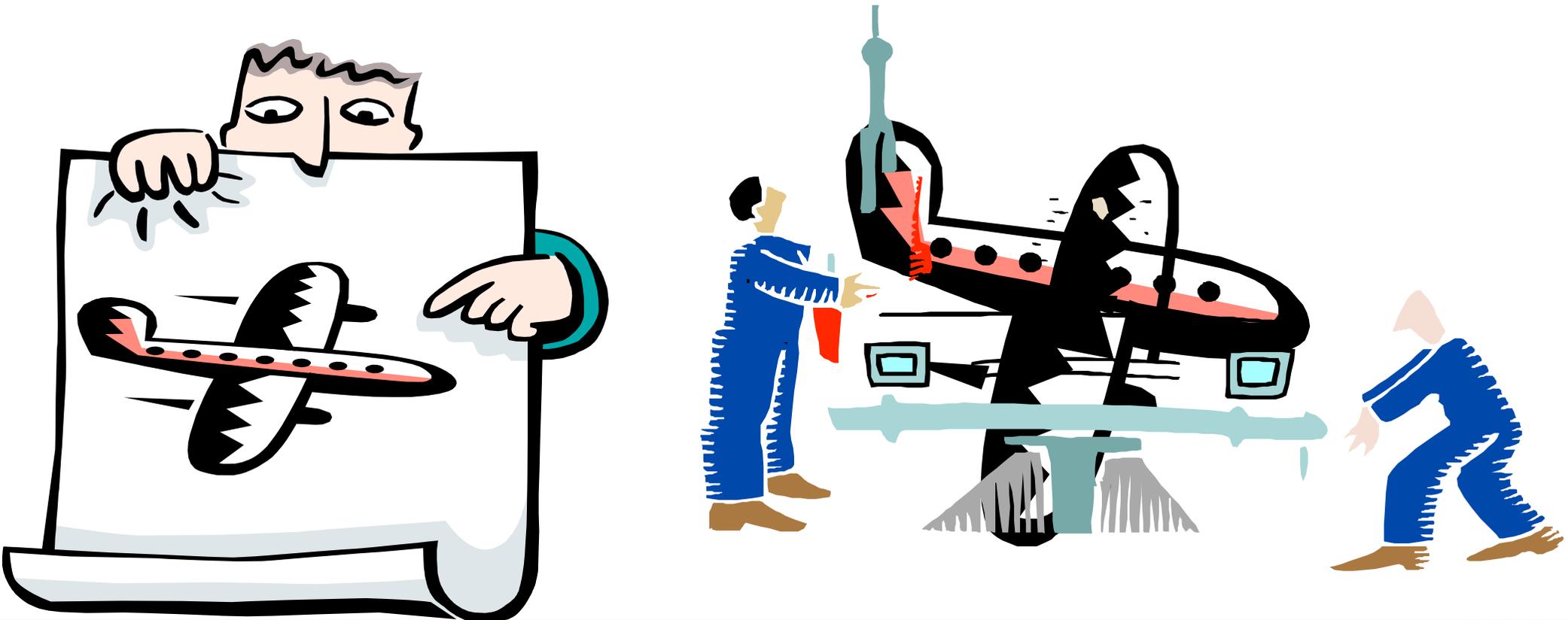


Towards factorization for jet production

Christian W Bauer, LBNL/UC Berkeley
Rutgers University
01/29/08

In collaboration with
Chris Lee, Sean Fleming, George Sterman
0801.???? [hep-ph]
Andrew Horning, Frank Tackmann

An artist's rendering



Motivation

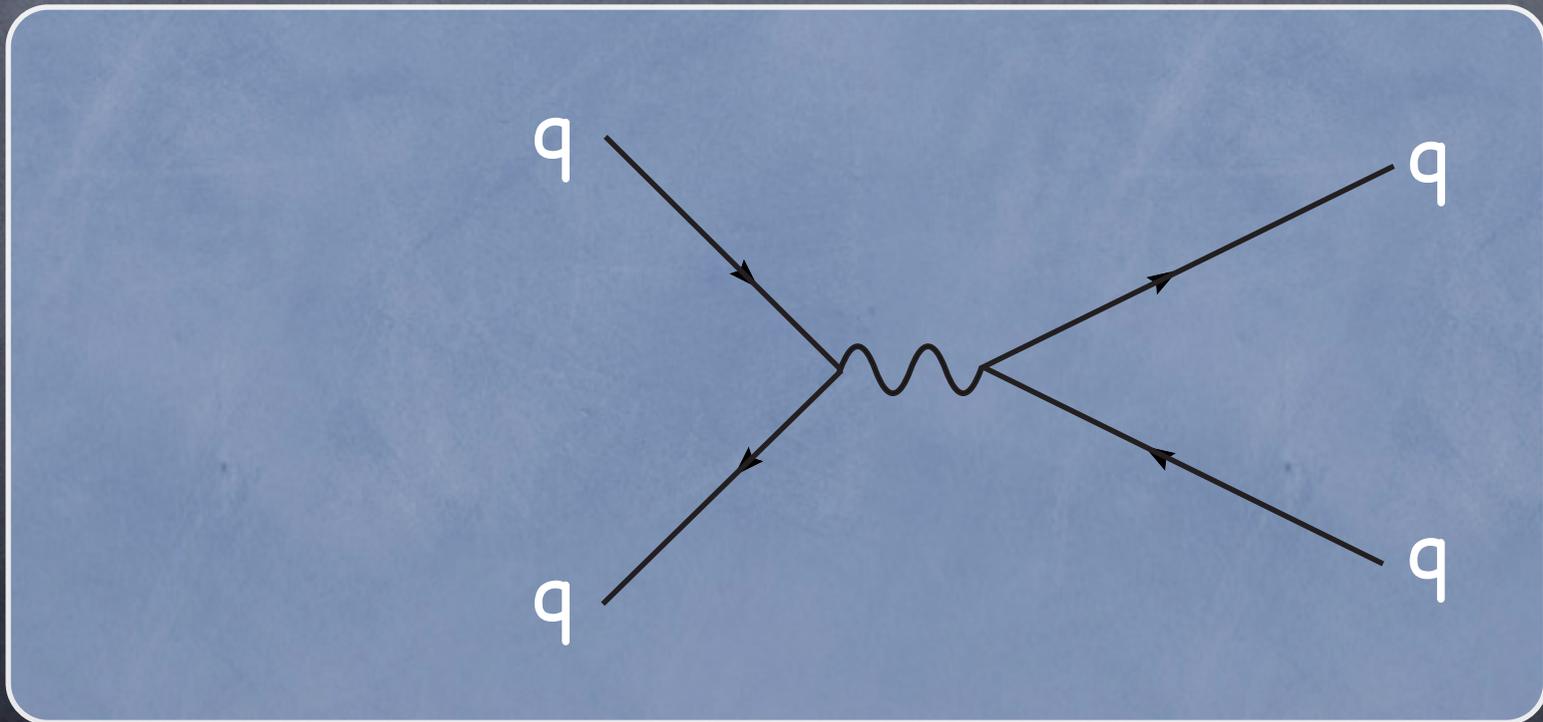
- Most interactions at hadron colliders produce multiple high energy partons in different directions
- Want to study distributions of these partons wrt to one another
- Clearly, will not see partons, and hadronization will be important
- Use jets of hadrons to identify the underlying parton

Questions

- How are partonic results related to jet observables?
- What is calculable in PT, and how does PT behave?
- What non-perturbative physics is needed?
- How do different jet definitions affect results?

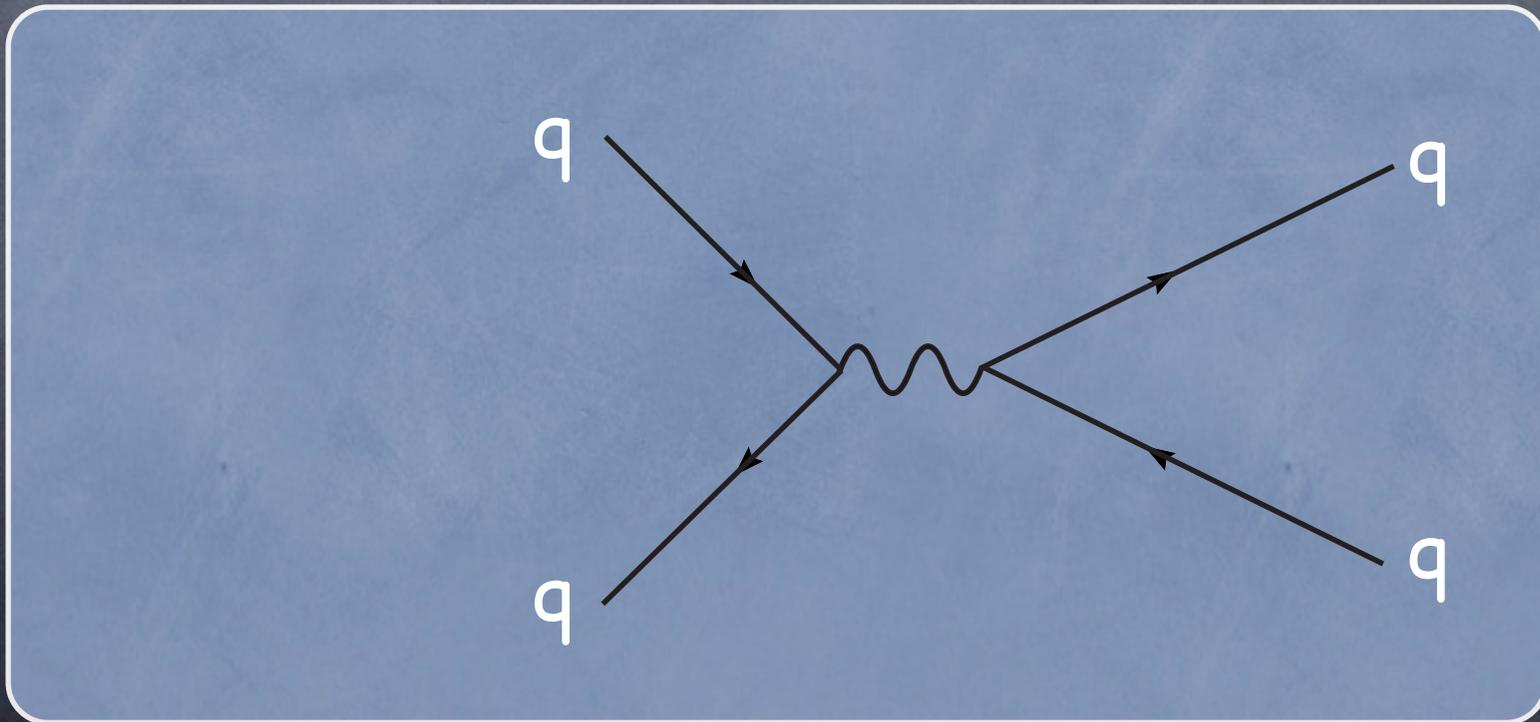
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- We clearly expect perturbative QCD to provide some understanding for jet production at high energies



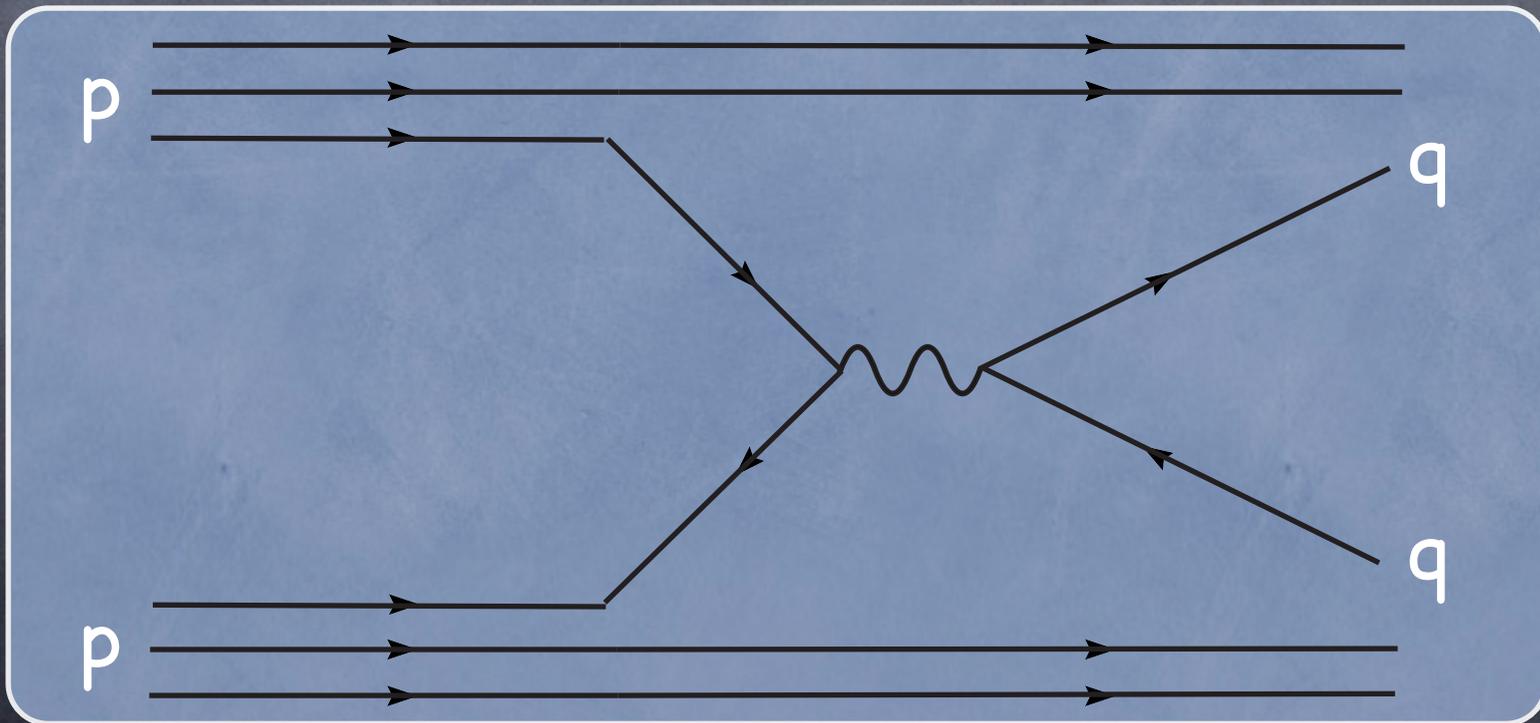
Motivation

- We clearly expect perturbative QCD to provide some understanding for jet production at high energies
- We also clearly expect non-perturbative physics to play a role, at least for hadron-hadron interactions



