Black hole determinants, quasinormal modes and the de Haas - van Alphen effect

Frederik Denef

Harvard

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with Sean Hartnoll and Subir Sachdev

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Summary

- Motivation:
  - Textbook result: de Haas - van Alphen oscillations probe structure of Fermi surface
  - What if there is no standard Landau Fermi liquid description?
  - Physical realization: high T_c superconductors in *normal* state
  - Exactly solvable model: large N CFT through holography

Summary

- Motivation:
  - Textbook result: de Haas - van Alphen oscillations probe structure of Fermi surface
  - What if there is no standard Landau Fermi liquid description?
  - Physical realization: high T_c superconductors in *normal* state
  - Exactly solvable model: large N CFT through holography
- Results:
  - Formula for 1 loop partition function in terms of quasinormal modes (dHvA is 1/N effect, ie 1-loop on gravity side)
  - dH-vA oscillations with same period but qualitatively different $T = 0$ behavior
  - Actual realizations: M-theory / Sasaki-Einstein 7-manifolds

Outline

1. Background
   - de Haas - van Alphen effect
   - Non-fermi liquids
   - AdS-CFT at finite T, μ and B
   - AdS-CFT at finite N
   - Quasinormal modes
2. AdS-BH determinants from quasinormal modes
3. de Haas - van Alphen from AdS-CFT
4. Models from string theory compactifications

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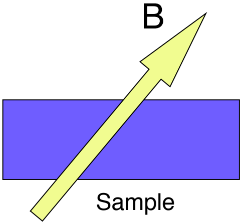
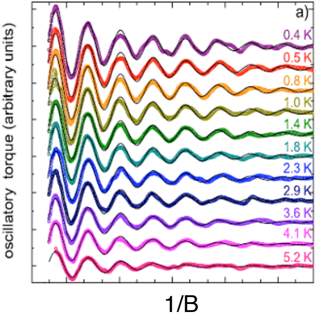
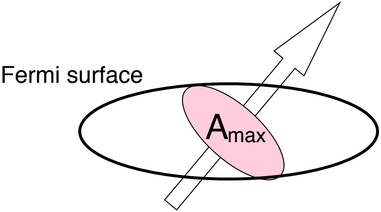
de Haas - van Alphen effect







$$\Delta\left(\frac{1}{B}\right) = \frac{\hbar}{A_{\max}}$$

Explanation: Landau level quantization

- Electron in magnetic field $B\,\hat{z}$:

$$\dot{p}_x = -B\dot{y}, \qquad \dot{p}_y = B\dot{x}$$

- Bohr-Sommerfeld quantization momentum space area

$$n\hbar = \oint p_x dx + p_y dy = \frac{1}{B}\oint p_x dp_y - p_y dp_x = \frac{A}{B}$$

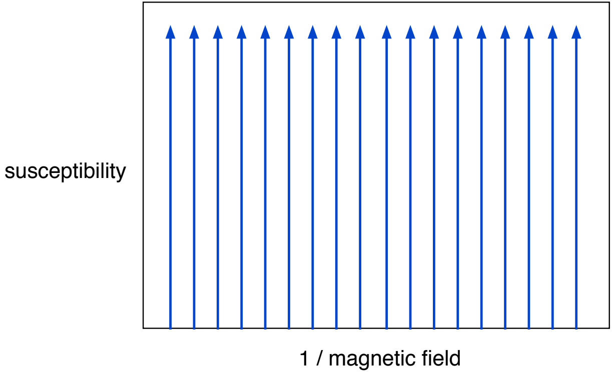
$\Rightarrow$ Fermi sea becomes collection of Fermi cylinders aligned with B, cross sectional areas quantized in units of $\hbar B$ (= Landau levels).

