NR effective theory for DM direct detection

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1008.1591(hep-ph)
Outline

- Overview of dark matter detection
- Motivation
- NR effective theory of DM direct detection
  - Scales and power counting rules
  - Examples of matching
- Recoil spectra
- Constraints from current direct detection experiments
- Conclusion
Overview

Measurement from CMB + supernovae + LSS indicates 23% of our universe is composed of DM;

Three ways to detect DM:

Direct detection  Collider production  Indirect detection
Overview

Measurement from CMB + supernovae

+LSS indicates 23% of our universe is composed of DM;

Three ways to detect DM:

Direct detection  Collider production  Indirect detection
Direct detection looks for signals from DM scattering off nucleus in the underground detector.

Dark matter

$v/c \approx 7 \times 10^{-4}$

$E_r \approx 10's$ KeV
**Direct detection kinematic regime**

\[ E_{\text{kin}} \sim \mathcal{O}(10 \text{ keV}) \]

\[ E_R \sim \mathcal{O}(1 - 10 \text{ keV}) \]

The scattering against the whole nucleus is described by non-relativistic quantum mechanics.
Current direct detection status

Direct detection rate

\[
\frac{dR}{dE_R} = N_T \frac{\rho \chi}{m_\chi} \int_{v_{\text{min}}}^{v_E} d^3v v f(v, v_E) \frac{d\sigma}{dE_R}
\]

Traditionally, signals are assumed to come from contact interactions independent of momentum transfer. For instance, for SI scattering,

\[
\frac{d\sigma}{dE_R} = \frac{m_N}{2v^2} \frac{\sigma_n}{\mu_n^2} A^2 F^2(E_R)
\]

Experimental results are presented as bounds on \( \sigma_n \) as a function of DM mass.
Spin-independent elastic scattering

DATA, listed up to bottom on plot:
- XENON1T: measured LSS from Xe cube
- CDMS: Sandia 2004-2009 Ge
- XENON100 2010 (95 kg·d)
- XENON10 projected sensitivity: 30 kg·d, 5-50 keV, 45% eff.
- XENON100 projected sensitivity: 60 kg·d, 5-30 keV, 45% eff.
- XENON100 projected sensitivity: 60 kg·d, 5-30 keV, 45% eff.
- XENON1T projected sensitivity: 3 ton·yr, 2-30 keV, 45% eff.
Continued: spin-dependent scattering
Summary

The current bounds on xsec per nucleon are

SI scattering: \((10^{-43} - 10^{-44}) \text{cm}^2\)

SD scattering: \((10^{-37} - 10^{-38}) \text{cm}^2\)
Motivation

How to extract more information about the underlying dark dynamics from the experimental data?

The simple assumption of contact interaction between DM with nucleons overlooks direct detection’s sensitivity to more general DM scenarios.

E.g: Momentum-dependent DM (Chang, Pierce and Weiner; Feldstein, Fitzpatrick and Katz 2009);

Dark electromagnetic moments (Chang, Weiner and Yavin; Barger, Keung and Marfatia; Fitzpatrick and Zurek; Banks, Fortin and Thomas 2010)
Top-down view: (from theory to direct detection experiment)

**UV complete DM Models** predict relic abundance, direct detection signals in terms of specific model parameters

**Effective field theory operators** (Kurylov and Kamionkowski 2003; Agrawal, Chacko, Kilic and Mishra 2010)

**Non-relativistic quantum mechanics theory**

**Model independent analysis**
Different models/effective ops lead to the same NR interaction for direct detection.

E.g:  Higgs exchange: \( \bar{\chi} \chi q \bar{q} q \)

Z exchange: \( \bar{\chi} \gamma^\mu \chi q \bar{q} \gamma_\mu q \)

gives the contact interaction in the NR limit.

Measured recoiling rate directly bounds the coupling of contact interaction.
From direct detection experiments to theories:

- Direct detection
- Non-relativistic effective potential
- Effective field theory operators
- UV complete DM Models
NR effective potential

- **Scales and simple power counting**

  Transferred momentum: $|q| \sim 100$ MeV

  Nucleus mass: $m_N \sim 10 - 100$ GeV

  DM mass: $m_\chi \sim 100$ GeV – 1 TeV

  Mediator mass: $m_o$ unfixed

- **Other scales:** e.g. DM-mediator interaction arises at nonrenormalizable level

  DM with electric dipole moment

\[
\frac{\bar{\chi} \sigma_{\mu \nu} \gamma_5 \chi F^{\mu \nu}}{\Lambda}
\]
Continued

Expansion parameters:

\[ \nu \sim 10^{-3} \]
\[ \frac{q}{m_N} \sim \frac{q}{m_\chi} \sim 10^{-3} \]
\[ \frac{q}{m_0}, \frac{q}{\Lambda} \text{ unfixed, can be as large as 0.1} \]