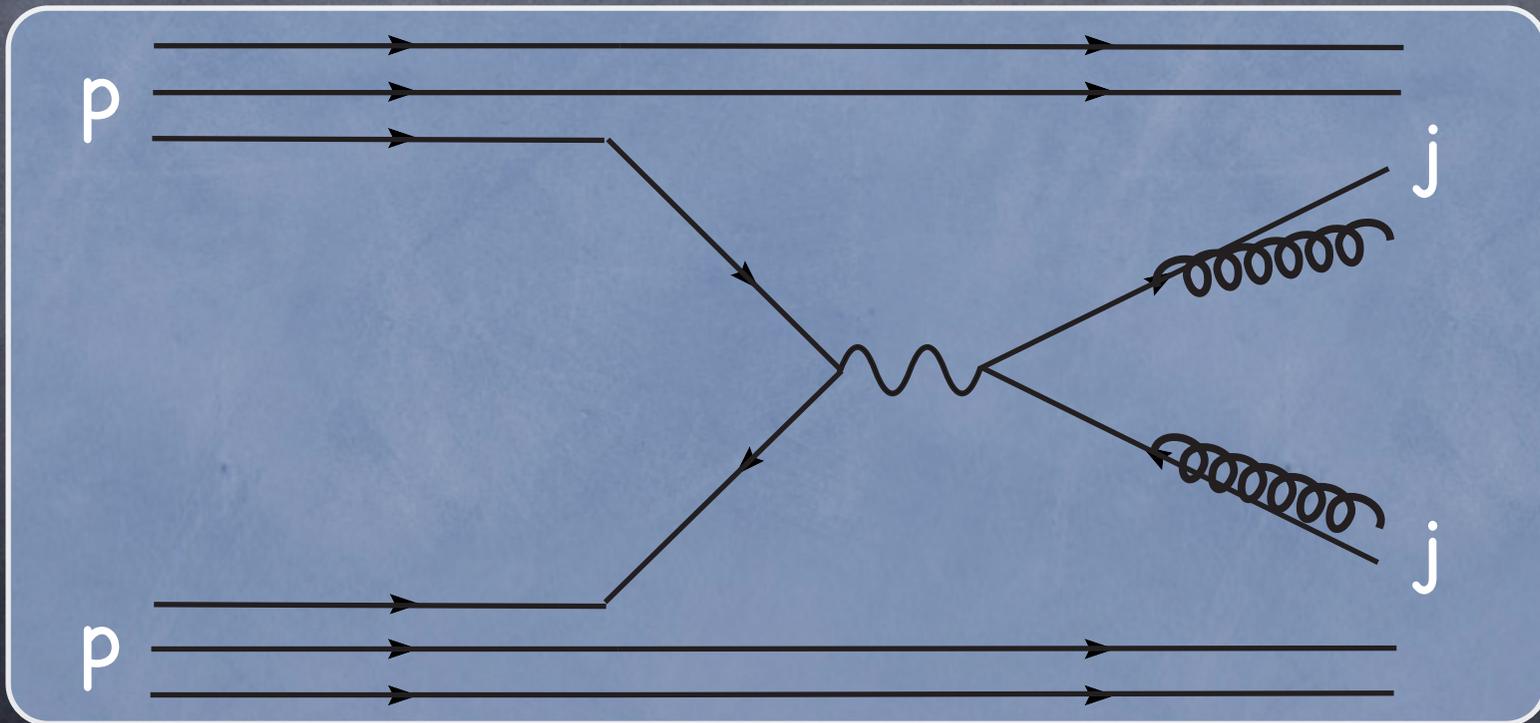
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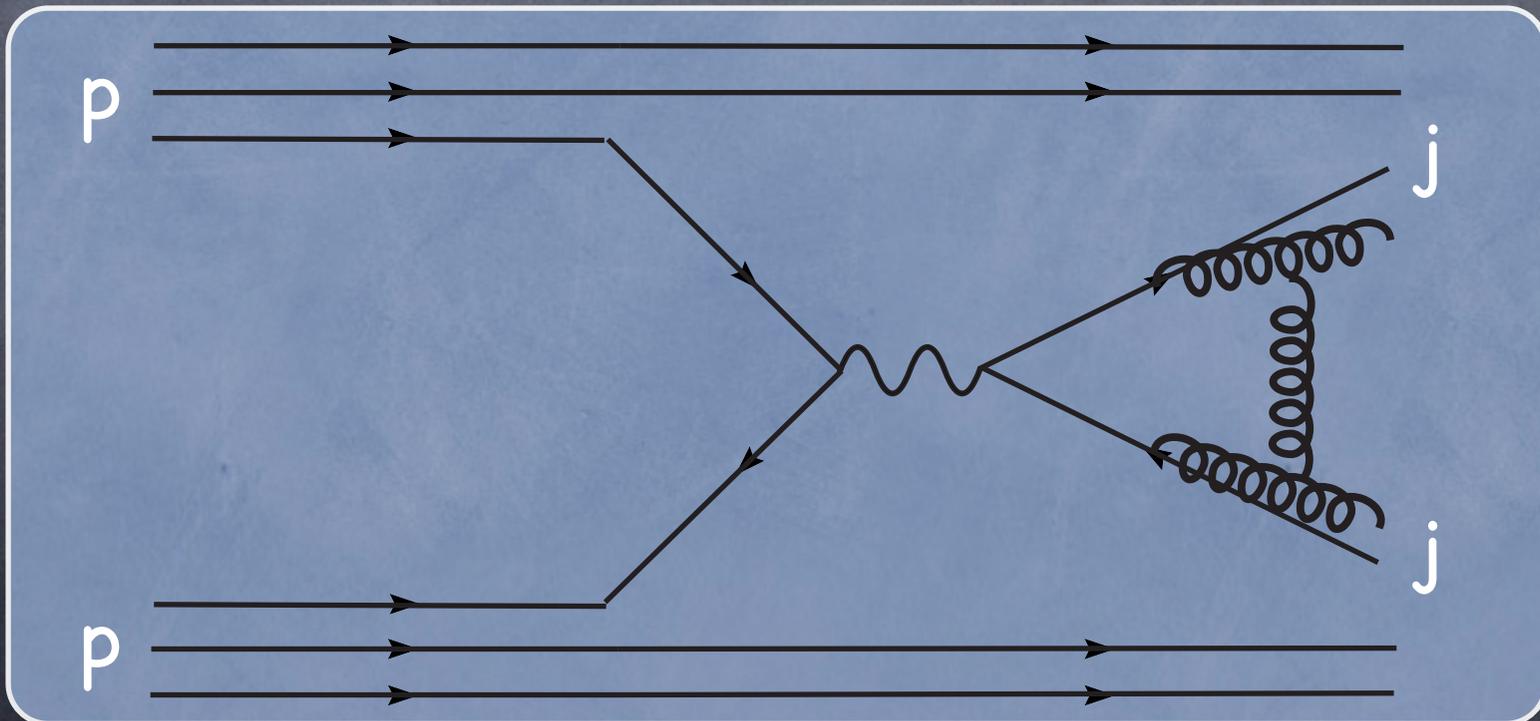
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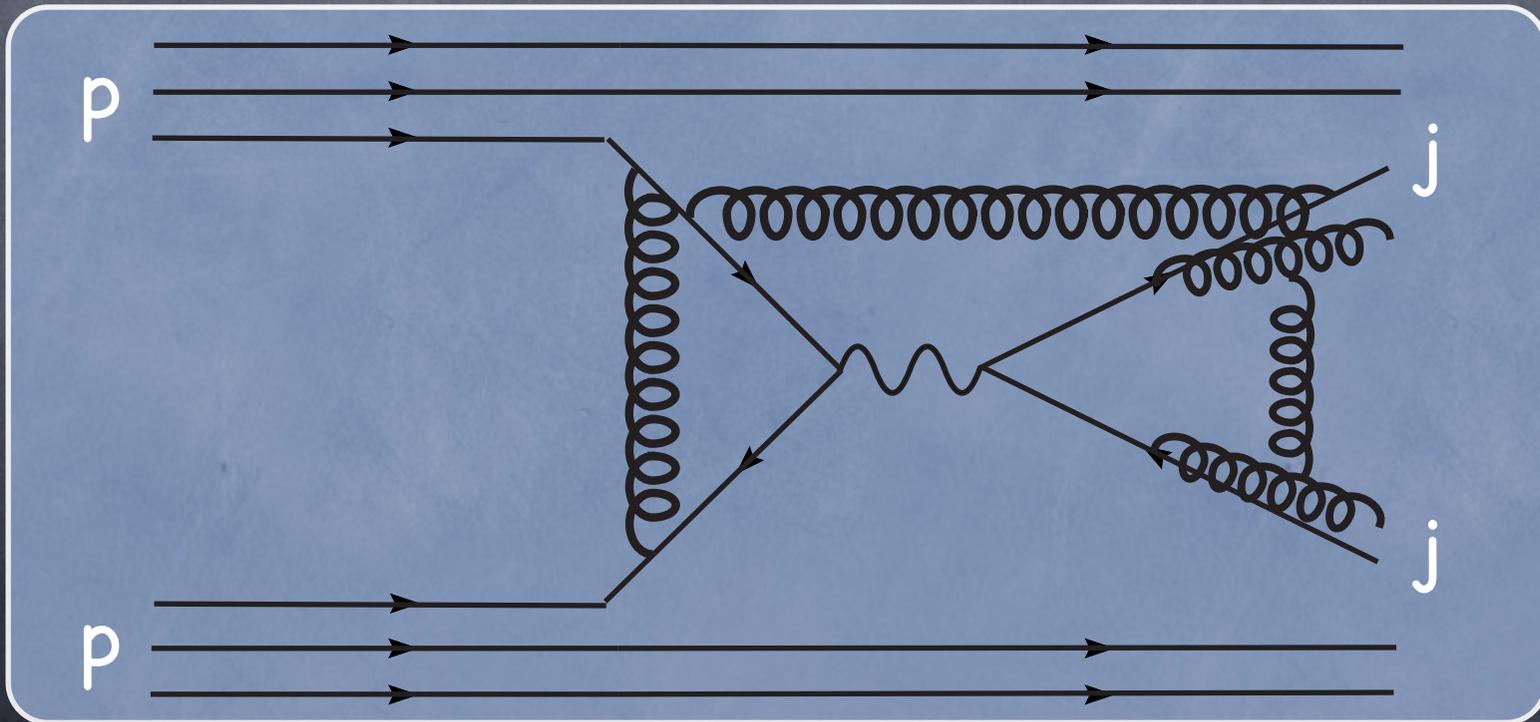
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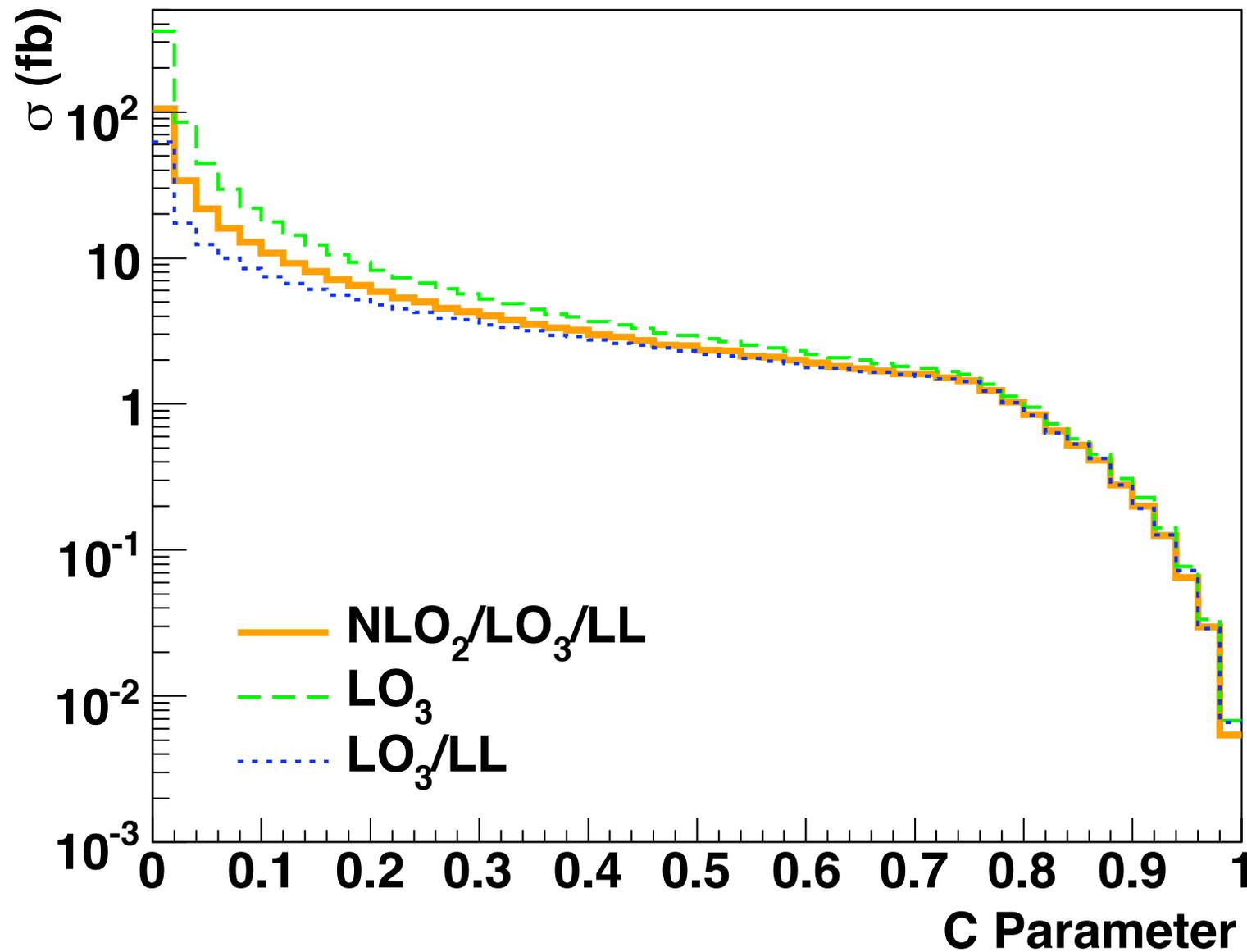
Motivation

- We usually think of QCD in terms of a perturbative expansion
- Presence of widely separated scales gives rise to logarithmic terms $\alpha_s^n \log^m(\Lambda_1/\Lambda_1)$
- Need to resum these terms to get precise theoretical prediction
- In jet physics, many different energy scales possible: $m(\text{jet}), E(\text{jet}), m(\text{jet}_1, \text{jet}_2), \dots$

No known way to sum these logarithms without factorization of process

Some output from GenEvA

CWB, Tackmann, Thaler ('08)



