Gluon Tagging and Quark & Gluon Samples
How well can we do at the 7 TeV LHC?

Jason Gallicchio

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12 April 2011
Biggest Motivation: Reject Gluey LHC Backgrounds

Part 1: The Gluon Tagger

Part 2: Finding Pure Samples of Quark and Gluon Jets
Gluon Tagging Motivation

Other than “Wouldn’t it be nice to know?”

Most *new physics* gives *quark* rather than *gluon* jets:
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- 8-jet Gluino event: $pp \rightarrow \tilde{g}\tilde{g}$ and each $\tilde{g} \rightarrow q\bar{q}\chi_1^0 + q\bar{q}\chi_2^0$

Tagging is especially important without $W$, $Z$, $\gamma$, $\ell^\pm$, $B$-Tags, or $E_T$
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Assignment: Your favorite example of gluon or non-$b$ quark signals.
Interesting *standard model physics* also tends to be quark-heavy

- Tops \((t\bar{t} \rightarrow 4\) or 6 quarks\)
- \(W\)'s decaying hadronically (there's no b-tag): \(W^+ \rightarrow u\bar{d}\) or \(c\bar{s}\)
- \(WW \rightarrow 4\) light quarks
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Motivation

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- The generic catchall: “Understand QCD”
- Q & G Jets as backgrounds to boosted top, $W$, $H$, etc.
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**NOTE!**
LEP found $b$-jets look more like gluon jets than light quark jets (in terms of size and particle count)
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Eventually combine Gluon-Tagging with B-Tagging and $\tau$-Tagging
But There’s a Lot of Glue to Get Stuck In

Chance EACH Jet is Quark

\[ p_T \text{ Cut on All Jets (GeV)} \]

So chance that all 4 jets \( \gtrsim 100 \text{ GeV} \) are quark: \( (30\%)^4 = 0.8\% \)
Outline

■ **Biggest Motivation:** Reject Gluey LHC Backgrounds

■ **Part 1:** The Tagger
  ■ Example Observables: Jet Mass and Charged Track Count
  ■ Evaluating the power of observables: Background Rejection
  ■ Familiar presentation of ‘Jet Shape’ and its problems
  ■ Measuring ‘size’ of jets
  ■ Combining 2 or more observables

■ **Part 2:** Finding Pure Samples of Quark and Gluon Jets
There will be *NO* single way to unambiguously sort jets.
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What’s different now than 20 years ago?
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What’s different now than 20 years ago?

- LHC calorimeter spatial resolution. (Sometimes 10x CDF or D0)
- LHC is very jetty and $pp$ is more gluey.
- The most difficult signals are buried under multi-jet events.
Goal

Which observables are useful:
- Individually?
- When combined?
Goal

Which observables are useful:

- Individually?
- When combined?

Interesting variables can be:

- Studied theoretically
- Verified experimentally
- Color Charge: $C_F$ vs $C_A$ → jet mass, size, track count
- Color Connections: 1 vs 2 → eccentricity and pull
- Electrical Charge → charge-weighted track $p_T$
- Spin: 1/2 vs 1 → not explicitly used
- ...

Differences in Quarks vs Gluons
Gluon has a greater effective color charge (squared) than quark:

Gluon adjoint’s $C_A$ vs Quark fundamental’s $C_F$

$$\frac{C_A}{C_F} = \frac{9}{4}$$
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Jet Mass in the small angle limit:

\[
\langle M^2 \rangle = C \frac{\alpha_s}{\pi} p_T^2 R^2
\]

where $C \sim C_A$ for gluon jets, and $\sim C_F$ for quark jets.
Normalizing by $p_T$ (200 GeV in this sample) generalizes better.
Evaluating the Observable: Sliding Cut

mass/Pt

Sliding Cut

Q

G

Q

G
ROC Curve for mass/Pt

Quark Signal Efficiency

Gluon Background Rejection
ROC Curve for mass/Pt

Gluon Background Rejection

Quark Signal Efficiency

50/50
Other Jet Sizes and $p_T$s

Rather than showing $10 \times 6 = 60$ ROC Curves, pick 80% point on each
Gluon adjoint’s $C_A$ vs Quark fundamental’s $C_F$

\[
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\]

Lore: quark jet first emits a gluon, and then it dominates the cascade.
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Multiplicity of any particle in a gluon jet should be $C_A/C_F = 9/4$ times greater (confirmed at LEP).