- Increasing B: shells pushed out of Fermi energy surface $E(k) = \mu$.
- Complete shell pushed out each time integral multiple of $\hbar B$ equals maximal cross sectional area $A_{\max}$ of $E(k) = \mu$ surface:

$$\frac{1}{B} = \frac{\hbar}{A_{\max}}\, n \, .$$

Zero temperature behavior in d=2

If effective description of weakly coupled (quasi)particles exist:



Delta function peak each time LL trajectory pushed out of $E(k) = \mu$.

Non-Fermi liquids

- Free fermion gas: Fermi sea
- Landau Fermi liquid theory: effectively Fermi surface with weakly interacting quasiparticle excitations. Dispersion relations:
  - $E \propto \delta k_\perp$
  - $\Gamma \propto \delta k_\perp^2$
- General RG argument explains success: effective theory IR free (except marginal BCS interaction)
- Nevertheless $\exists$ deviations: non Fermi liquids = non-Landau:
  - still Fermi surface in sense of existence of gapless fermionic excitations at sharp momentum shell.
  - but behavior different from Landau liquid, e.g. different dispersion relations, temperature dependence of resistivity, specific heat, ...

Phenomenology

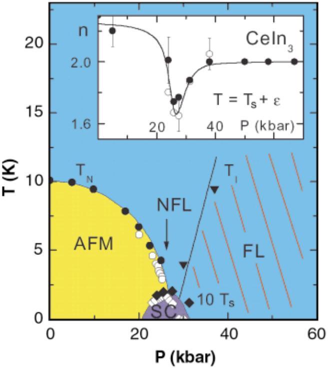


Fig. 59: (T, P) phase diagram of the antiferromagnet $CeIn_3$. T_N and T_I are respectively the Néel temperature and the crossover temperature to the FL regime. The full Symbols are CEA Grenoble data, the open ones the Cambridge results [3]. The inset shows the pressure variation of the exposant n derived from the low temperature fitting of the electrical resistivity, $\rho = \rho_0 + A_n T^n$. NFL behaviour ($n \neq 2$) is observed just at P_c.

Strongly-correlated Systems

Review by J.P. Sanchez

- Strong electron interactions $\rightsquigarrow$ NFL behavior
- Near quantum critical point $\rightsquigarrow$ interacting CFT in IR.
- Same strong interactions $\rightsquigarrow$ unconventional superconductivity.

Theoretical model?

We want exactly solvable model in $d = 1 + 2$, retaining following elements:

- Strongly interacting.
- Perturbation of quantum critical point by nonzero temperature, charge density, magnetic field.

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$\rightsquigarrow$ AdS-CFT

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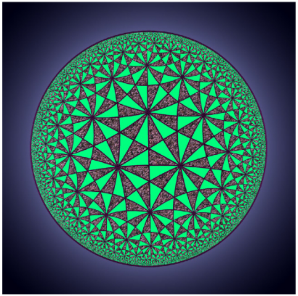
$\rightsquigarrow$ AdS-CFT

We want to ask in particular:

- Are there still de Haas - van Alphen oscillations?
- If so, what are differences with Landau liquid theory?

AdS-CFT at finite T, μ and B



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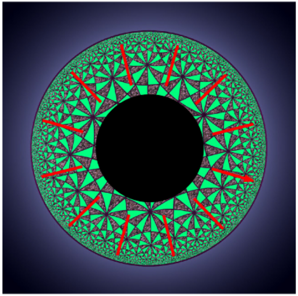


- Pure AdS_4 = quantum critical point described by CFT_3
- Dyonic black hole deforms away from QCP by turning on

$$T = T_H, \qquad \mu = A_0, \qquad B = B$$

AdS-CFT at finite N

- We will see: dH-vA effect only arises at subleading order in $1/N$.
- CFT partition function to subleading order in $1/N$ expansion = gravity partition function to subleading order in $\hbar$ (loop) expansion:

$$Z \approx \left(\det S''[\phi_{\rm cl}]\right)^{\#} \, e^{-S[\phi_{\rm cl}]}$$

  Here $S'' = D$ is typically operator of Laplace type, $\det D = \prod_i \lambda_i$.
- For massive fields: universal expression for local contributions in terms of curvature invariants, from heat kernel expansion.
- dHvA determined by nonlocal contributions: harder, but UV finite, so do not need full string theory.