---

**SI experimental bounds**

\[ \sigma \sim 10^{-44} \, cm^2 \]

**SD experimental bounds**

\[ \sigma \sim 10^{-38} \, cm^2 \]

compared to a typical weak process xsec

\[ \sigma_W \sim 10^{-36} \, cm^2 \]

|q| suppressed operator can still be relevant if they are the leading operator for direct detection.
Consider two limits of mediator masses $m_0$:

a. $m_0 \gg |q|$  \hspace{1cm} \text{Contact interaction}

b. $m_0 \ll |q|$  \hspace{1cm} \text{Long-range interaction}

Assume all expansion parameters of order $10^{-3}$

a. Contact interaction: Operators suppressed by a single $|q|$

b. Long-range interaction: Operators suppressed by $|q|^3$

may still be relevant for direct detection if they are the leading operator.
Effective NR potential

\[
V_{\text{eff}} = V_{\text{eff}}^{\text{SI}} + V_{\text{eff}}^{\text{SD}}
\]

\[
V_{\text{eff}}^{\text{SI}} = h_1 \delta^3(\vec{r}) - h_2 \vec{s}_\chi \cdot \vec{\nabla} \delta^3(\vec{r}) + l_1 \frac{1}{4\pi r} + l_2 \frac{\vec{s}_\chi \cdot \vec{r}}{4\pi r^3}
\]

\[
V_{\text{eff}}^{\text{SD}} = h'_1 \vec{s}_\chi \cdot \vec{s}_N \delta^3(\vec{r}) - h'_2 \vec{s}_N \cdot \vec{\nabla} \delta^3(\vec{r}) + l'_1 \frac{\vec{s}_\chi \cdot \vec{s}_N}{4\pi r} + l'_2 \frac{\vec{s}_N \cdot \vec{r}}{4\pi r^3}
\]
In the Born approximation, the matrix element of the scattering is
\[ \mathcal{M}(\vec{q}, \vec{v}) = -\int d^3\vec{r} e^{i\vec{q} \cdot \vec{r}} V_{\text{eff}}(\vec{r}, \vec{v}) \]

For the numerical studies, we only consider elastic scattering;

Only list static potential, or, \( v \)-independent potential;

\( v \)-dependent potential produces nearly identical recoil spectrum to the static one for the elastic scattering;

Only list potential leads to interaction suppressed by a single \( |q| \);

The coefficients are dimensionful; direct detection bounds on combinations of couplings and scales.

We factor out the nuclear form factor and suppress the spin indices.
Example of matching: Femionic DM(SI)

**SI NR operator complete set:** up to a scalar function $f(q^2, v^2)$

In general, four building blocks to construct rotational invariants

<table>
<thead>
<tr>
<th>Momentum space (w/o mediator)</th>
<th>Position space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{O}_1^{(++)}$</td>
<td>$\mathcal{O}_1$ = $\delta^3(\vec{r})$, $\frac{1}{4\pi r}$</td>
</tr>
<tr>
<td>$\mathcal{O}_2^{(--)}$</td>
<td>$\mathcal{O}<em>2$ = $\vec{s}</em>\chi \cdot \vec{P}$</td>
</tr>
<tr>
<td>$\mathcal{O}_3^{(++)}$</td>
<td>$\mathcal{O}<em>3$ = $(\vec{s}</em>\chi \times \vec{v}) \cdot \delta^3(\vec{r})$, $\frac{(\vec{s}_\chi \times \vec{v}) \cdot \vec{r}}{4\pi r^3}$</td>
</tr>
<tr>
<td>$\mathcal{O}_4^{(--)}$</td>
<td>$\mathcal{O}<em>4$ = $(\vec{s}</em>\chi \times \vec{v}) \cdot \delta^3(\vec{r})$, $\frac{(\vec{s}_\chi \times \vec{v}) \cdot \vec{r}}{4\pi r^3}$</td>
</tr>
</tbody>
</table>

$m, v, s$ \chi, $\vec{s}_N$

$\mathcal{O}_1^{(++)} = 1$

$\mathcal{O}_2^{(--)} = i\vec{s}_\chi \cdot \vec{q}$

$\mathcal{O}_3^{(++)} = i\vec{s}_\chi \cdot (\vec{P} \times \vec{q})$

$\mathcal{O}_4^{(--)} = i\vec{s}_\chi \cdot (\vec{P} \times \vec{q})$

$(\vec{P} = \mu_N \vec{v} + \vec{q}/2)$
Mapping of effective field theory operators to the NR potential

\( O_{1}^{(++)} = 1 \)

\( O_{2}^{(-+)} = i \vec{s}_\chi \cdot \vec{q} \)