$$\frac{\langle N_g \rangle}{\langle N_q \rangle} = \frac{C_A}{C_F}$$
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$$\frac{\langle N_g \rangle}{\langle N_q \rangle} = \frac{C_A}{C_F} \quad \frac{\sigma^2_g}{\sigma^2_q} = \frac{C_A}{C_F}.$$ 

(Calculated to $N^3$LO by Capella, et al. hep-ph/9910226)

No detector simulation, but require charged particles $p_T > 500$ MeV.
...this old favorite does quite well at high $p_T$:

Higher $p_T$ means more charged tracks and more ‘time’ to establish $C_A/C_F$. 
Accumulate 3 million back-to-back dijet events

Quark Jets

Gluon Jets

(Same total amount of $p_T$, which is hidden by logarithmic color bands.)
Integrated Jet Shape out to $r = 0.1$ for 100 GeV

- Distribution is *not* narrow gaussian around average
- Correlations *between* bins is also useful
Radial Moment – a measure of the “girth” of the jet

Weight $p_T$ deposits by distance from jet center

Radial Moment, or Girth:  
\[ g = \sum_{i \in \text{jet}} \frac{p_T^i}{p_T^{\text{jet}}} |r_i| \]
Jet Angularities: 

\[ A_a = \sum_{i \in \text{jet}} E_i f_a(\theta) \]

\[ f_a(\theta) = \sin^a \theta (1 - \cos \theta)^{1-a} \]

with \( \theta = \frac{\pi |r_i|}{2R} \) or just \( \theta \)

Normalization? none, mass, jet energy, jet \( p_T \).
Angularities

Angularity vs $a$ for different Jet Sizes

Small $R=0.2$ Jets (red) perform better than Large $R=1.0$ Jets (pink)
Radial Moments and Their Kernels

Linear Moment

Quadratic Moment

Tapered Moment

Tapered Radial Kernel
- Positive kernel weights mean gluon-like.
- Overall vertical shift or scaling leads to same distribution.
- Quarks have most of their $p_T$ near the center.
Optimal Kernel for different $p_T$’s and $R = 1.0$. 

1600 GeV $\rightarrow$ 50 GeV
Types of Variables

The menu, including varying jet size

- Distinguishable particles/tracks/subjets
  - multiplicity, $\langle p_T \rangle$, $\sigma_{p_T}$, $\langle k_T \rangle$,
  - charge-weighted $p_T$ sum

- Moments
  - mass, girth, broadening
  - angularities
  - optimal kernel
  - 2D: pull, planar flow

- Subjet properties
  - Multiplicity for different algorithms and $R_{\text{sub}}$
  - First subjet’s $p_T$, 2nd’s $p_T$, etc.
  - Each subjet’s mass
  - Splitting $k_T$ scale
Best Variables in Each Category

**LHC 200: Background Rejection**

- charged ct $R=0.5$
- subjet ct $R_{sub}=0.1$
- mass/Pt $R=0.3$
- $\cos$ moment $R=0.5$
- ang $a=1$ $R=0.5$
- girth $R=0.5$
- 1st subjet $p_T$
- decluster $k_T$
- optimal $R=1.0$
- pull $\eta$ $R=0.3$
- —pull— $R=0.3$
- planar flow $R=0.3$
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Story So Far...

- Keep only 40% of gluons while keeping 80% quarks
- For multiple jets, these fractions exponentiate
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Where to put the Cut? 80%?
How do we rank the variables?

- Always demand 80% quark, see how much glue you can reject?
- Optimize $S/B$?
- Optimize $S/\sqrt{B}$?
Cutting gives some signal efficiency and some background efficiency.
Cutting and $S/\sqrt{B}$

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If you start with $S$ signal events and $B$ background events,

$$\frac{S}{B} \rightarrow \frac{S\epsilon_s}{B\epsilon_b}$$
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Improvement in statistical significance scales differently

$$\sigma = \frac{S}{\sqrt{B}} \rightarrow \frac{S\epsilon_s}{\sqrt{B\epsilon_b}} = \sigma \frac{\epsilon_s}{\sqrt{\epsilon_b}}$$
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Cutting can improve significance only to a point...
Significance Improvements for Best Single Variables