Quasinormal modes

- Long history in astrophysics.
- For us: nonlocal 1-loop effects captured by quasinormal modes.
- E.g. Schwarzschild-AdS:

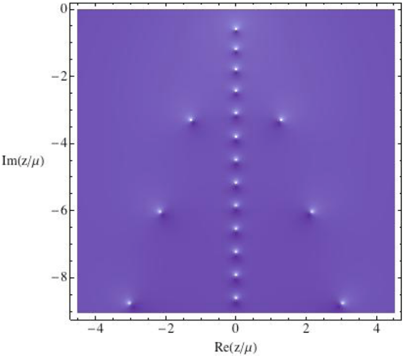
$$ds^2 = -V(r)\,dt^2 + \frac{1}{V(r)}dr^2 + r^2\,d\Omega_2^2\,, \qquad V(r) = 1 - \frac{M}{r} + \frac{r^2}{L^2}$$

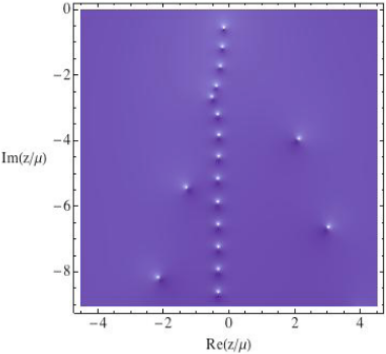
- Scalar field QNMs: solutions to eom $D\phi = 0$ in this background, with $\phi(r) \sim r^{-\Delta}$ b.c. at $r \to \infty$ and infalling b.c. at horizon $r \to R$, i.e.

$$\phi(r) \sim e^{-i\omega(x+t)}\,, \qquad x = \log(r-R)^{1/2}\,.$$

- Like a resonance in QM: ω complex; stability requires $\mathrm{Im}\,\omega < 0$.
- CFT: poles in G^{ret}, excitations at finite T, lifetime $\sim 1/|\mathrm{Im}\,\omega|$

Example: QNMs of charged scalars in RN-AdS





$T/\mu = 0.075,\ q = 0, 1$ respectively.

Determinants from quasinormal modes

- Wick rotate $\tau = it$. Work in units with $T_H = 1/2\pi$.
- Near horizon geometry: $ds^2 = \rho^2 d\tau^2 + d\rho^2 + ds_\perp^2$.
- Solutions eom $D\phi = 0$ for $\rho \to 0$:

$$\phi_{\omega_\star} \sim \rho^{-i\omega_\star}\, e^{-\omega_\star \tau}\,, \qquad \phi_{\tilde{\omega}_\star} \sim \rho^{+i\tilde{\omega}_\star}\, e^{-\omega_\star \tau}$$

  Lorentzian cont.: correspond to ingoing resp. outgoing boundary conditions $\rightsquigarrow$ $\omega_\star$ = QNM, $\tilde{\omega}_\star$ = anti-QNM frequencies.
- Then we claim, by matching poles and zeros:

$$\det D = e^{P(\Delta)} \prod_{\omega_\star} \prod_{n \geq 0} (\omega_\star - in) \prod_{\tilde{\omega}_\star} \prod_{n<0} (\tilde{\omega}_\star + in)$$

  where $P(\Delta)$ is polynomial in Δ, containing only local contributions, obtained from heat kernel expansion coefficients.