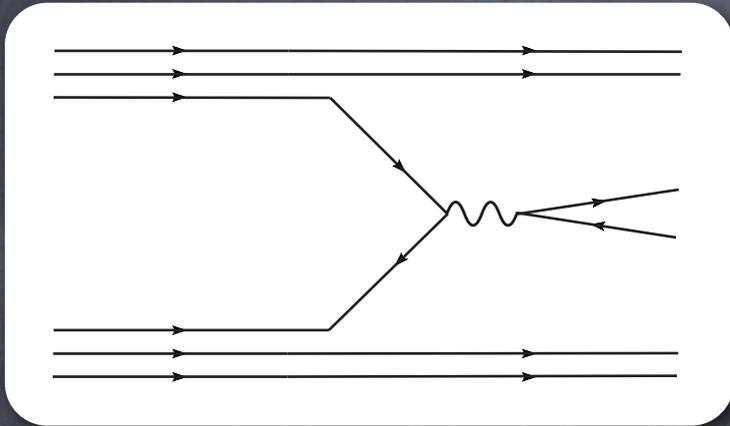
Previous work

- First proofs of factorization based on pioneering work of Collins, Soper and Sterman Collins, Soper, Sterman (80s)
- Study properties of Feynman diagrams to separate long and short distance physics
- Very well understood for bread&butter physics (DIS, DY, ...)
- Much work for more general processes For a review, see Sterman's TASI lectures
- Effective field theory treatment possible since invention of SCET CWB, Fleming, Pirjol, Rothstein, Stewart ('02)

Introduction to Factorization

Introduction to Factorization

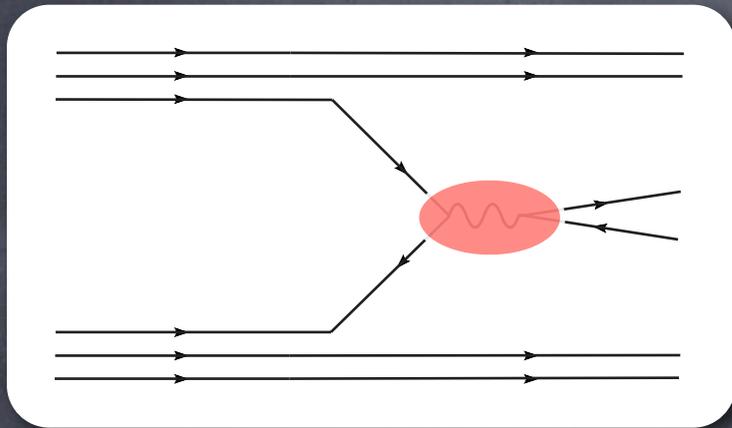
Drell Yan: $p + p \rightarrow X + e^- + e^+$



$$\sigma(p+p \rightarrow X+e^-+e^+) =$$

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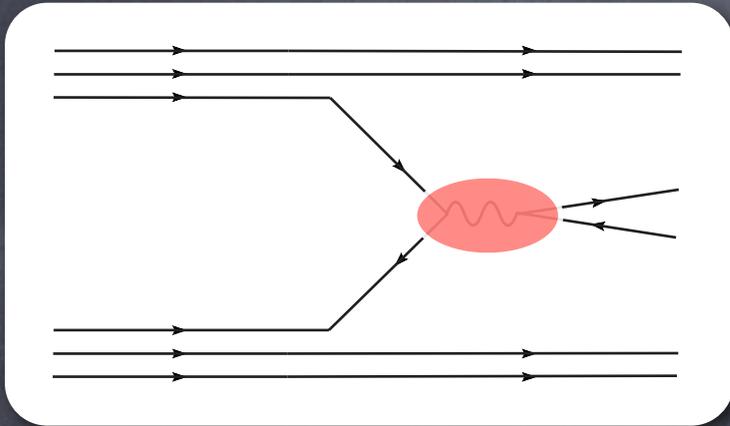
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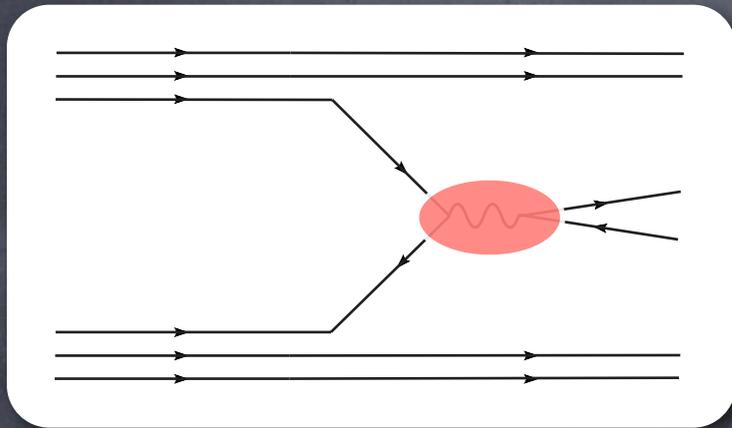
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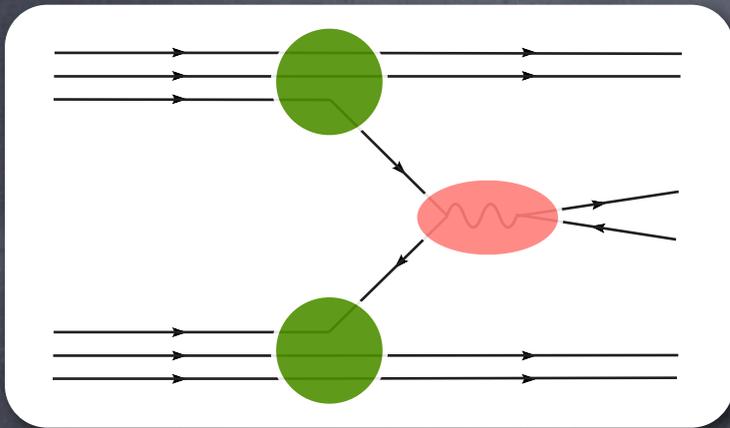


$$\sigma(p+p \rightarrow X+e^-+e^+) = \sigma(q+q \rightarrow e^-+e^+)$$

- Partonic cross section
- Short distance
- Perturbative

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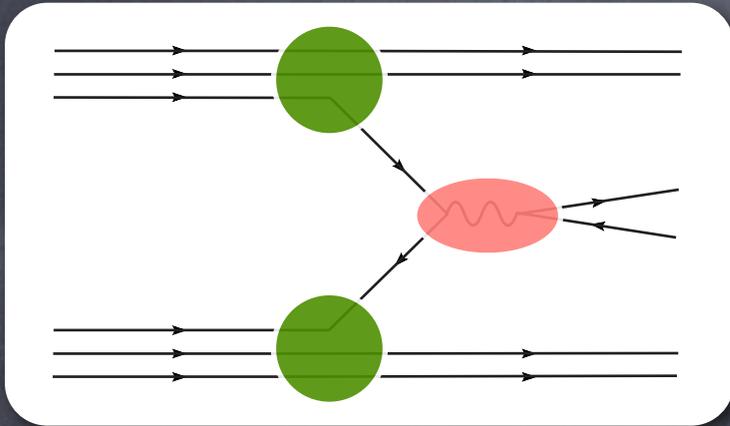


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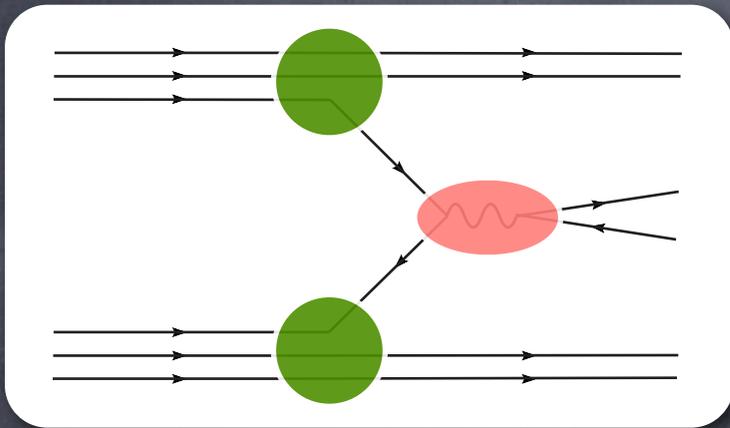


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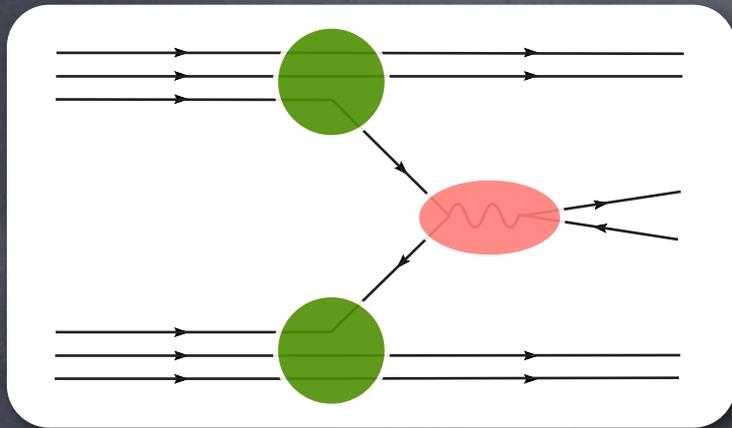
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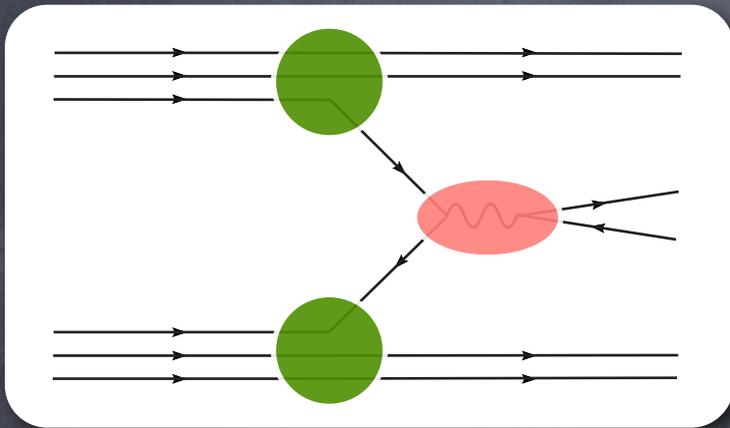
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How do we get non-perturbative information?

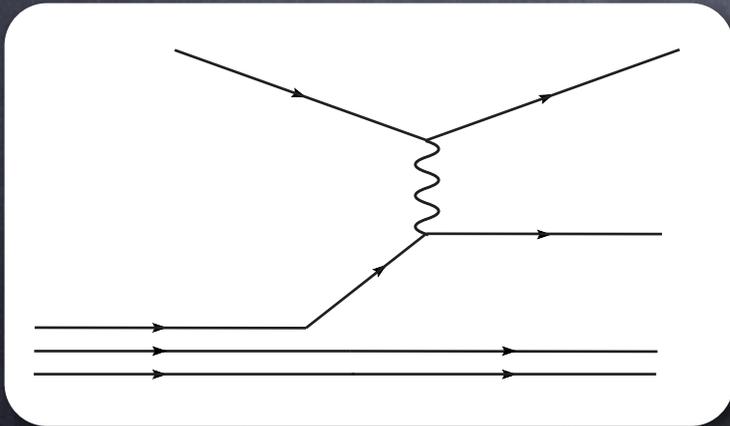
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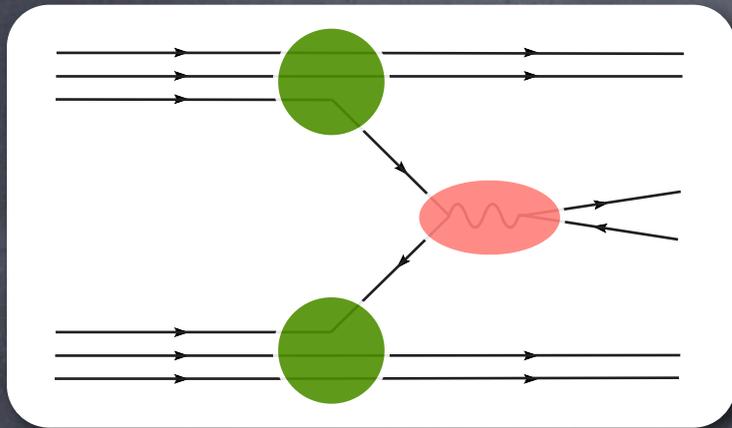
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DIS: $p + e^- \rightarrow X + e^-$



