h^2}{2} + \lambda v \psi^\dagger \psi h + \lambda \frac{v^2}{2} \psi^\dagger \psi \]

- Case 4: \( \psi \) is dark matter
  - Direct detection

\[
\sigma_{p,n,SI} = \frac{a}{\pi (m_\psi + m_p)^2} \frac{m_p^2}{m_h^4} \frac{9y_t^4 m_p^2}{m_h^2} f^2 \\
f = \frac{6}{27} + \frac{21}{27} (f_{Tu} + f_{Td} + f_{Ts})
\]

- a=4 for a real scalar
- a=1 for a complex scalar or majorana fermion
- a=1/4 for a dirac fermion
Quartic Interactions

- Case 4: Dark matter

Direct Detection

Blue = Top divergence canceled
Red = Gauge divergence canceled
Solid = Complex scalar
Dashed = Dirac fermion
Quartic Interactions

\[ \mathcal{L} \supset \lambda \psi^\dagger \psi HH^\dagger \]

\[ = \lambda \psi^\dagger \psi \frac{h^2}{2} + \lambda v \psi^\dagger \psi h + \lambda \frac{v^2}{2} \psi^\dagger \psi \]

• Case 4: \( \psi \) is dark matter
  
  • Indirect detection
  
  • Cross sections in the large mass, small velocity limit

\[
\langle \sigma_{\text{fermion}} v \rangle_{v=0} = 0
\]

\[
\langle \sigma_{\text{scalar}} v \rangle_{v=0} = \frac{9y_t^4}{16\pi m_{\psi}^2}
\]
Quartic Interactions

- Case 4: Dark matter

Indirect Detection

Blue = Top divergence canceled
Red = Gauge divergence canceled
Solid = Complex scalar dark matter

Figure 1: The abundance, direct and indirect detection bounds on dark matter particles which cancels quadratic divergences. The indirect detection bound is the Fermi bound on $XX \rightarrow bb$ [27], while the annihilation channel is into two Higgses. As the Higgs decays dominantly into bottoms, the bounds are expected to be similar in strength so the plot can used to find the approximate indirect detection bound. The direct detection bound is taken from XENON100 [26] and the relic abundance is taken from [37]. Bounds are placed assuming $X$ makes up all of the dark matter. The blue line signals a cancelation of the top quadratic divergence while a red line is the gauge quadratic divergence. The black lines are the current bounds. Solid lines are complex scalar dark matter while dashed lines are dirac fermion dark matter.
Quartic Interactions

- Case 4: Dark matter

Abundance

Blue = Top divergence canceled
Red = Gauge divergence canceled
Solid = Complex scalar
Dashed = Dirac fermion

Figure 1: The abundance, direct and indirect detection bounds on dark matter particles which cancels quadratic divergences. The indirect detection bound is the Fermi bound on $XX \rightarrow \bar{b}b$ [27], while the annihilation channel is into two Higgses. As the Higgs decays dominantly into bottoms, the bounds are expected to be similar in strength so the plot can used to find the approximate indirect detection bound. The direct detection bound is taken from XENON100 [26] and the relic abundance is taken from [37]. Bounds are placed assuming $X$ makes up all of the dark matter. The blue line signals a cancelation of the top quadratic divergence while a red line is the gauge quadratic divergence. The black lines are the current bounds. Solid lines are complex scalar dark matter while dashed lines are dirac fermion dark matter.
Minimal Naturalness
Summary

• Yukawa terms
  • 4th generation and electroweakino like signals

• Quartic terms
  • New Higgs decays if $\psi$ is light enough
  • Modified decays to gauge bosons if $\psi$ is charged under the SM
  • Suppressed Higgs couplings and either a SM-like heavy higgs or a scalar that decays to pairs of Ws/Zs/hs
  • Direct and Indirect detection signals if $\psi$ is dark matter
  • A measurable correlation if $\psi$ is a scalar
Outline

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  • SUSY theories
Mass Mixing

• Is there some other way to tie the quartic interaction with a decay channel without assuming a symmetry?