More specialized formulae

If background is PT invariant (still with $\beta = 2\pi$):

$$\det D = e^P \prod_{\omega_\star} |\omega_\star| |\Gamma(i\omega_\star)|^2 \,.$$

If background has no horizon so $\omega_\star$ are normal modes:

$$(\det D)^{-1} = e^P \prod_{\omega_\star} \frac{e^{-\pi|\omega_\star|}}{1 - e^{-2\pi|\omega_\star|}} = \mathrm{Tr}\, e^{-2\pi H} \,.$$

Example: AdS_3 / BTZ

- AdS_3: $\omega_\star = \pm\frac{2n+\ell+\Delta}{L}$

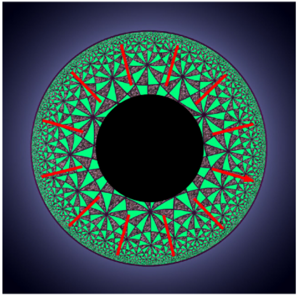
$$\det D = e^{-\int d^3x\sqrt{g}\,\frac{(\Delta-1)^3}{12\pi L^3}-\frac{(\Delta-1)^2\Lambda}{8\pi^{3/2}L^2}}\prod_k \left(1-q^{k+\Delta}\right)^{k+1},$$

where $q = e^{2\pi i\tau}, \tau = \frac{i}{2\pi LT}$.

- BTZ: $\omega_\star = \pm\frac{p}{L} - 2\pi i T(\Delta + 2n)$,

same result with $\tau \to -1/\tau$.

AdS-CFT at finite T, μ and B



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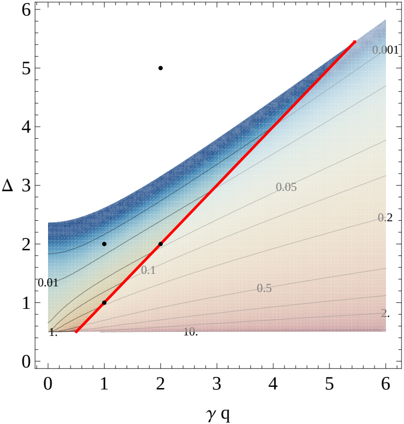
Minimal model

Planar Dyonic black hole in theory with

- gravity with negative cosmological constant
  $\rightsquigarrow c \sim L^2/G_N$.
- Maxwell $U(1)$
  $\rightsquigarrow \rho \sim g^2$.
- Minimally coupled charged scalar
  $\rightsquigarrow$ dual to possible 'Cooper boson' leading to superconducting phase
- Minimally coupled charged fermion
  $\rightsquigarrow$ dual to possible gapless fermionic excitations of Fermi surface

Normal vs superconducting phase: T_c/μ plot



- Δ = conformal dimension operator dual to scalar ($m^2 = \Delta(\Delta - 3)/L^2$)
- q = quantized charge scalar
- $\gamma = \sqrt{c\rho} = g\,L/\sqrt{G_N}$

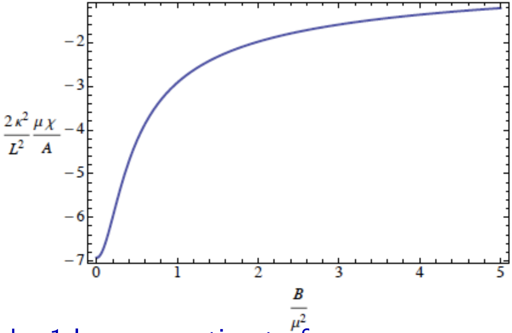
No dH-vA oscillations at leading order

- Susceptibility to leading order:

$$\chi = -\partial_B^2 F = -T\partial_B^2 S_{\rm cl}$$

where $S_{\rm cl}$ is the classical Euclidean dyonic black hole action.