\( O_{3}^{(--)} = \vec{s}_\chi \cdot \vec{P} \)

\( O_{4}^{(++)} = i \vec{s}_\chi \cdot (\vec{P} \times \vec{q}) \)

\( \bar{\chi} \chi \bar{q} q, \quad \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q \)

\( \bar{\chi} \gamma^5 \chi \bar{q} q, \quad \bar{\chi} \sigma^{\mu \nu} \gamma^5 D_\mu \chi \bar{q} \gamma_\nu q \)

\( \bar{\chi} \gamma^5 \chi \bar{q} q, \quad \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q \)

\( \bar{\chi} \sigma^{\mu \nu} D_\mu \chi \bar{q} \gamma_\nu q \)
Examples of simple theories for each potential:

\[ \delta^3(\vec{r}) \]  
Higgs exchange

\[ \vec{s}_\chi \cdot \vec{\nabla} \delta^3(\vec{r}) \]  
DM dark EDM off nucleus charge

\[ \frac{1}{r} \]  
exchange of a light boson

\[ \frac{\vec{s}_\chi \cdot \vec{r}}{r^3} \]  
DM EDM off nucleus charge
Recoil spectrum

The NR theory highlights the possibility of having qualitatively different recoil energy spectrum.

<table>
<thead>
<tr>
<th>SI NR operators</th>
<th>SD NR operators</th>
<th>$E_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^3(\vec{r})$</td>
<td>$\vec{s}_N \cdot \vec{\nabla} \delta^3(\vec{r})$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\vec{s}_X \cdot \vec{\nabla} \delta^3(\vec{r})$</td>
<td>$\vec{s}_X \cdot \vec{s}_N \delta^3(\vec{r})$</td>
<td>$E_R$</td>
</tr>
<tr>
<td>$\frac{1}{4\pi r}$</td>
<td>$\frac{\vec{s}_X \cdot \vec{s}_N}{4\pi r}$</td>
<td>$E_R^{-2}$</td>
</tr>
<tr>
<td>$\frac{\vec{s}_X \cdot \vec{r}}{4\pi r^3}$</td>
<td>$\frac{\vec{s}_N \cdot \vec{r}}{4\pi r^3}$</td>
<td>$E_R^{-1}$</td>
</tr>
</tbody>
</table>
Sample Spectra

Sample spectrum for Germanium.

Left: spectrum with contribution from one operator;

Right: spectrum with contributions from two operators;
Constraints from direct detection (CDMS, Xenon10, Xenon100 for SI direct detection)

\[ \delta^3(\vec{r}) \quad h_1 \lesssim 10^{-8} \text{ GeV}^{-2} = \frac{10^{-4}}{(100 \text{ GeV})^2} \]

\[ \vec{s}_\chi \cdot \vec{\nabla} \delta^3(\vec{r}) \quad h_2 \lesssim 10^{-7} \text{ GeV}^{-3} = \frac{10^{-1}}{(100 \text{ GeV})^3} \]

\[ \frac{1}{r} \quad l_1 \lesssim 10^{-11} \]

\[ \frac{\vec{s}_\chi \cdot \vec{r}}{r^3} \quad l_2 \lesssim 10^{-9} \text{ GeV}^{-1} = \frac{10^{-7}}{(100 \text{ GeV})}. \]
Example 1 \[ \delta^3(\vec{r}) \quad h_1 \lesssim 10^{-8} \text{GeV}^{-2} = \frac{10^{-4}}{(100 \text{ GeV})^2} \]

Higgs exchange: \[ y_s \sim 10^{-4} \]

Z exchange: \[ \bar{\chi} \gamma^\mu \chi h^\dagger D_{\mu} h \quad (\nu_{EW}/\Lambda)^2 \sim 10^{-4} \]

Example 2 \[ \vec{s}_\chi \cdot \vec{r}/r^3 \quad l_2 \lesssim 10^{-9} \text{GeV}^{-1} = \frac{10^{-7}}{(100 \text{ GeV})} \]

(Pseudo)scalar exchange: \[ l_2 \sim g/m_\chi \quad g \lesssim 10^{-7} \]

DM dipole moment: \[ l_2 = d \quad d \lesssim 10^{-23}(e \cdot cm) \]
**Bound on contact interaction**

**momentum independent**

**momentum dependent**

Black: CDMS; Green: Xenon 10; Purple: Xenon100
**Bound on long range interaction**

**Coulomb potential**

**Dipole potential**

Xenon 10 has strongest bound as it has the lowest energy threshold;

Threshold calibration is crucial for not only light DM but also long range interaction mediated by light bosons.
Conclusion

We present a model-independent framework based on NR operators to analyze data from direct detection.

If near future direct detection sees DM, it will not only shed information on DM mass, overall scattering cross section but also DM interaction from recoiling spectrum.
Thank you!