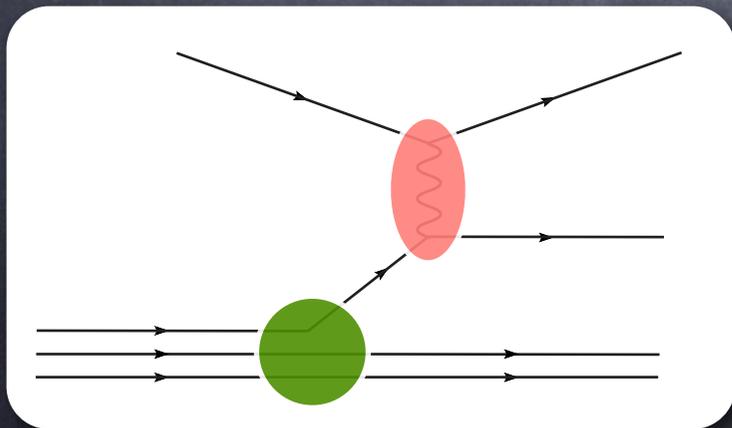
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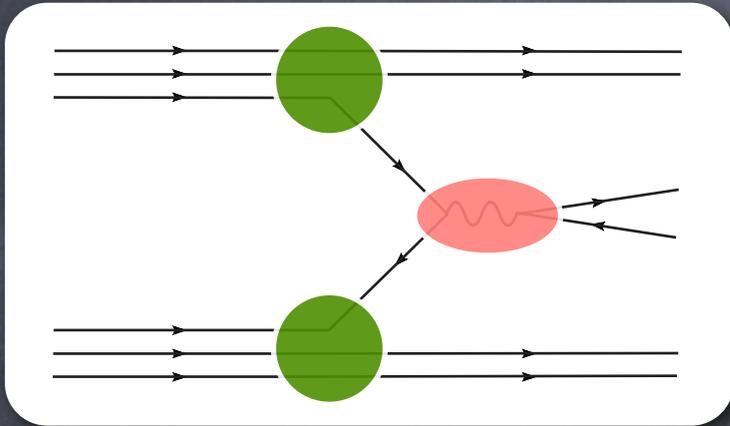
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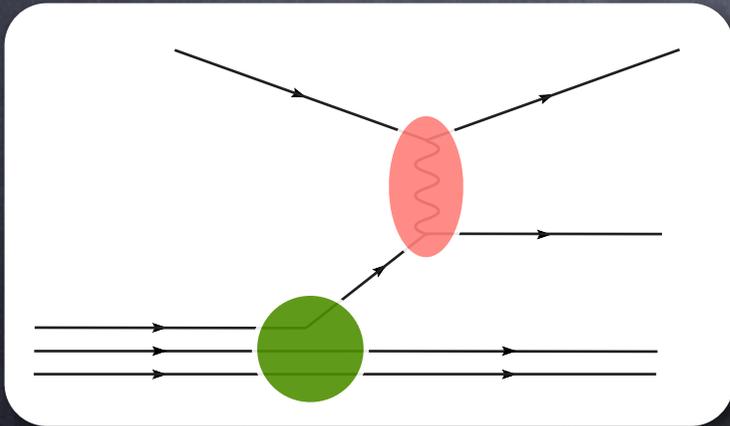
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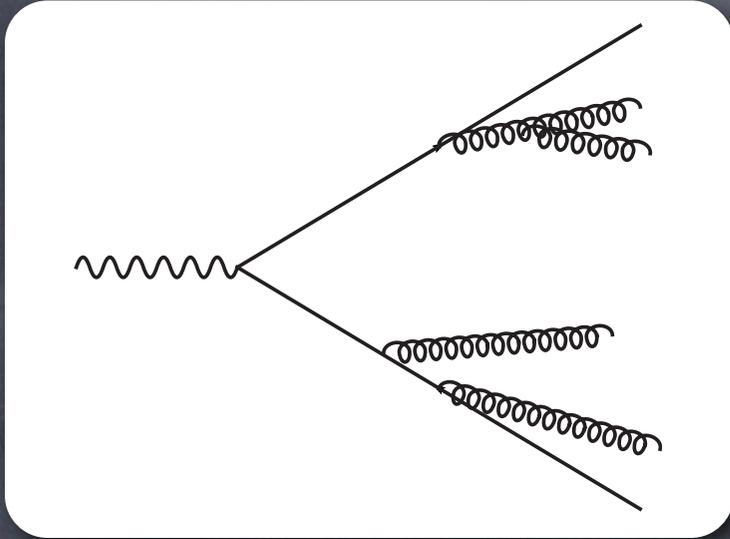
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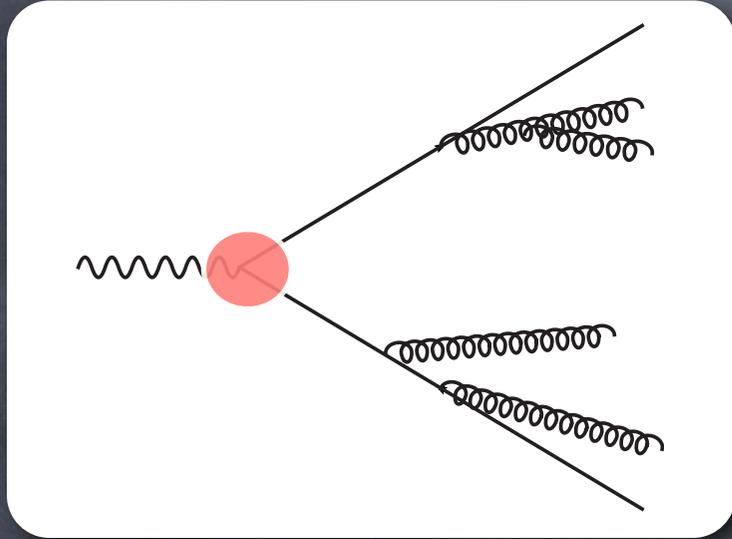
Event shapes: $e^+ + e^- \rightarrow \text{hadrons}$



$$\sigma(e^+e^- \rightarrow \text{hadrons})$$
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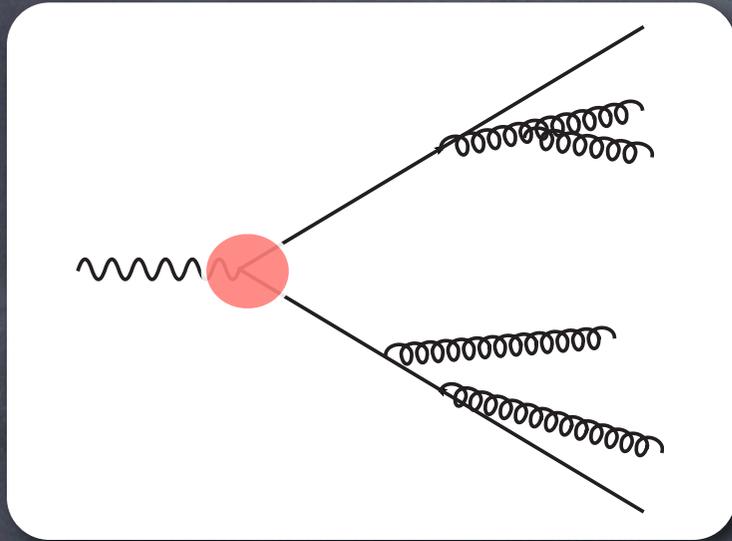
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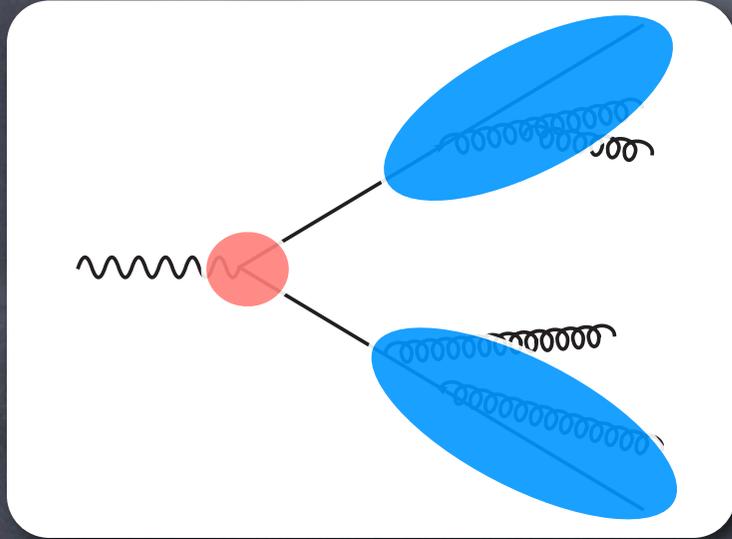
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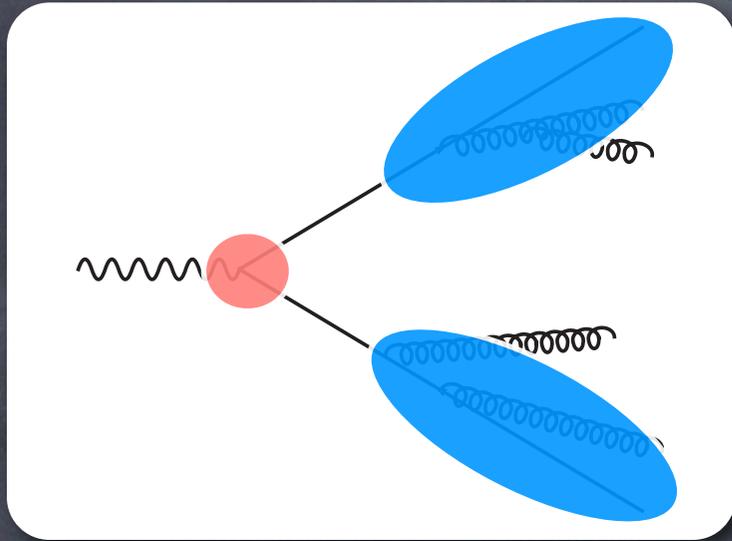
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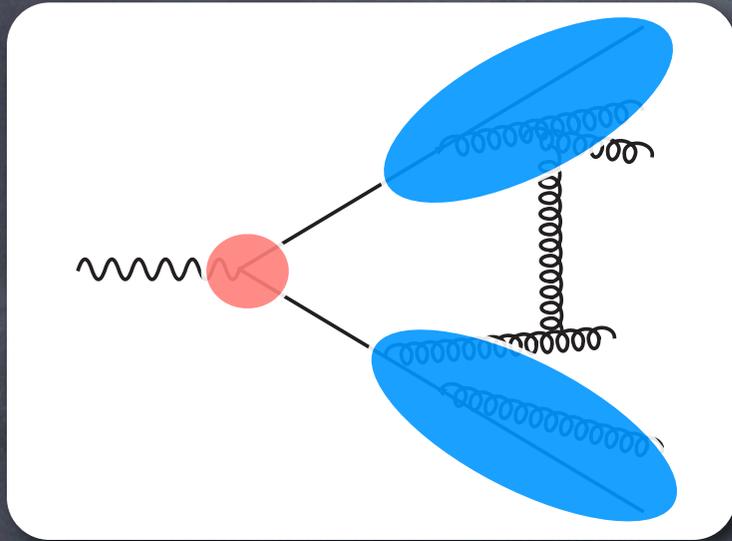
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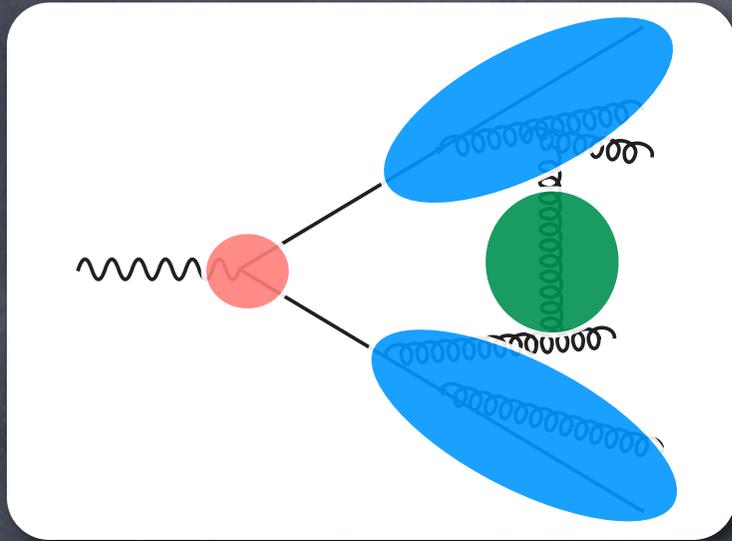
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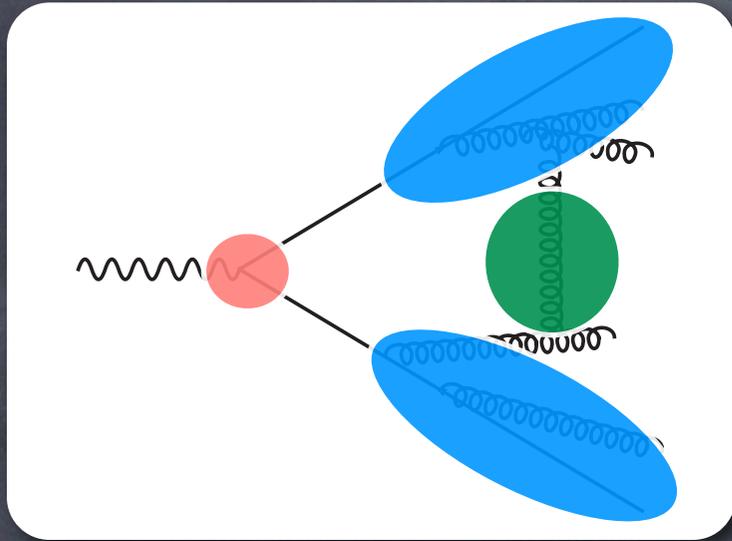
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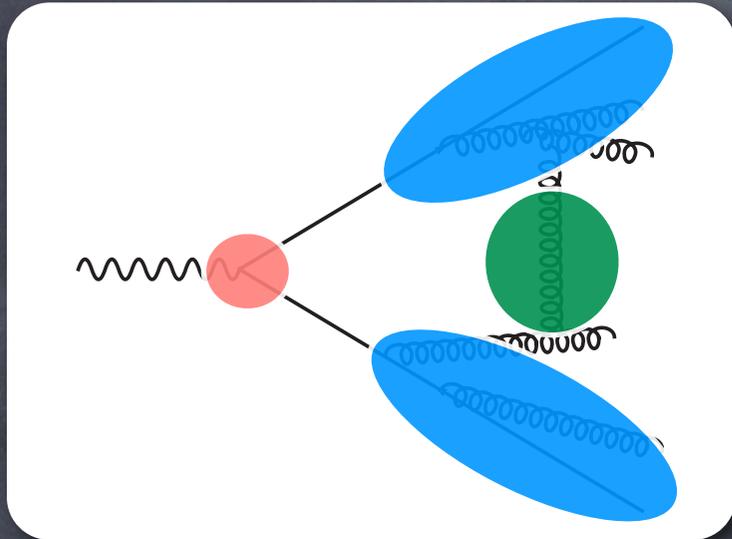
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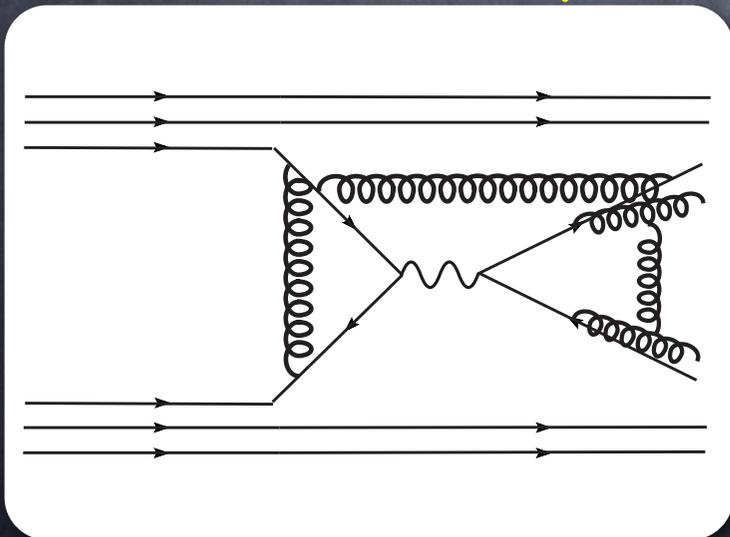
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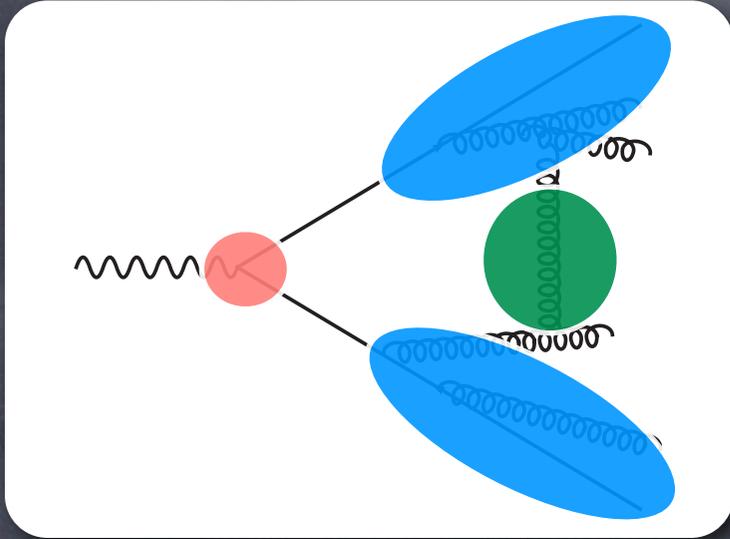
Jet production: $p + p \rightarrow \text{jets}$