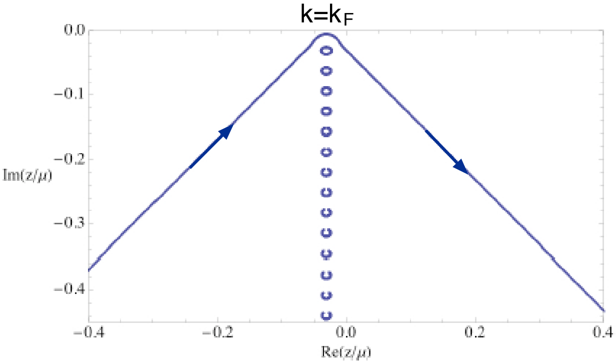
- This does not oscillate:



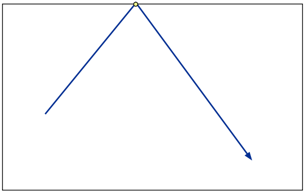
$\Rightarrow$ must consider 1-loop correction to free energy.

Fermionic quasinormal modes

- [Faulkner-Liu-McGreevy-Vegh]: At $B = 0$, particular quasinormal frequencies of spinor field show interesting low T behavior as function of transverse planar momentum k:



Fermionic quasinormal modes at $T = 0$



- In $T \to 0$ limit: $\omega(k) \sim (k - k_F)^z - i\,(k - k_F)^\delta$ where exponents z and δ depend in simple way on effective AdS_2 conformal dimension

$$\nu = \sqrt{\Delta^2 + \frac{k_F^2}{\mu^2} - \gamma^2 q^2}$$

- At $k = k_F$: gapless quasiparticle excitations $\rightsquigarrow$ Fermi surface
- Expoinents different from Landau liquid $z = 1$, $\delta = 2$.

$T = 0$ oscillations at 1 loop

- Behavior QNMs at nonzero B: just replace $k^2 \to nB$, $n \in \mathbb{Z}$.
- From expression 1-loop free energy in terms of QNMs, we extract $T = 0$ nonalyticity χ:
  - Singularity occurs whenever

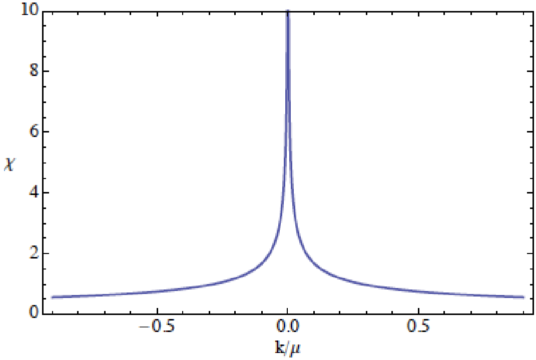
$$\frac{1}{B} = \frac{n}{k_F^2} \quad \Rightarrow \quad \Delta\left(\frac{1}{B}\right) = \frac{1}{A_F} \quad \checkmark$$

  - Exponent of divergence:

$$\chi \sim \delta B^{-2+\frac{1}{2\nu}}$$

    different from delta functions of free case.

Periodic $T = 0$ divergence suscpetibility



Models from string theory compactifications

M-theory on Sasaki-Einstein 7-manifolds [FD-Hartnoll]:

- SE_7 = base CY_4 cone $\rightsquigarrow$ huge landscape $\rightsquigarrow$ distributions, ...
- Dual to $\mathcal{N} = 2$ M2 SCFTs.
- Consistent truncation to $U(1)_R$ Maxwell + Einstein (!).
- Identified modes leading to minimal models (!!).
- Superconducting phase at $T = 0$ generic for 'skew-whiffed' orientation SE_7, possibly generic for susy orientation.
- Fermi surface properties not yet investigated (normal phase at sufficiently low T?)

Conclusions

- AdS-CFT leads to tractable models of non-Fermi liquid behavior
- de Haas - van Alphen oscillations persist whenever there is a Fermi-surface ($T = 0$ gapless modes at $k = k_F$).
- Same period, different amplitudes (see also finite T results [Hartnoll-Hoffman]).
- Determinant formula, possibly of broader use.
- Sasaki-Einstein compactifications has right ingredients to give string realizations.