• Adding a generic term in the Lagrangian will generate their own signatures
Mass Mixing

- Mass mixing is unique in that it allows the quartic to become a decay channel
Decays via a Higgs

- Obviously not direct evidence as the term could have been in the Lagrangian from the start
- Unique low energy assumption for generating an observable signature at the LHC!
- Decays through the Higgs but NOT gauge bosons.
- Cascades through the Higgs and only the Higgs inform us about quartics responsible for naturalness!
Decays via a Higgs

- This simplified model can be reached as a limit of Little Higgs models
- Assume a single vector like new particle canceling the top quadratic divergence.

\[ \mathcal{L} = f \lambda_1 \psi_1 t_R^c + f \lambda_2 \psi_1 \psi_1^c - \lambda_1 Q_3 H t_R^c + \frac{\lambda_1}{2f} H H^\dagger t_R^c \psi_1 \]
Decays via a Higgs

- Go to mass basis in the small $v$ limit

$$\mathcal{L}_{\text{mixing}} = m_U U U^c + \lambda^i_U U^c H Q_i + \lambda^i_{SM} u^c_i H Q_j$$

$$+ \frac{\lambda_{UU}}{m_U} U^c U H H^\dagger + \frac{\lambda^i_{Uu}}{m_U} u^c_i U H H^\dagger$$

- In Little Higgs models, the quartics are related by a rotation angle.

- In the past, $\lambda^i_U$ was assumed to dominate the phenomenology

- At the level of 2 body decays, only true if $\lambda^i_U > \lambda^i_{Uu} \frac{v}{m_U}$
Decays via a Higgs

• Quartic dominating decays
  
  • $\lambda^i_U$ related to flavor physics. Typical assumptions of Little Higgs and Fourth generation models are a flavor texture that makes these terms small so that FCNC are not an issue. Assume that making them small also makes them irrelevant for phenomenology.

  • If there are no ad hoc cancelations $\lambda^i_U$ is of similar size to the SM particle being mixed with while $\lambda^i_{U \bar{U}}$ is of similar size to the quadratic divergence being canceled. Mixing with light partners while canceling top quadratic divergences yields phenomenology based off of the quartic.
Phenomenology of Quartic

\[ \frac{\lambda_{Uu}^i}{m_U} u_i^c U H H^\dagger \]

- Quartic gives four decay channels

\[ U \rightarrow u_i + h \]
\[ U \rightarrow u_i + h + h \]
\[ U \rightarrow u_i + Z + Z \]
\[ U \rightarrow u_i + W^+ + W^- \]
Phenomenology of Quartic

Blue = a single Higgs
Yellow = a pair of Ws
Green = a pair of Zs
Red = a pair of Higgses
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SUSY Quartics

• Current collider signatures of SUSY are independent of the cancelation of the top quadratic divergence.

• SUSY relates top yukawa to various quartics

\[ W = y_t Q_3 H_u t^c - y_b Q_3 H_d b^c \]
Only fermionic symmetries can relate yukawas to scalar quartics

- Measure any scalar quartic to test supersymmetry

Focus has been on

\[ \mathcal{L} \supset \tilde{t}\tilde{t}^\dagger H_u H_u^\dagger = \frac{v}{\sqrt{2}} h\tilde{t}\tilde{t}^\dagger + \cdots \]

- Not LHC observable
SUSY Quartics

- Any LHC observable quartics?

\[ F_Q = y_t H_u \tilde{t}_R + y_b H_d \tilde{b}_R \]

\[ \mathcal{L} \supset y_t y_b^* H_u \tilde{t}_R H_d^\dagger \tilde{b}_R^\dagger + h.c. = v y_t y_b^* H^- \tilde{t}_R \tilde{b}_R^\dagger + \cdots \]

- Mediates decays between right handed stop and sbottoms!
SUSY Quartics

\[ \mathcal{L} \supset y_t y_b^* H_u \tilde{t}_R H_d^\dagger \tilde{b}_R^\dagger + h.c. \]

- No tree level term in the MSSM, soft or not that allows for a right handed stop to decay to a right handed sbottom decay!

- Unlike the well studied quartic, demonstrating the existence of this operator or measuring its value is doable at the LHC

- Just observe the decay
Easy Simplified Models

• Light right handed stop, sbottom and LSP
• Naturalness requires light stops
  • Many models of natural SUSY have NLSP stops/sbottoms
  • Left handed stops tend to be heavier due to RG effects of the gaugino
  • In fact making the 3rd generation squarks not tachyonic can be a strong constraint
• Also provides an interesting way to study the charged Higgs
Easy Simplified Models

Wino LSP

Model W1

\[ \tilde{t}_R, \tilde{b}_R, H^+, \tilde{W}^{\pm,0} \]

mass

t/b

Model W2

\[ \tilde{b}_R, \tilde{t}_R, H^+, \tilde{W}^{\pm,0} \]

mass

t/b
Easy Simplified Models

Gravitino LSP

Model G1

\[ b \rightarrow \tilde{b}_R \rightarrow H^+ \rightarrow \tilde{t}_R \rightarrow \tilde{t}_R + \tilde{b}_R + \tilde{t}_R + 2\tilde{G} \]