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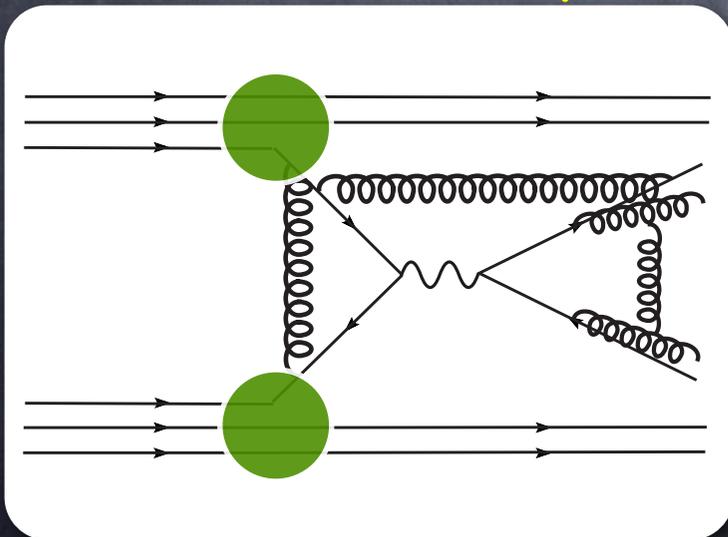
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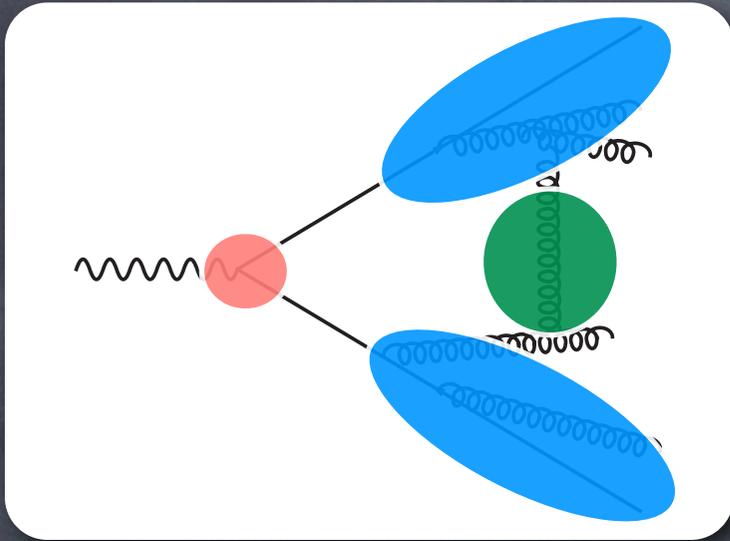
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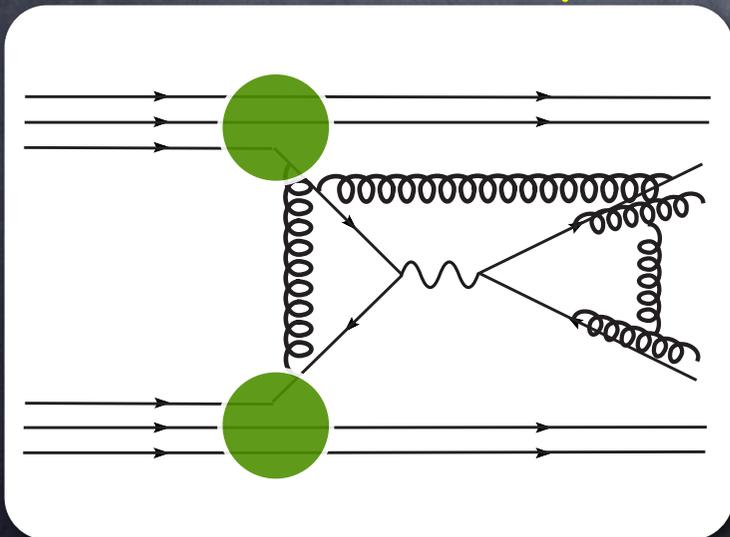
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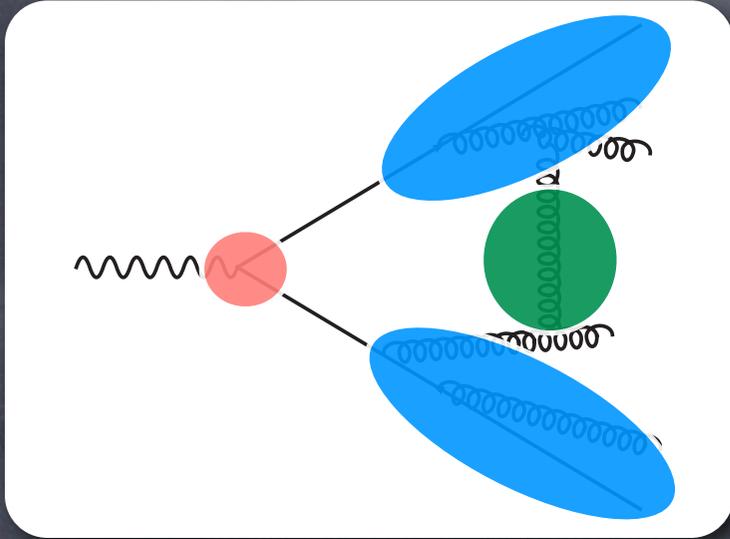
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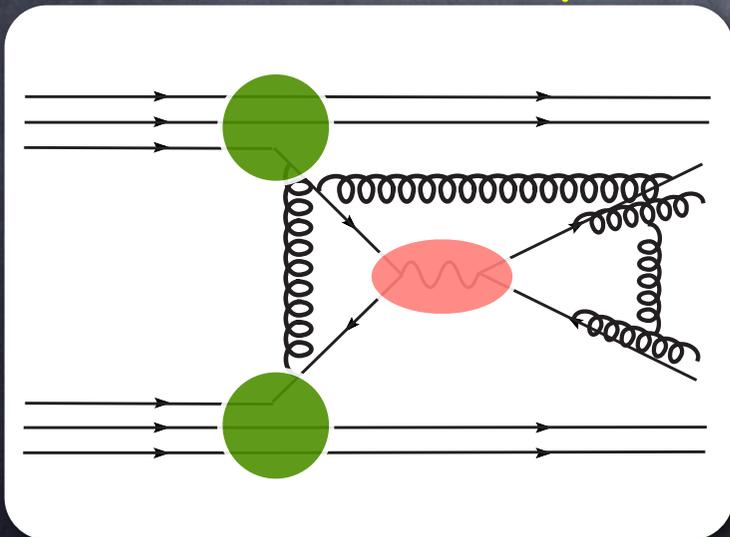
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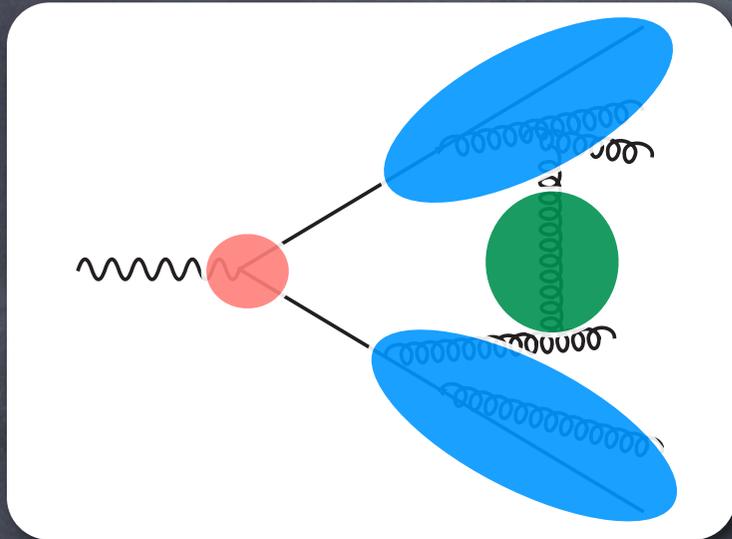
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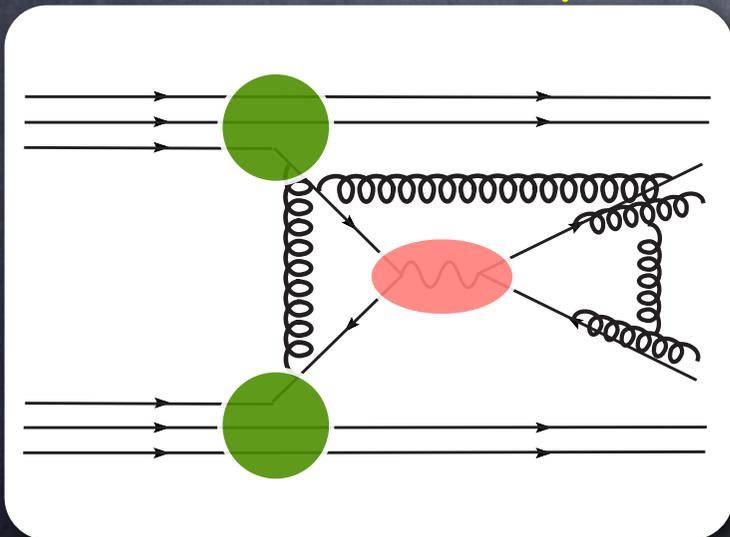
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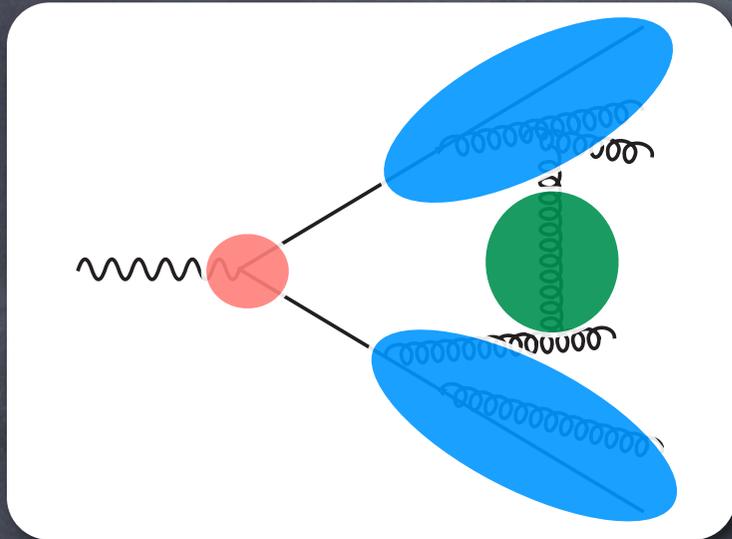
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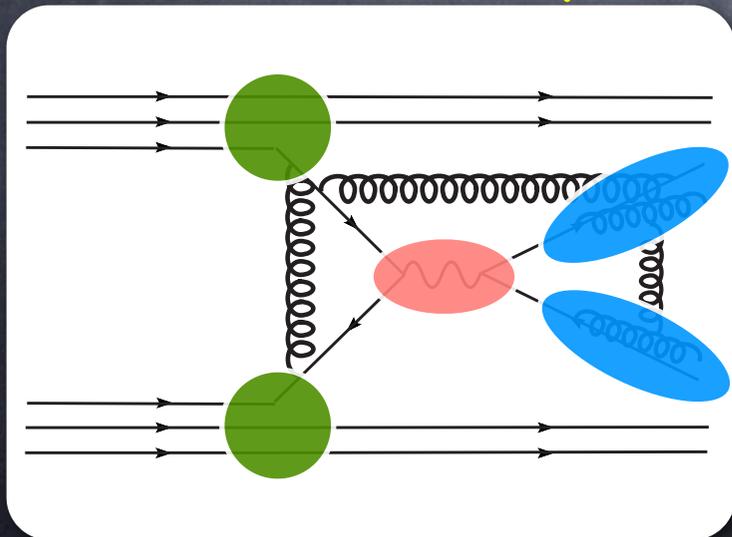
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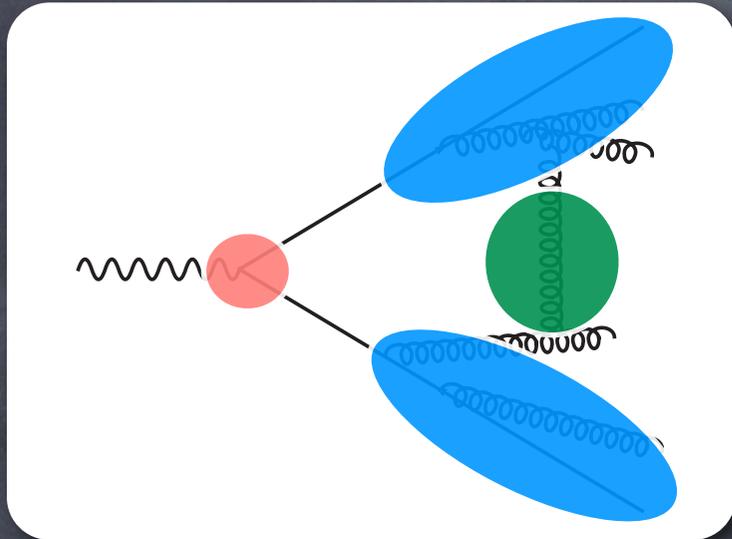
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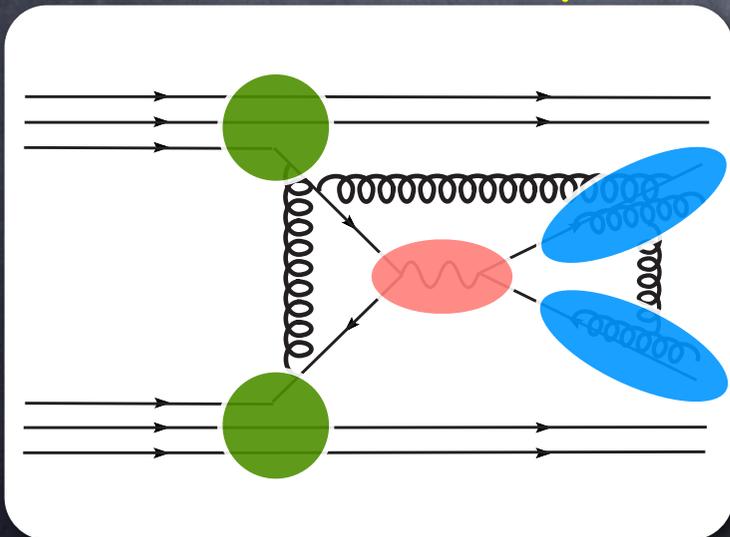
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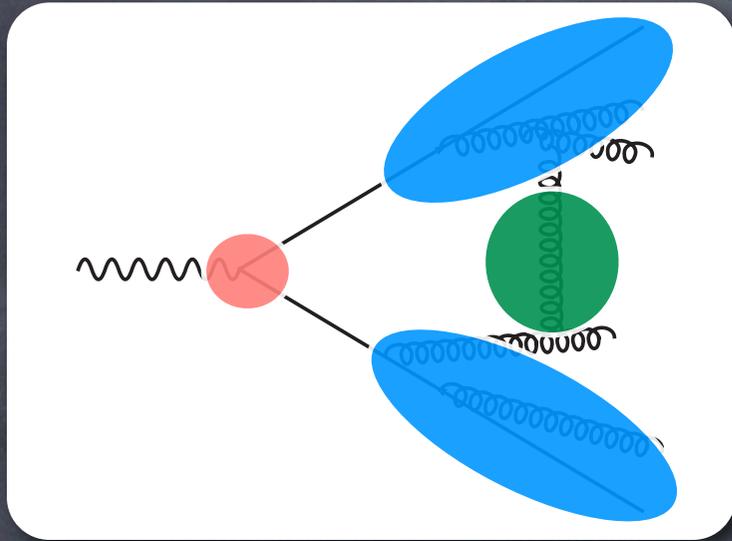
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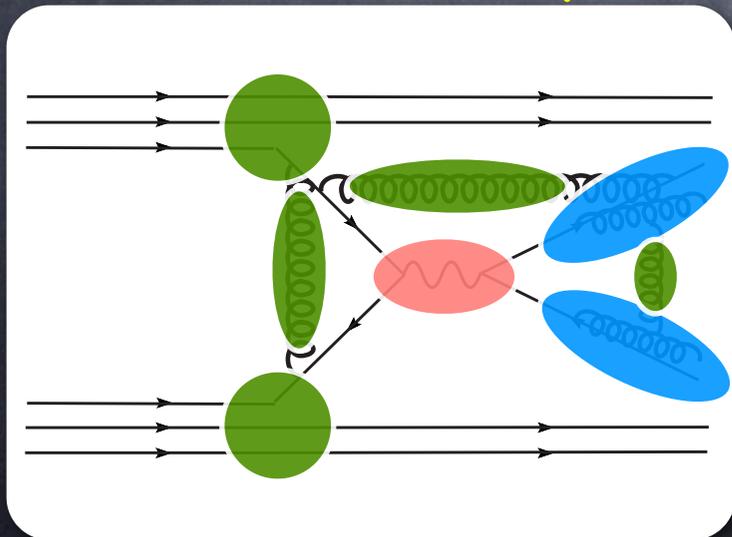
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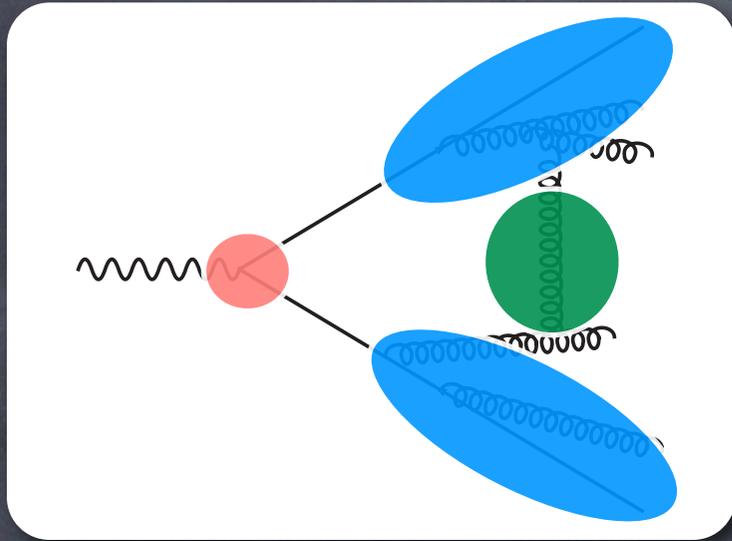
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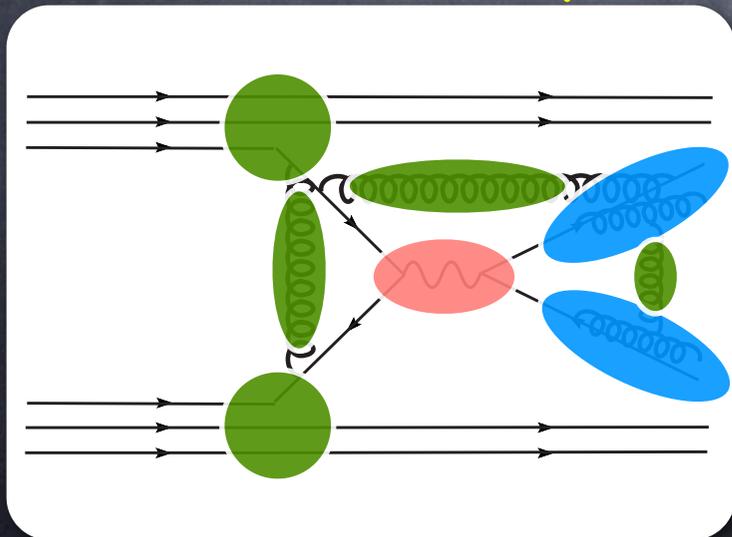
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Quick introduction to SCET

