Orientiholes

Frederik Denef, Mboyo Esole and Megha Padi, arXiv:0901.2540
Outline

Motivation and basic idea

Review of $\mathcal{N} = 2$ black hole bound states

Type IIA orientiholes
Motivation and basic idea
The IIB (F-theory) landscape

Central in modern string phenomenology (Fenomenology)

- GUT model building [Beasley-Heckman-Vafa,...]
- models of inflation [Baumann-Dymarsky-Klebanov-McAllister,...]
- moduli stabilization [Kachru-Kallosh-Linde-Trivedi,...]
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- Missing: global picture
  - genuine, fully consistent compactifications
  - systematics
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- In principle simple [Vafa]:
  F-theory compactification with 4 susies in 4 dim
  ⇔
  elliptically fibered CY 4-fold + 4-flux
All CY4 hypersurfaces in weighted $\mathbb{C}P^5$

Total number $= 1,100,055$ [Lynker-Schimmrigk-Wisskirchen]:

$$h_+ \equiv h^{3,1} + h^{1,1} \text{ versus } h_- \equiv h^{3,1} - h^{1,1}$$

$$N_{D3} = \frac{\chi}{24} \approx \frac{h_+}{4}.$$
Elliptically fibered subset

At least 49,751:

\[ h_+ \equiv h^{3,1}_+ + h^{1,1}_+ \text{ versus } h_- \equiv h^{3,1}_- - h^{1,1}_- \]
Number of flux vacua

Continuum estimate for number of vacua for fixed CY4 within region $\mathcal{M}$ of complex structure moduli space [Ashok-Denef-Douglas]:

$$N_{\text{vac}} = \text{Vol} \left[ S^{b_4} \big| R^2 = \frac{\chi}{12} \right] \int_{\mathcal{M}} e(D)$$

where

$$\text{Vol} \left[ S^{b} \big| R^2 = \frac{\chi}{12} \right] = \frac{(\pi R^2)^{b}}{(b/2)!}$$

Example with largest $\chi$:

$$w = (1, 1, 84, 516, 1204, 1806) , \quad h^{3,1} = 303148 , \quad h^{1,1} = 252 , \quad b_4 = 1, 819, 942 , \quad \chi = 1, 820, 448$$

has

$$N_{\text{vac}} \propto \text{Vol} = 10^{139598}.$$
CY3 orientifold with

- O7 + O3
- D7 + D3
- RR + NSNS 3-flux + worldvolume 2-flux
Weakly coupled IIB picture

[Sen]

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Virtually all vacuum degeneracy arises from D-brane d.o.f.
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  - D7 + D3
  - RR + NSNS 3-flux + worldvolume 2-flux

- Virtually all vacuum degeneracy arises from D-brane d.o.f.

- Constructing all = intractable. Instead: how many D-brane vacua in different sectors? [Douglas]
Problems with conventional approach

- ADD-formula far outside of regime of asymptotic validity
  \[ Q_{D3} \gg b_4 \text{ (because } Q_{D3} \sim \frac{b_4}{24} \text{)} \]

- D7-D3 bound states not taken into account

- No systematic enumeration of different sectors of D7 configuration space

- Even more basic issues such as K-theory constraints and D-term stability have not systematically been addressed.
Basic idea

Start with IIB O7/O3 orientifold (no bulk flux) and perform following procedure:
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$$IIB \to IIA, \quad D7/D3 \to D4/D0, \quad O7/O3 \to O4/O0.$$
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   $$IIB \to IIA, \quad D7/D3 \to D4/D0, \quad O7/O3 \to O4/O0.$$ 
4. Decompactify $\tilde{T}^3 \to \mathbb{R}^3$
5. Take $g_s^{(4)}$ up again
Result: Orientiholes

Key fact:

Witten index vacua $\Leftrightarrow$ index of BPS states
Estimate numbers of vacua in various sectors landscape by "measuring" (refined) Bekenstein-Hawking entropy of various mesoscopic black hole configurations: $N_{\text{vac}} \sim e^{S_{BH}}$
“Experimental” landscapeology

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- Finer enumeration from multiparticle quantum states (the fuzzball approach to landscapeology)
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- brane-brane open string indices $\iff$ angular momenta

- Subtle $\mathbb{Z}_2$ “tadpoles” on IIB side = charge measurable by Aharonov-Bohm experiment
Other motivations

1. funky spacetimes, where you can take a walk around the center of the universe and come back as your mirror image.