LHC 200: Significance

charged ct R=0.5
subjet ct R_{sub}=0.1
mass/Pt R=0.3
cos moment R=0.5
ang a=1 R=0.5
girth R=0.5
1st subjet \( p_T \)
decluster \( k_T \)
optimal R=1.0
pull \( \eta \) R=0.3
—pull— R=0.3
planar flow R=0.3
Combining Variables: Ex. Girth vs Charged Count

**Quark**

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<thead>
<tr>
<th>Charged Count</th>
<th>Girth</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>7</td>
<td>0.05</td>
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**Likelihood:** \( q/(q + g) \)
ROC Curves for Best Combinations

LHC 0100: Background Rejection

1: optimal kernel $R=0.3$
1: angularity $a=+1$
1: optimal kernel $R=0.4$
2: optimal and girth
2: $a=+1$ and $<k_T>$
2: optimal and charged count
3: optimal and charged count
3: $a=+1$ and $<k_T>$
3: optimal and charged count
Gluon Background Rejection

Quark Signal Efficiency

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Significance Improvement

Final Results: Best Pairs

Quark Signal Efficiency

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Summary of Gluon Tagging

Can reject 80% of gluons while keeping 80% quarks
Can reject 95% of gluons while keeping 50% quarks
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We can improve $S/\sqrt{B}$ on quark-only signals by a factor of 2x-3x.
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For 4-jets, all of these get raised to the 4th for glue-heavy background. Differences in these variables are worth verifying & calculating.
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- Integrate these variables into FastJet or SpartyJet.
Biggest Motivation: Reject Gluey LHC Backgrounds

Part 2: The Gluon Tagger

Part 2: Finding Pure Samples of Quark and Gluon Jets

- Goal: High Purity with a high Cross Section
- Use MADGRAPH tree-level samples
- Find kinematic variables to cut on
- 2D combinations and/or Multivariate Techniques
- 99% Quark purity from $\gamma + 2$ jets
- 95% Gluon purity from 3 jets
Detailed Multi-Jet Composition

Result of Parton Distribution Functions
X+2jet Composition

\[\gamma+2\text{jets}\]

\[p_T \text{ Cut on All Jets (GeV)}\]

\[\text{QQ, QG, GG}\]

\[\text{b+2jets}\]

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Starting Samples

Chance EACH Jet is Quark

- $\gamma + 1j$
- $Z/W + 1j$
- $Z/W + 2j$
- $\gamma + 2j$
- $b + 2j$
- $bb + j$
- $p_T$ Cut on All Jets (GeV)
Kinematics are too similar to do much:

- \( gq \rightarrow \gamma q \): Quark signal
- \( q\bar{q} \rightarrow \gamma g \): Gluon background
When the softer jet is quark, the photon is often radiated off of it, rather than the harder jet.
Other useful kinematics (?)
Quark Purification in $\gamma+2$jet: Look at Softer Jet

2D version of the same

![Quark and Gluon Distributions](image.png)
Quark Purification in $\gamma+2\text{jet}$: Look at Softer Jet

2D version of the same

**Quark**

**Gluon**

**Likelihood**
Approximating the Likelihood Contours with $f(x, y) = xy$
Approximating the Likelihood Contours with $f(x, y) = xy$
Quark Purification in $\gamma+2\text{jet}$: Look at Softer Jet

Do it again to find an even better combination

$$\eta_\gamma \eta_{j_1} + \Delta R_{\gamma j_2}$$
Automating the process with Boosted Decision Trees

Some totally crazy illustrative example from my Higgs+Z work:
Automating the process with Boosted Decision Trees

Root node

- $x_i > c_1$
- $x_i < c_1$

- $x_j > c_2$
- $x_j < c_2$

- $x_j > c_3$
- $x_j < c_3$

- $x_k > c_4$
- $x_k < c_4$

- B
- S

- S
- S

- B
Do Not Fear Boosted Decision Trees

BDT 2

BDT 8

BDT 32

BDT 64

BDT 256

Likelihood

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Quark Purity for Different $p_T$

Cross Section in pb

70% 80% 90% 100%

Quark Purity

50 γjj
100 γjj
200 γjj
400 γjj
800 γjj
1600 γjj
100 γj
50 γj
200 GeV Gluon Purity

Cross Section in pb vs. Gluon Purity

- 2jet
- b+jet
- 3jet softest
- 3jet mid
- 3jet hardest
- b+2jet hardest
- b+2jet softest

