Towards factorization for jet production

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In collaboration with
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0801.???? [hep-ph]
Andrew Horning, Frank Tackmann

An artist's rendering

Motivation

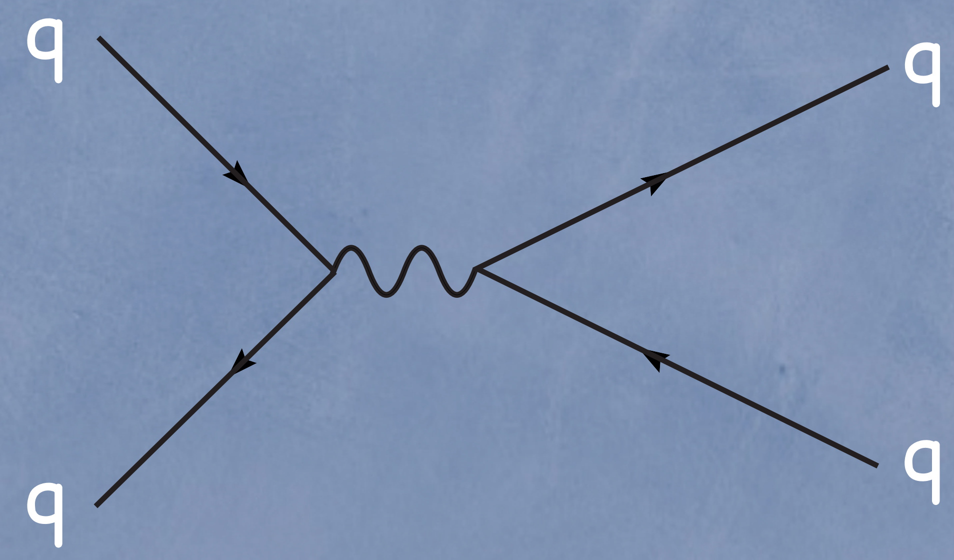
- Most interactions at hadron colliders produce multiple high energy partons in different directions
- Want to study distributions of these partons wrt to one another
- Clearly, will not see partons, and hadronization will be important
- Use jets of hadrons to identify the underlying parton

Questions

- How are partonic results related to jet observables?
- What is calculable in PT, and how does PT behave?
- What non-perturbative physics is needed?
- How do different jet definitions affect results?

Motivation

- We clearly expect perturbative QCD to provide some understanding for jet production at high energies



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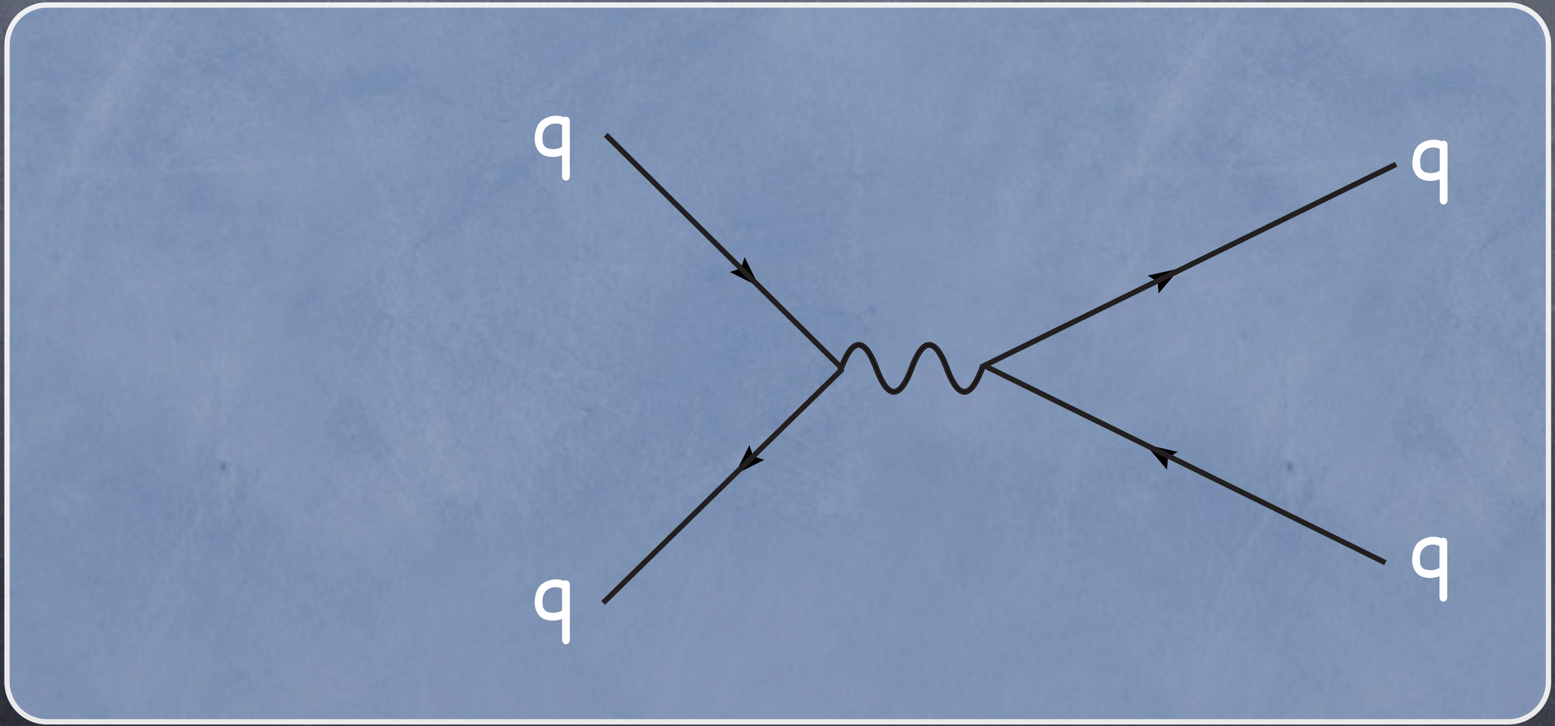
- We clearly expect perturbative QCD to provide some understanding for jet production at high energies
- We also clearly expect non-perturbative physics to play a role, at least for hadron-hadron interactions