2. new invariants and associated modular forms, wall crossing formulae, ...

3. new version of the OSV conjecture: $\mathcal{Z}_{OH} \sim \mathcal{Z}_{top}$ (linear!)
Review of $\mathcal{N} = 2$ black hole bound states
Single centered black holes

Spherically symmetric BPS black hole of charge $\Gamma \equiv (p^\Lambda, q_\Lambda)$:

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} d\vec{x}^2$$
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Solutions $\iff$ attractors [Ferrara-Kallosh-Strominger]:

Radial inward flow of vector multiplet moduli $A(r)$ is gradient flow of central charge $|Z(\Gamma, t)|$.

BH entropy:

$$S(\Gamma) = \pi \min_t |Z(\Gamma, t)|^2$$
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BPS black hole molecules

More general BPS solutions exist: multi-centered bound states:

\[ ds^2 = -e^{2U(\vec{x})} \left( dt - \omega_i dx^i \right)^2 + e^{-2U(\vec{x})} d\vec{x}^2. \]
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- Stationary but with intrinsic spin from e.m. field
Explicit multicentered BPS solutions

- \( N \)-centered solutions characterized by harmonic function \( H(\vec{x}) \) from 3d space into charge space:

\[
H(\vec{x}) = \sum_{i=1}^{N} \frac{\Gamma_i}{|\vec{x} - \vec{x}_i|} + H_\infty
\]

with \( H_\infty \) determined by \( t_{|\vec{x}|=\infty} \) and total charge \( \Gamma \).
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\[
\sum_{j=1}^{N} \frac{\langle \Gamma_i, \Gamma_j \rangle}{|\vec{x}_i - \vec{x}_j|} = 2 \operatorname{Im} \left( e^{-i\alpha} Z(\Gamma_i) \right)_{|\vec{x}|=\infty}
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where \(\langle \Gamma_1, \Gamma_2 \rangle = \Gamma_1^m \cdot \Gamma_2^e - \Gamma_1^e \cdot \Gamma_2^m\) and \(\alpha = \arg Z(\Gamma)\).
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- All fields can be extracted completely explicitly from the entropy function $S(\Gamma)$ on charge space, e.g.

$$e^{2U(\vec{x})} = \frac{\pi}{S(H(\vec{x}))}$$
2-centered example and decay at marginal stability

\[ \Gamma_1 \quad \Gamma_2 \]

Equilibrium distance from position constraint:

\[ |\vec{x}_1 - \vec{x}_2| = \langle \Gamma_1, \Gamma_2 \rangle \]

\[ |Z_1 + Z_2| \Im(\overline{Z_1}Z_2) \]

\[ |\vec{x}| = \infty \]

When MS wall is crossed: RHS \( \to \infty \) and then becomes negative: decay

Spin:

\[ J = \langle \Gamma_1, \Gamma_2 \rangle \]
2-centered example and decay at marginal stability

\[ |\vec{x}_1 - \vec{x}_2| = \frac{\langle \Gamma_1, \Gamma_2 \rangle}{2} \frac{|Z_1 + Z_2|}{\text{Im}(Z_1 \overline{Z_2})} \bigg|_{|\vec{x}|=\infty} \]
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Example: pure $D4 = D6 - \overline{D6}$ molecule

- Pure D4 with D4-charge $P$ has

$$Z \sim -P \cdot \frac{t^2}{2} - \frac{P^3}{24}.$$
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$Z(t) = 0$ at $t \sim iP \Rightarrow$ No single centered solution.
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- Instead: realized as bound state of single D6 with $U(1)$ flux $F = P/2$ and anti-(single D6 with flux $F = -P/2$):

$$\text{D6}[P/2] \quad \text{D6}[P/2]$$

Stable for $\text{Im} \ t > \mathcal{O}(P)$. 
**Transition between** \( g_s |\Gamma| \gg 1 \) **and** \( g_s |\Gamma| \ll 1 \) **pictures**