Field content of SCET

CWB, Fleming, Luke ('00)

CWB, Fleming, Pirjol, Stewart ('00)

Light cone coordinates: $p^\mu = (n \cdot p, \bar{n} \cdot p, p^\perp)$

$\frac{1}{2}(p_0 - p_3)$ ← $\frac{1}{2}(p_0 + p_3)$ → p_i

Degrees of freedom

| Type | (p^+, p^-, p^\perp) | Fields |
|-----------|-------------------------------------|---------------|
| collinear | $(\lambda^2, \mathbf{1}, \lambda)$ | χ_n, A_n |
| soft | $(\lambda^2, \lambda^2, \lambda^2)$ | q_s, A_s |

Construct the most general operators with required field content to given order in λ

Interactions in SCET

Leading order collinear Lagrangian

$$\mathcal{L} = \sum_n \bar{\chi}_n \left[i n \cdot D_n + g n \cdot A_s + i \not{D}_n^\perp \frac{1}{i \bar{n} \cdot D_n} i \not{D}_n^\perp \right] \frac{\not{n}}{2} \chi_n$$

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Collinear fields

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Interactions in SCET

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Collinear fields

Soft gluon

Interactions in SCET

Leading order collinear Lagrangian

$$\mathcal{L} = \sum_n \bar{\chi}_n \left[i n \cdot D_n + g n \cdot A_s + i \not{D}_n^\perp \frac{1}{i \bar{n} \cdot D_n} i \not{D}_n^\perp \right] \frac{\not{n}}{2} \chi_n$$

Collinear fields

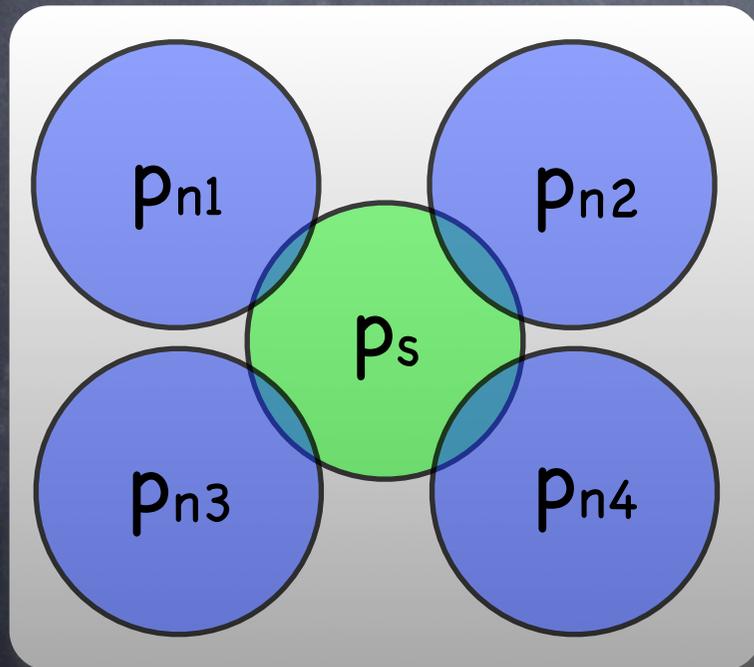
Soft gluon

- No interactions between collinear fields of different directions
- Interaction between collinear and soft fields only via one single term

Soft/collinear decoupling

CWB, Pirjol, Stewart ('00)

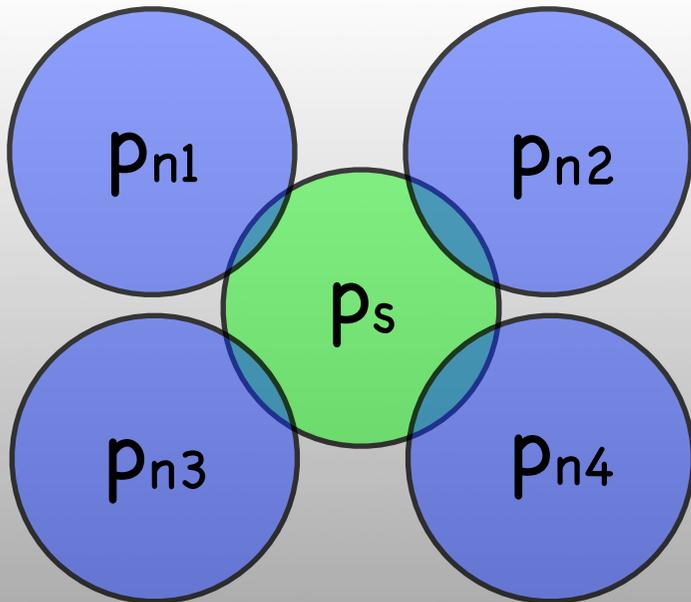
$$\mathcal{L} = \sum_n \bar{\chi}_n \left[i\vec{n} \cdot D_n + g_n \cdot A_s + i\not{D}_n^\perp \frac{1}{i\vec{n} \cdot D_n} i\not{D}_n^\perp \right] \frac{\not{n}}{2} \chi_n$$



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Perform field redefinition

$$\chi_n = Y_n \chi_n^{(0)} \quad A_n = Y_n A_n^{(0)} Y_n^\dagger$$

$$Y_n = \text{P exp} \left[ig \int_0^\infty ds n \cdot A_s(ns) \right]$$

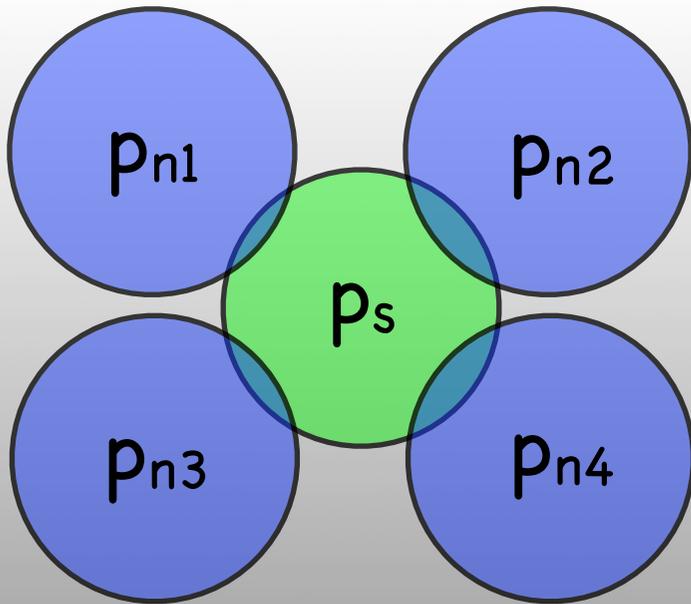
$$Y_n Y_n^\dagger = 1$$

$$i n D Y_n = Y_n i n \partial$$

Soft/collinear decoupling

CWB, Pirjol, Stewart ('00)

$$\mathcal{L} = \sum_n \bar{\chi}_n^{(0)} \left[i\bar{n} \cdot D_n + i\not{D}_n^\perp \frac{1}{i\bar{n} \cdot D_n} i\not{D}_n^\perp \right] \frac{\not{n}}{2} \chi_n^{(0)}$$



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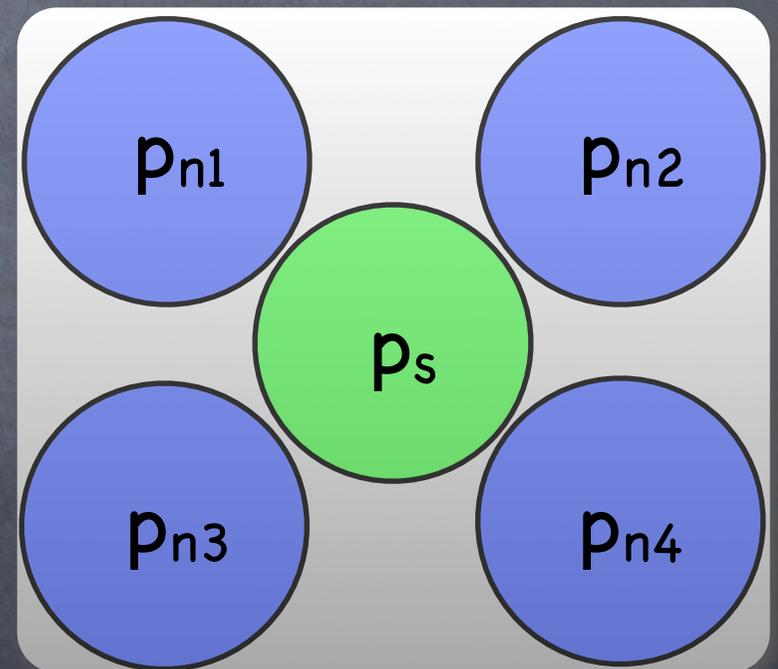
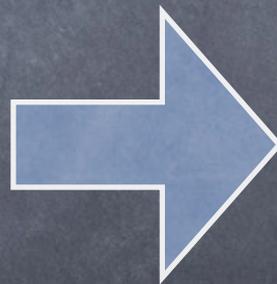
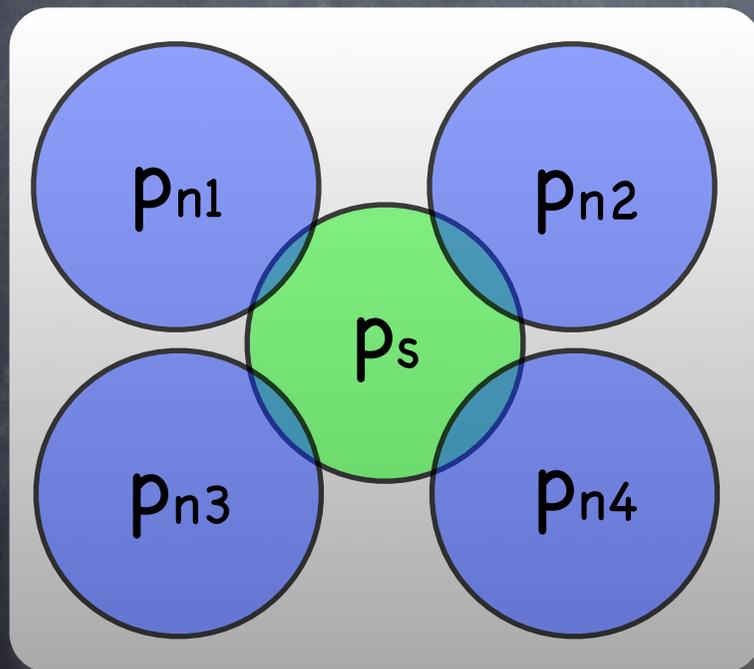
$$Y_n Y_n^\dagger = 1$$

$$i\bar{n} D Y_n = Y_n i\bar{n} \partial$$

Soft/collinear decoupling

CWB, Pirjol, Stewart ('00)

$$\mathcal{L} = \sum_n \bar{\chi}_n^{(0)} \left[i\vec{n} \cdot D_n + i\not{D}_n^\perp \frac{1}{i\vec{n} \cdot D_n} i\not{D}_n^\perp \right] \frac{\not{n}}{2} \chi_n^{(0)}$$