Model G2

\[ t \rightarrow \tilde{t}_R \rightarrow H^+ \rightarrow \tilde{b}_R \rightarrow \tilde{b}_R + \tilde{t}_R + \tilde{b}_R + 2\tilde{G} \]
Easy Simplified Models

- For some regions of parameter space current searches have good sensitivity
- If the charged Higgs is heavy, it decays into 3rd gen. quarks
- Signal has 2-4 tops and 2-4 bottoms for which searches already exist
- Assume $m_{H^+} = 300 \text{GeV}$, $m_H = m_L + 350 \text{GeV}$
Easy Simplified Models

Purple: many tops/bottom ATLAS search applied Simplified Models
Red: direct sbottom production bounds
Blue: direct stop production bounds

Figure 6: 95% confidence level bounds on the NLSP/LSP parameter space for the simplified models discussed in Sec. 2. The heavier squark is 350 GeV more massive than the lighter squark and the charged Higgs has a mass of 300 GeV. Bounds on the cascade are shown in solid purple while the bounds on the sbottom/stop NLSPs are shown in dotted red/blue (dashed, line-dot-line, and thick solid depending on the search). We see that current searches already place bounds on the cascade that are comparable with searches for the direct production of the lighter squark. There are regions of parameter space where the cascade would be seen before the direct production of the NLSP.

We see for both simplified models that the bounds on the cascades are non-negligible. In most of parameter space, discovery of the cascade would happen shortly after the discovery of the direct searches for the NLSP. In squeezed regions, the cascade is seen first. This effect is enhanced when the NLSP decays primarily through a top.

Conclusion

In this article, we discussed a decay mode that results from a supersymmetric scalar quartic. Discovery of this decay mode and its possible measurement would show the existence of a scalar quartic interaction which is related to a yukawa interaction by a symmetry. Finding this relationship would prove that our underlying theory is supersymmetric. This quartic interaction is $H_uH_d^\dagger\tilde{t}_R\tilde{b}_R^\dagger$ and gives decays via a...
Conclusion

• Should approach naturalness in a model independent manner
  • 4th generation and electroweakino like signals
  • Precision Higgs measurements
  • Extra Higgs-like scalars
  • Direct and Indirect detection signals for dark matter
  • A measurable correlation for scalars
Conclusion

- Decays involving only scalars are important
  - Mass mixing the quartic results in an observable decay that involves only the Higgs - seen in Little Higgs models
  - Decays involving charged higgses and not W bosons: evidence that the stop sector is supersymmetric
Goldstone Equivalence Theorem

\[ \mathcal{L} \supset -m_T T T^c + \lambda H H^\dagger T u_3^c \]

In the large mass limit, the decay to two W/Z should be described by this term.
Goldstone Equivalence Theorem

\[ |M(T \rightarrow thh)|^2 \sim \frac{\lambda^2}{2} p_{T,\mu} p_{t}^{\mu} \]

\[ |M(T \rightarrow tW^+W^-)|^2 \sim 4\lambda^2 m_W^4 p_{T,\mu} p_{t}^{\mu} \frac{1}{((p_T - p_t)^2 - m_h^2)^2} \frac{(p_W^+ \cdot p_W^-)^2}{m_W^4} \]
Little Higgs Model

• Toy Little Higgs model describing SU(3) breaking down to SU(2)

\[ \Sigma = \exp \left( \frac{i}{f} \begin{pmatrix} 0 & \bar{H} \\ H^\dagger & 0 \end{pmatrix} \right) \begin{pmatrix} 0 \\ f \end{pmatrix} \]

• Collective symmetry breaking dictates that the Lagrangian is

\[ \mathcal{L} \supset \lambda_1 u_3^c \Sigma \chi + \lambda_2 f u'^c u' \]

• Lowest order terms are

\[ \mathcal{L} \supset f (\lambda_1 u_3^3 + \lambda_2 u'^c) u' - \lambda_1 u_3^c HQ_3 + \frac{\lambda_1}{2f} HH^\dagger u_3^c u' \]
Little Higgs Model

- Diagonalize in the small $v$ limit

\[
\mathcal{L} \supset \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} t_3^c H Q_3 + \frac{\lambda_1^2}{\sqrt{\lambda_1^2 + \lambda_2^2}} T^c H Q_3 + \frac{\lambda_1^2}{2m_T} H H^\dagger T^c T + \frac{\lambda_1 \lambda_2}{2m_T} H H^\dagger t_3^c T
\]

- Comparing the two and three body decays give

\[
\frac{\lambda_2}{\lambda_1} \frac{\sqrt{\lambda_1^2 + \lambda_2^2} v}{2m_T} = \frac{\lambda_2^2 v}{2 y m_T}
\]