- Mass squared lightest bosonic modes of open strings between \( \Gamma_1 \) and \( \Gamma_2 \):

\[
\frac{M^2}{M_s^2} \sim \frac{|\vec{x}_1 - \vec{x}_2|^2}{\ell_s^2} + \Delta \alpha \\
= c(t) g_s^2 + \Delta \alpha
\]
Transition between $g_s|\Gamma| \gg 1$ and $g_s|\Gamma| \ll 1$ pictures

- Mass squared lightest bosonic modes of open strings between $\Gamma_1$ and $\Gamma_2$:

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\]

- On stable side of MS wall $\Delta \alpha < 0$, so if $g_s$ gets sufficiently small, open strings become tachyonic and branes condense into single centered D-brane.
Establishing existence of multicentered BPS configurations not easy: position constraints, $S(H(\vec{x})) \in \mathbb{R}^+ \ \forall \vec{x}$, ...
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Conjecture: Branches of multicentered configuration moduli spaces in 1-1 correspondence with attractor flow trees:
Establishing existence of multicentered BPS configurations not easy: position constraints, \( S(H(\vec{x})) \in \mathbb{R}^+ \forall \vec{x}, \ldots \) However:

Conjecture: Branches of multicentered configuration moduli spaces in 1-1 correspondence with attractor flow trees:

- Much simpler to check & enumerate!
Flow tree decomposition of BPS Hilbert space

- Flow trees can also be given microscopic interpretations (decay sequences / tachyon gluing).
Flow tree decomposition of BPS Hilbert space

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Hilbert space of BPS states of charge $\Gamma$ in background $t$ has canonical decomposition in attractor flow tree sectors:

$$\mathcal{H}(\Gamma, t) =$$
The BPS index

Hilbert space of BPS states in 4d $\mathcal{N} = 2$ theories:

$$\mathcal{H}(\Gamma, t) = \left( \frac{1}{2}, 0, 0 \right) \otimes \mathcal{H}'(\Gamma, t)$$

Index:

$$\Omega(\Gamma, t) = \text{Tr}_{\mathcal{H}'(\Gamma, t)} (-1)^{2J'_3} = (-1)^{\dim_{\mathbb{C}} M} \chi(M).$$
Wall crossing formula for primitive splits

Near marginal stability wall $\Gamma \to \Gamma_1 + \Gamma_2$ (with $\Gamma_1$ and $\Gamma_2$ primitive), the decaying part of $\mathcal{H}'(\Gamma, t)$ has following factorized form:

$$\Delta \mathcal{H}'(\Gamma, t) = (J) \otimes \mathcal{H}'(\Gamma_1, t) \otimes \mathcal{H}'(\Gamma_2, t)$$

with $J = \frac{1}{2}(\langle \Gamma_1, \Gamma_2 \rangle - 1)$. 

Spin factor:
- macroscopically from intrinsic angular momentum monopole-electron system ($-1/2$ from spin-magnetic coupling)
- microscopically from quantizing open string tachyon moduli space $\mathcal{M}_{\text{susy}} = \mathbb{C}P^2$. 

Implies index jump $\Delta \Omega = (-2J(2J + 1)\Omega(\Gamma_1, t_{\text{ms}})\Omega(\Gamma_2, t_{\text{ms}})$.
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Type IIA orientiholes
Solutions

$\mathcal{N} = 2$ solutions invariant under $\tau'$. Two cases:

\[
\tau_{04/00} = \Omega \sigma^* P^*
\]
\[
\tau_{06/02} = \Omega (-1)^{F_L} \sigma^* P^* .
\]

where $P : \vec{x} \rightarrow -\vec{x}$ and $\sigma$ is internal involution.