Deep inelastic scattering

Very simple proof

$$\sigma = \sum_X \langle p | \begin{array}{c} \text{wavy line} \\ \nearrow \text{---} \end{array} | X \rangle \langle X | \begin{array}{c} \text{wavy line} \\ \searrow \text{---} \end{array} | p \rangle$$

Very simple proof

$$\sigma = \sum_X \langle p | \begin{array}{c} \text{wavy line} \\ \nearrow \quad \longrightarrow \end{array} | X \rangle \langle X | \begin{array}{c} \text{wavy line} \\ \longleftarrow \quad \searrow \end{array} | p \rangle$$

$$= \langle p | \begin{array}{c} \text{wavy line} \\ \nearrow \quad \longrightarrow \quad \longleftarrow \quad \searrow \end{array} | p \rangle$$

Very simple proof

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Partonic kinematics

$$p_Y = Q (-1, 1, 0)$$

$$p_p = Q (x m_p^2 / Q^2, 1/x, 0) \Rightarrow p_p^2 = m_p^2 \approx \Lambda^2$$

$$p_X = Q (-1 - x m_p^2 / Q^2, (1-x)/x, 0) \Rightarrow p_X^2 \approx Q^2$$

Very simple proof

$$\sigma = \sum_X \langle p | \begin{array}{c} \text{wavy} \\ \nearrow \quad \rightarrow \end{array} |X\rangle \langle X| \begin{array}{c} \text{wavy} \\ \leftarrow \quad \searrow \end{array} |p\rangle$$

$$= \langle p | \begin{array}{c} \text{wavy} \\ \nearrow \quad \text{red} \rightarrow \quad \searrow \\ \text{wavy} \end{array} |p\rangle$$

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$$= \langle p | \begin{array}{c} \text{wavy} \\ \text{red} \\ \text{blue} \end{array} \begin{array}{c} \text{wavy} \\ \text{red} \\ \text{blue} \end{array} | p \rangle$$

$$= H \otimes \langle p | \begin{array}{c} \text{wavy} \\ \text{blue} \end{array} | p \rangle$$

$$\sigma = H \otimes f_q$$

Event shapes near endpoint

What are event shapes?

Simple example is thrust T

$$T = \frac{1}{Q} \max \sum_{i \in X} |\mathbf{t} \cdot \mathbf{p}_i|$$

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For 2-jet events $T=1$



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More generally, define event shape e

$$e = \frac{1}{Q} \sum_{i \in X} |\mathbf{p}_i^T| f_e(\eta_i)$$

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More generally, define event shape e

$$e = \frac{1}{Q} \sum_{i \in X} |\mathbf{p}_i^T| f_e(\eta_i)$$

$$f_T = \exp(-|\eta|)$$

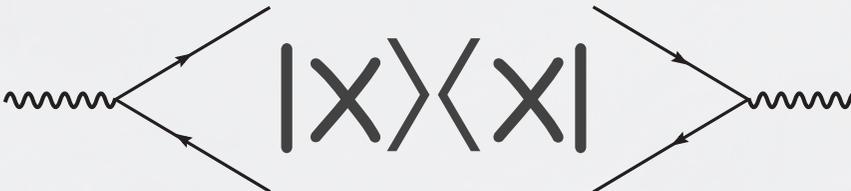
$$f_C = 3/\cosh(\eta)$$

Differences from DIS

- DIS is completely inclusive process
 - Observables formed from leptonic variables
 - Every final state contributes same to final observable
- **Allows to perform the sum over final states**
- Event shape is weighted cross section
 - Observables formed from hadronic variables
 - Different final states contribute with different weight to final observable

Sum over final states not possible

4 steps to the factorization

$$\sigma(e) = \sum_X \langle 0 | \text{---} \langle X | \langle X | \text{---} | 0 \rangle \delta(e - e(X))$$


1. Write $e(X)|X\rangle = \hat{e}|X\rangle$ and sum over states
2. Match onto operators in SCET
3. Use decoupling of Lagrangian to factorize operator
4. Factorize the matrix element and obtain final result

1: Define \hat{e} and sum over states

$$\sigma(e) = \sum_X \langle 0 | \text{---} \langle X | \text{---} | 0 \rangle \delta(e - e(X))$$

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$$= \sum_X \langle 0 | \text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right| \delta(e - \hat{e}) | X \rangle \langle X | \begin{array}{l} \nearrow \\ \searrow \end{array} \text{wavy} | 0 \rangle$$

1: Define \hat{e} and sum over states

$$\sigma(e) = \sum_X \langle 0 | \text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right| X \rangle \langle X | \left. \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \text{wavy} | 0 \rangle \delta(e - e(X))$$

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Next step: match onto SCET

2: Match onto SCET

$$\sigma(e) = \langle 0 | \text{---} \delta(e - \hat{e}) \text{---} | 0 \rangle$$

Match full QCD current onto SCET

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Match full QCD current onto SCET

$$[\bar{q} \Gamma^\mu q] = C [\bar{X}_n \Gamma^\mu X_{\bar{n}}]$$

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Match full QCD current onto SCET

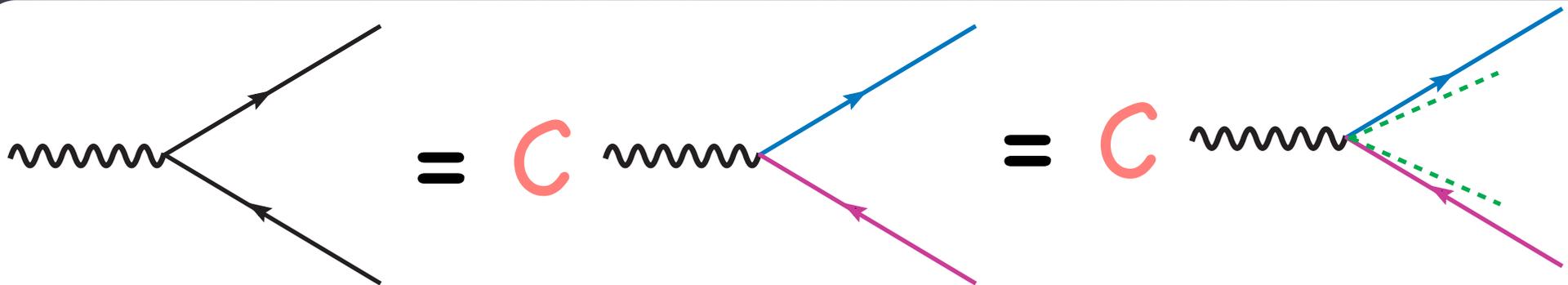
$$[\bar{q} \Gamma^\mu q] = C [\bar{X}_n \Gamma^\mu X_{\bar{n}}] = C [\bar{X}_n^{(0)} Y_n^\dagger \Gamma^\mu Y_{\bar{n}} X_{\bar{n}}^{(0)}]$$

2: Match onto SCET

$$\sigma(e) = \langle 0 | \text{wavy line} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \delta(e - \hat{e}) \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \text{wavy line} | 0 \rangle$$

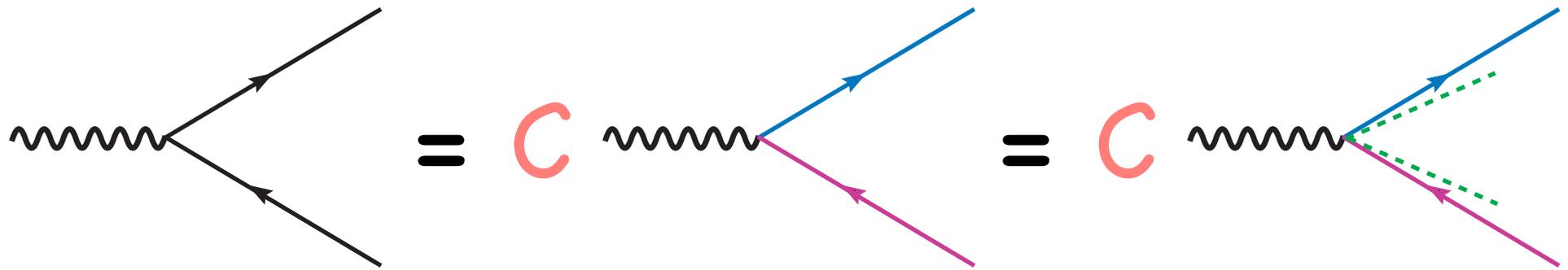
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3: Match onto SCET

$$\sigma(e) = \langle 0 | \text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \delta(e - \hat{e}) \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \text{wavy} | 0 \rangle$$



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$$\text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle = C \text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle = C \text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle$$

The diagram shows a sequence of three Feynman diagrams. The first diagram has a wavy line on the left and two solid black lines on the right. The second diagram has a wavy line on the left, a blue line on the right, and a magenta line on the left. The third diagram has a wavy line on the left, a blue line on the right, a magenta line on the left, and a dashed green line on the right. Each diagram is preceded by an equals sign and a red letter 'C'.

This gives for the cross section

$$\sigma(e) = |C|^2 \langle 0 | \text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \delta(e - \hat{e}) \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \text{wavy} | 0 \rangle$$

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The diagram shows a sequence of three Feynman diagrams. The first diagram has a wavy line on the left and two solid black lines on the right. The second diagram has a wavy line on the left, a blue line on the right, and a magenta line on the right, with a red 'C' between them. The third diagram has a wavy line on the left, a blue line on the right, a magenta line on the right, and a dashed green line on the right, with a red 'C' between them.

This gives for the cross section

$$\sigma(e) = |C|^2 \langle 0 | \text{wavy} \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \delta(e - \hat{e}) \left\langle \begin{array}{l} \nearrow \\ \searrow \end{array} \right\rangle \text{wavy} | 0 \rangle$$

What is \hat{e} ?

4. Factorize \hat{e}

$$\hat{e} = \frac{1}{Q} \int_{-\infty}^{\infty} d\eta f_e(\eta) \mathcal{E}_T(\eta; \hat{t})$$

Transverse energy flow operator defined as

$$\mathcal{E}_T(\eta) = \frac{1}{\cosh^3 \eta} \int_0^{2\pi} d\phi \lim_{R \rightarrow \infty} R^2 \int_0^{\infty} dt \hat{n}_i T_{0i}(t, R\hat{n})$$

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Lagrangian completely decoupled

$$L = L_n + L_{\bar{n}} + L_s \Rightarrow e = \hat{e}_n + \hat{e}_{\bar{n}} + \hat{e}_s$$

Allows to write

$$\delta(e - \hat{e}) = \int de \int de \int de \delta(e - e - e - e)$$

$$\delta(e - \hat{e}) \delta(e - \hat{e}) \delta(e - \hat{e})$$

5: Factorize matrix element

$$\sigma(e) = |C|^2 \langle 0 | \text{wavy} \delta(e - \hat{e}) \text{wavy} | 0 \rangle$$

The diagram shows a wavy line connecting the bra state $\langle 0|$ to the operator $\delta(e - \hat{e})$, and another wavy line connecting the operator to the ket state $|0\rangle$. On the left wavy line, a blue arrow points up-right and a magenta arrow points down-right. On the right wavy line, a blue arrow points up-left and a magenta arrow points down-left. Dotted lines also connect the wavy lines to the operator.

Using operator identity from previous slide

$$\langle 0 | \text{wavy} \delta(e - \hat{e}) \text{wavy} | 0 \rangle = \int de \int de \int de$$

The diagram is identical to the one above, showing the matrix element with wavy lines and colored arrows.

$$\times \langle 0 | \text{wavy} \begin{matrix} \delta(e - \hat{e}) \\ \delta(e - \hat{e}) \\ \delta(e - \hat{e}) \end{matrix} \text{wavy} | 0 \rangle \delta(e - e - e - e)$$

The diagram shows a wavy line connecting the bra state $\langle 0|$ to a central point, and another wavy line connecting the central point to the ket state $|0\rangle$. From this central point, three lines branch out: a blue line to the top, a green line to the middle, and a magenta line to the bottom. Each line ends with a $\delta(e - \hat{e})$ operator. The operators are colored blue, green, and magenta respectively. A fourth line, colored black, extends to the right and ends with a $\delta(e - e - e - e)$ operator. The wavy lines and colored arrows are the same as in the previous diagrams.