Logarithmic scale for Cross Section in pb

Percentages along the x-axis:
- 50%
- 60%
- 70%
- 80%
- 90%
- 100%
Best Samples for Gluon Purity

Cross Section in pb

Gluon Purity

0% 20% 40% 60% 80% 100%

50 jj
100 jj
200 jj
400 jj
800 jj
1600 jj
50 jjj
100 jjj
200 jjj
400 jjj
800 jjj
50 bjj
100 bjj
200 bjj
400 bjj
800 bjj
- **Quark** samples at 99% purity for $\gamma+$jet
- **Gluon** samples at 90%-95% purity for 3jets
Summary of Finding Samples

- **Quark** samples at 99% purity for $\gamma$+jet
- **Gluon** samples at 90%-95% purity for 3jets

- **Gluon** samples at 95%-99% purity for $b$+2jets with perfect B-Tagging and B-Anti-Tagging
Summary of Finding Samples

- **Quark** samples at 99% purity for $\gamma+\text{jet}$
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- **Gluon** samples at 95%-99% purity for $b+2\text{jets}$ with perfect B-Tagging and B-Anti-Tagging

Now go forth and use these tools for good.... Thanks!
In case waving my hands proves insufficient ...
- Number of charged tracks
- Jet Shape (shown)
- LEP $e^+e^- \rightarrow Zg \rightarrow b\bar{b}g$
Jet $p_T$: anti-$k_T$ $R=0.5$

Hard Parton $p_T$
Covariance Tensor: \[ C = \sum_{i \in \text{jet}} \frac{p_T^i}{p_T^{jet}} \begin{pmatrix} \Delta \eta_i \Delta \eta_i & \Delta \eta_i \Delta \phi_i \\ \Delta \phi_i \Delta \eta_i & \Delta \phi_i \Delta \phi_i \end{pmatrix} \]
Covariance Tensor:
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Combination of Eigenvalues
- Girth: \( g = \sqrt{a^2 + b^2} \)
- Determinant: \( \text{det} = a \cdot b \)
- Ratio: \( b/a \)
- Eccentricity: \( \epsilon = \sqrt{a^2 - b^2} \)
- Planar Flow: \( \text{pf} = \frac{4ab}{(a+b)^2} \)
- Orientation: \( \theta \)

Not useful for Q vs G: first emission sets this shape, and has similar 2-body kinematics.
Subjets – Smaller is Better

- Subjet Algorithm: anti-$k_T$, CA, $k_T$
- Subjet Size: Darkest is $R_{sub} = 0.1$, lightest $R_{sub} = R_{jet}$

Teamwork & Collaboration

Gluon Tagging and Quark & Gluon Samples

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(components_jet0_ak_sub__count)

(Background Rejection at 50% Quark vs Initial Jet Size)
Explosion of Variables

- Different Jet sizes ($R = 0.1, 0.2, 0.4, 0.7, 1.0, 1.4, ...$)
- Different Jet definitions (anti-kt, kt, CA, SisCone)
- Different Generators: Pythia vs Herwig
- Different Samples: Dijet vs $\gamma$+jet vs 8-Jets
- Different Subjet sizes and types
- Different Powers in the various moments
- Charged Tracks or Calorimeter deposits?

And different variables are better for different Jet $p_T$ ranges.
Jet Broadening similar to linear moment for small-angles: \( k_T \approx p_T r \)

\[
B_{\text{jet}} = \frac{\sum_i |\vec{p}_i \times \hat{n}_{\text{jet}}|}{\sum_i |\vec{p}_i|} = \frac{\sum_i |\vec{k}_T i|}{\sum_i |\vec{p}_i|}
\]
Compare to ATLAS B-Tagging

(a) Non-purified light jets

(b) Purified light jets

(c) c-jets

(d) τ-jets
Gluon Purification: Lesson about Harsh Cuts

\[ \eta_{j3} \]

![Plot of \( \eta_{j3} \)]

\[ |\eta_{j3}| - |\eta_{j1} - \eta_{j2}| \]

![Plot of \( |\eta_{j3}| - |\eta_{j1} - \eta_{j2}| \)]
Trijet Sample with Different Kinematic Cuts

Cross Section in pb

Gluon Purity

800 jjj

50 jjj

400 jjj

200 jjj

100 jjj

η_{j3}

|η_{j3} - |η_{j1} - η_{j2}|