E.g. O4/O0 one modulus case:

\[
\Gamma_1 = (P^0, P^1, Q_1, Q_0)_1 , \quad \Gamma_{-1} = \Gamma'_1 = (-P^0, P^1, -Q_1, Q_0)_1 , \quad \Gamma_0 = (0, P^1, 0, Q_0)_0 .
\]
Main difference

Phase $\alpha_\infty$ is fixed by choice of orientifold projection:

\[ \alpha_\infty = 0 \quad (O4/O0), \quad \alpha_\infty = -\frac{\pi}{2} \quad (O6/O2) \]

Consequence: if $\alpha_\infty = \pi + \arg Z$: neg. mass, grav. repulsive, inverted attr. flow, attr. point $\rightarrow$ repulsor point, sol. singular.
Basic bound state

- Simplest possibility:
  \[ \Gamma = \Gamma_1 + \Gamma_0 + \Gamma'_1 \]
  i.e. bound state of charge with its own image (+ charge in the middle of the universe)

- From integrability constraint:
  \[ \frac{I(\Gamma_1, \Gamma_0)}{|\vec{x}_1|} = 2 \text{Im}[e^{-i\alpha Z_1}]_\infty . \]
  where
  \[ I(\Gamma_1, \Gamma_0) := \frac{\langle \Gamma_1, \Gamma'_1 \rangle}{2} + \langle \Gamma_1, \Gamma_0 \rangle = 2J \]
  \[ \Rightarrow \text{Stability condition:} \]
  \[ I(\Gamma_0, \Gamma_1) \text{Im}[e^{-i\alpha Z_1}]_\infty > 0 . \]
Wall crossing formula

Index counting orientifold invariant BPS states:

\[ \Omega_{\text{inv}}(\Gamma, t) = \text{Tr}_{\mathcal{H}'_{\text{inv}}(\Gamma, t)} (-)^{2J_3} = (-1)^{\dim \mathcal{M}_{\text{inv}}} \chi(\mathcal{M}_{\text{inv}}). \]
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Corollary, by comparing to microscopic picture:

angular momentum of pair = open string Witten index
\(\mathbb{Z}_2\) torsion charge

- For charge odd under \(\tau\) (e.g. D6 in O4/O0 case): Aharonov Bohm experiment can distinguish between odd and even number of dipoles.
- Related to subtle anomalies on IIB side.
Example: pure D7 branes in degree 8 hypersurface in $\mathbb{CP}^4_{4,1,1,1,1}$ O3/O7 orientifolded by reflection of first coordinate.

Complicated story (cf. previous talk at Rutgers)

D7 (possibly with flux) obtained from two D9-D9′ pairs with $a, b$ units of flux.
Application to counting basic D7 vacua

Orientihoole split flow:

Predicts Euler characteristics moduli spaces:

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Note: decays at quite large vol.!
General tadpole canceling D7 vacua

\[ \Gamma = -8\Gamma_{O4} + \Gamma_{O4} = (0, 28, 0, \frac{133}{3}) \]
General tadpole canceling D7 vacua

- $\Gamma = -8 \Gamma_{O4} + \Gamma_{O4} = (0, 28, 0, \frac{133}{3})$

- Single centered black holes solutions exist with $S = \frac{S_{BH}}{2} = 1789$ in large volume approx. but at $t_* \approx 2i$ $\nabla$ can’t trust result.
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- Do single centered solutions exist in fully corrected theory?
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\[ S = \frac{S_{BH}}{2} = 1789 \] in large volume approx. but at \( t_* \approx 2i \) \( \sim \)
 can’t trust result.

Do single centered solutions exist in fully corrected theory?
Physical argument: scaling solutions exist (for \( a = 2 \)).
General tadpole canceling D7 vacua

Fat multicentered solutions:

\[ S = 1540 \text{ in large vol. approx. (ok } \text{Im } t_* \approx 7) \]

⇒ In this sector \( N_{\text{vac}} \approx 10^{668} \).
Directions for future work

- lift to M-theory
- solutions in $T^3$. MS = ??
- corrections to $Z$
- map different landscape sectors to different kinds of BH configurations
- implications stability issues for phenomenology
- nonprimitive wall crossing
- OSV, modular forms
- bulk fluxes
- nonsusy