5: Factorize matrix element

$$\sigma(e) = |C|^2 \langle 0 | \text{wavy} \delta(e - \hat{e}) \text{wavy} | 0 \rangle$$

The diagram shows a wavy line connecting the bra state $\langle 0|$ to the delta function $\delta(e - \hat{e})$, and another wavy line connecting the delta function to the ket state $|0\rangle$. Blue arrows point from the wavy lines towards the delta function, and pink arrows point from the delta function towards the wavy lines.

$$\sigma(e) = |C|^2 \int de \int de \int de \delta(e - e - e - e)$$

$$\langle 0 | \delta(e - \hat{e}) | 0 \rangle \langle 0 | \delta(e - \hat{e}) | 0 \rangle$$

The diagram shows two separate delta function matrix elements. The first is $\langle 0 | \delta(e - \hat{e}) | 0 \rangle$ with blue arrows pointing from the bra to the delta and from the delta to the ket. The second is $\langle 0 | \delta(e - \hat{e}) | 0 \rangle$ with pink arrows pointing from the bra to the delta and from the delta to the ket.

$$\langle 0 | \delta(e - \hat{e}) | 0 \rangle$$

The diagram shows a single delta function matrix element $\langle 0 | \delta(e - \hat{e}) | 0 \rangle$ with dashed green lines connecting the bra state to the delta function and the delta function to the ket state.

$$\sigma(e) = H \otimes J_1 \otimes J_2 \otimes S$$

The same in equations

$$\sigma(e) = H \otimes J_1 \otimes J_2 \otimes S$$

$$\frac{1}{\sigma_0} \frac{d\sigma}{de} = |C_2(Q; \mu)|^2 \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s) J_n(e_n; \mu) J_{\bar{n}}(e_{\bar{n}}; \mu) S(e_s; \mu)$$

with

The same in equations

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$$\frac{1}{\sigma_0} \frac{d\sigma}{de} = |C_2(Q; \mu)|^2 \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s) J_n(e_n; \mu) J_{\bar{n}}(e_{\bar{n}}; \mu) S(e_s; \mu)$$

$$J_n(e_n; \mu) \equiv \int \frac{dk^+}{2\pi} \mathcal{J}_n(e_n, k^+; \mu)$$

with

$$J_{\bar{n}}(e_{\bar{n}}; \mu) \equiv \int \frac{dl^-}{2\pi} \mathcal{J}_{\bar{n}}(e_{\bar{n}}, l^-; \mu)$$

$$S(e_s; \mu) \equiv \int \frac{d^4r}{(2\pi)^4} S(e_s, r; \mu)$$

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$$\langle 0 | \chi_n(x)_\beta \delta(e_n - \hat{e}_n) \bar{\chi}_n(0)_\gamma | 0 \rangle \equiv \int \frac{dk^+ dk^- d^2k_\perp}{2(2\pi)^4} e^{-ik \cdot x} \mathcal{J}_n(e_n, k^+; \mu) \left(\frac{\not{n}}{2} \right)_{\beta\gamma}$$

$$\langle 0 | \bar{\chi}_{\bar{n}}(x)_\alpha \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}) \chi_{\bar{n}}(0)_\delta | 0 \rangle \equiv \int \frac{dl^+ dl^- d^2l_\perp}{2(2\pi)^4} e^{-il \cdot x} \mathcal{J}_{\bar{n}}(e_{\bar{n}}, l^-; \mu) \left(\frac{\not{\bar{n}}}{2} \right)_{\delta\alpha}$$

$$\frac{1}{N_C} \text{Tr} \langle 0 | \bar{Y}_{\bar{n}}(x) Y_n^\dagger(x) \delta(e_s - \hat{e}_s) Y_n(0) \bar{Y}_{\bar{n}}(0) | 0 \rangle \equiv \int \frac{d^4r}{(2\pi)^4} e^{-ir \cdot x} S(e_s, r; \mu)$$

What can we do with this?

Lee, Stermann ('06)

- Results splits up into several simpler functions
- Operator definition of non-perturbative contributions

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \delta \left(e - \frac{1}{Q} \int d\eta f_e(\eta) \mathcal{E}(\eta) \right) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle$$

Using boost along \mathbf{n} direction with rapidity η' , show

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \delta \left(e - \frac{1}{Q} \int d\eta f_e(\eta) \mathcal{E}(\eta + \eta') \right) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle$$

Choose $\eta' = -\eta$ and define $F_e = \int d\eta f_e(\eta)$

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \delta \left(e - \frac{F_e}{Q} \mathcal{E}(0) \right) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle$$

What can we do with this?

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \delta \left(e - \frac{F_e}{Q} \mathcal{E}(0) \right) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle$$

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Calculable parameter

$$F_T=2, F_C=3\pi$$

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Universal operator

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Calculable parameter
 $F_T=2, F_C=3\pi$

Universal operator

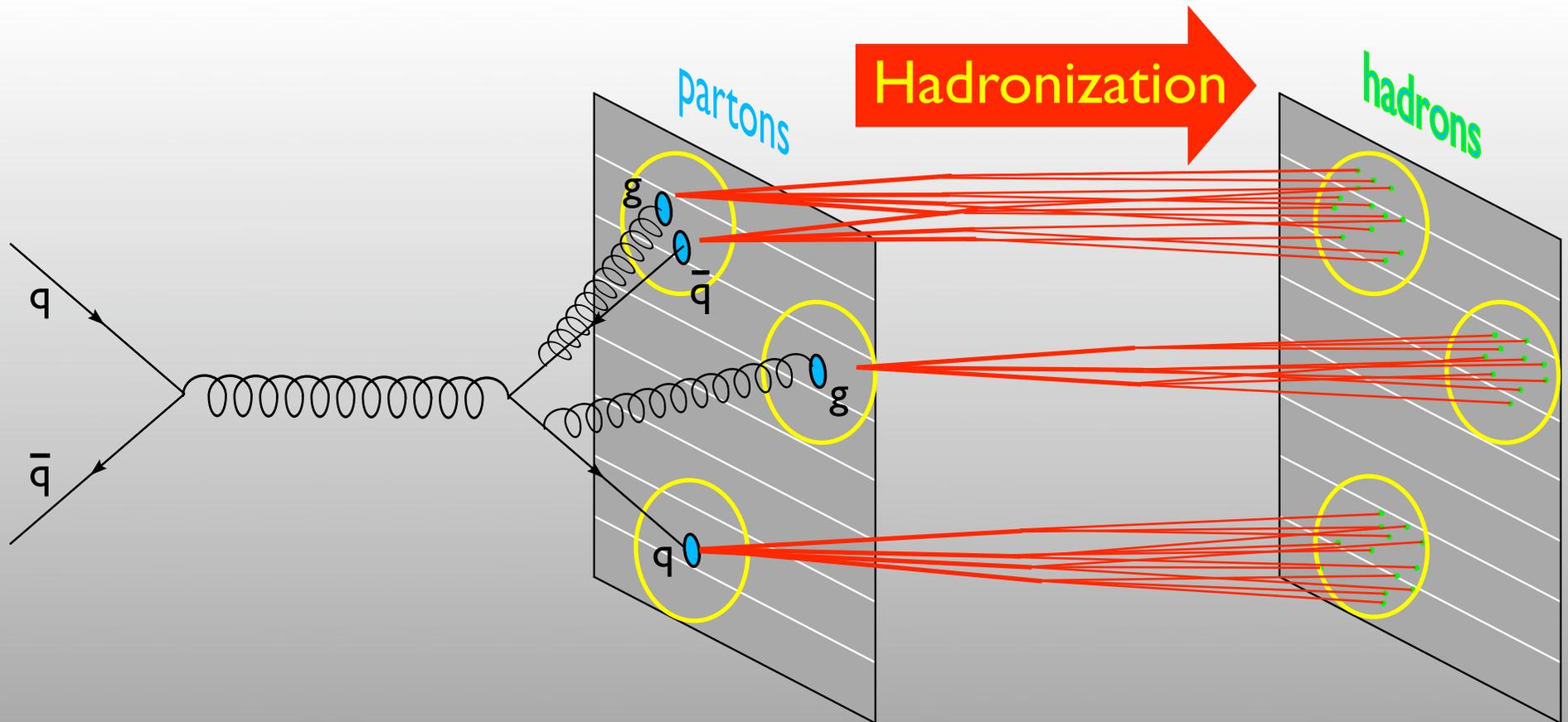
For example, expand in $1/Q$ to find

$$S(e) = \delta(e) + \delta'(e) \frac{F_e}{Q} \left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \mathcal{E}(0) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle + \dots$$

Non-perturbative matrix element independent on which event shape is considered

Towards factorization for jet production

How to deal with jets



How to deal with jets

- Jets: collection of hadrons which are “close together”
- How can we quantify this statement?
- Need jet algorithms, many possibilities

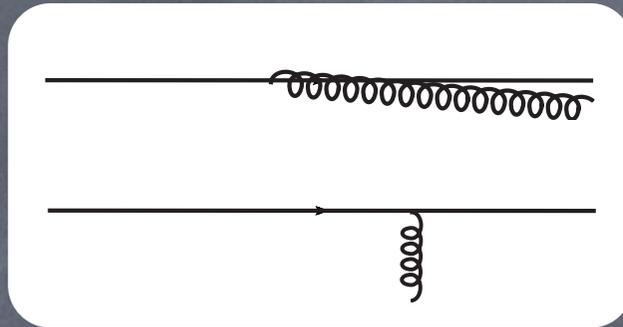
Jet algorithms groups all particles into jets, and returns the four-momentum of every jet

Requirements on jet algorithms

- Has to be efficient
- Needs to be IR safe

Requirement for IR safety

QCD has collinear and soft divergences



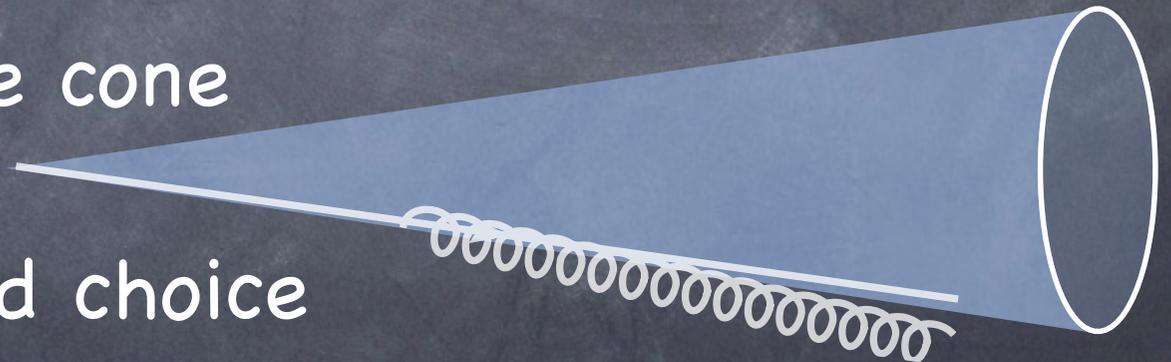
Collinear gluon

Soft gluon

Both emissions have large couplings in QCD

Jet algorithm should not be sensitive to either

For example, a naive cone based algorithm would not be a good choice



Total momentum of jet is unaffected by soft or collinear emission

Jet observables

1. Jet algorithm groups particles into jets...

$$\{p_1, p_2, \dots, p_N\} = \{\{p_1, \dots, p_{n_1}\}, \{p_1, \dots, p_{n_2}\}, \dots\}$$

2...and returns total four momentum of each jet

$$J(\{p_1, p_2, \dots, p_N\}) = \{P_1, P_2, \dots\}$$

3. Observables are formed out of the jet momenta

$$o = O(\{P_1, P_2, \dots\}) = O[J](\{p_1, p_2, \dots, p_N\})$$

$$\sigma(o) = \sum_X \langle pp | \text{---} | X \rangle \langle X | \text{---} | pp \rangle \times \delta(o - O[J](\{p_1(X), \dots, p_N(X)\}))$$

Need operator that picks out momenta of particles

Using the operator idea again

A simple operator that is available

$$\mathcal{E}(\hat{n}) |X\rangle = \sum_{i \in X} \omega_i \delta^3(\hat{n} - \hat{n}_i) |X\rangle$$

$$\mathcal{E}(\hat{n}) = \lim_{R \rightarrow \infty} R^2 \int_0^\infty dt \hat{n}_i T_{0i}(t, R\hat{n})$$

As before

$$\hat{\xi} = \hat{\xi} + \hat{\xi} + \hat{\xi}$$

Allows to write

$$\delta(\varepsilon - \hat{\xi}) = \int d\varepsilon \int d\varepsilon \int d\varepsilon \delta(\varepsilon - \varepsilon - \varepsilon - \varepsilon)$$
$$\delta(\varepsilon - \hat{\xi}) \delta(\varepsilon - \hat{\xi}) \delta(\varepsilon - \hat{\xi})$$

Factorize matrix element

$$\sigma(o) = |H|^2 \int d\varepsilon \int d\varepsilon \int d\varepsilon O[J][\varepsilon + \varepsilon + \varepsilon]$$

$$\times \langle 0 | \begin{array}{c} \nearrow \searrow \\ \nearrow \searrow \end{array} | 0 \rangle \langle 0 | \begin{array}{c} \searrow \nearrow \\ \searrow \nearrow \end{array} | 0 \rangle$$

$$\times \langle 0 | \begin{array}{c} \nearrow \delta(\varepsilon - \hat{\varepsilon}) \searrow \\ \nearrow \delta(\varepsilon - \hat{\varepsilon}) \searrow \end{array} | 0 \rangle \langle 0 | \begin{array}{c} \searrow \delta(\varepsilon - \hat{\varepsilon}) \nearrow \\ \searrow \delta(\varepsilon - \hat{\varepsilon}) \nearrow \end{array} | 0 \rangle$$

$$\times \langle 0 | \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} \delta(\varepsilon - \hat{\varepsilon}) \begin{array}{c} \text{---} \text{---} \\ \text{---} \text{---} \end{array} | 0 \rangle$$

Need to study behavior of $O[J]$. For reasonable jet def's, depends at leading order either on ε or ε

$$\sigma(o) = O[J] \otimes H \otimes f_q \otimes f_q \otimes J_1 \otimes J_2 \otimes S$$

Is the soft function relevant?

$$\sigma(o) = |H|^2 \int d\varepsilon \int d\varepsilon \int d\varepsilon O[J][\varepsilon + \varepsilon + \varepsilon] \\ \times f_q \otimes f_q \otimes J_1(\varepsilon) \otimes J_1(\varepsilon) \otimes S(\varepsilon)$$

If

$$O[J](\varepsilon + \varepsilon + \varepsilon) = O[J](\varepsilon + \varepsilon)$$

Example is $m(j_1, j_2)$

$$\sigma(o) = |H|^2 \int d\varepsilon \int d\varepsilon O[J][\varepsilon + \varepsilon] \\ \times f_q \otimes f_q \otimes J_1(\varepsilon) \otimes J_1(\varepsilon) \otimes \int d\varepsilon S(\varepsilon)$$

Is the soft function relevant?

$$\begin{aligned} & \int d\varepsilon \langle 0 | \text{---} \delta(\varepsilon - \hat{\varepsilon}) \text{---} | 0 \rangle \\ &= \langle 0 | \text{---} \text{---} | 0 \rangle \\ &= \langle 0 | [Y_1 Y_2^\dagger Y_3 Y_4^\dagger] [Y_4 Y_3^\dagger Y_2 Y_1^\dagger] | 0 \rangle \\ &= 1 \end{aligned}$$

Soft function only important if observable sensitive to soft momenta

Conclusions

- Understanding factorization is crucial to make theoretical predictions for experimental observables
- Factorization can be understood using SCET
- Factorization simple for totally inclusive processes
- For weighted cross sections, need operator statement about restricted final states
- Allows to understand factorization in event shapes without assumption about parton-hadron duality
- Similar methods applicable to jet production at hadron colliders, but some more work required