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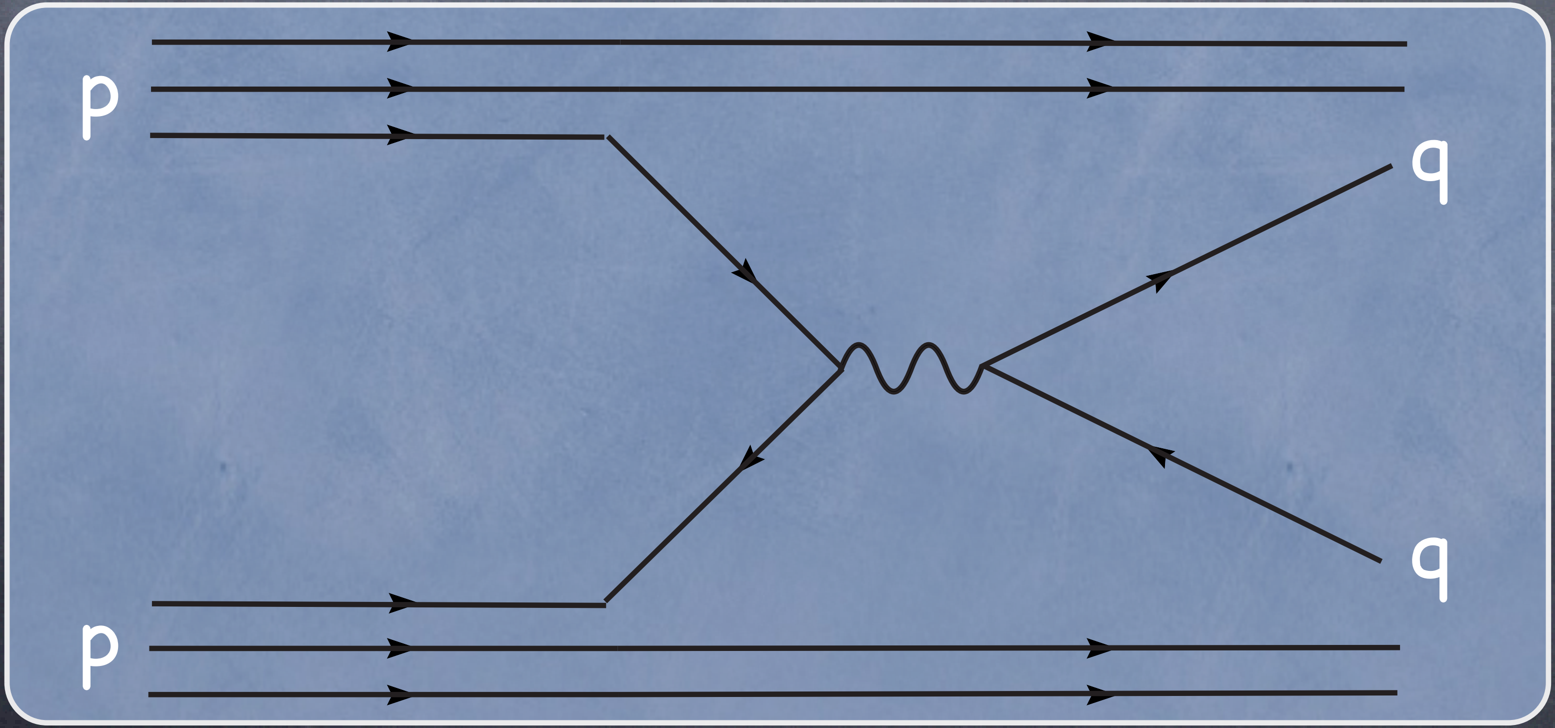

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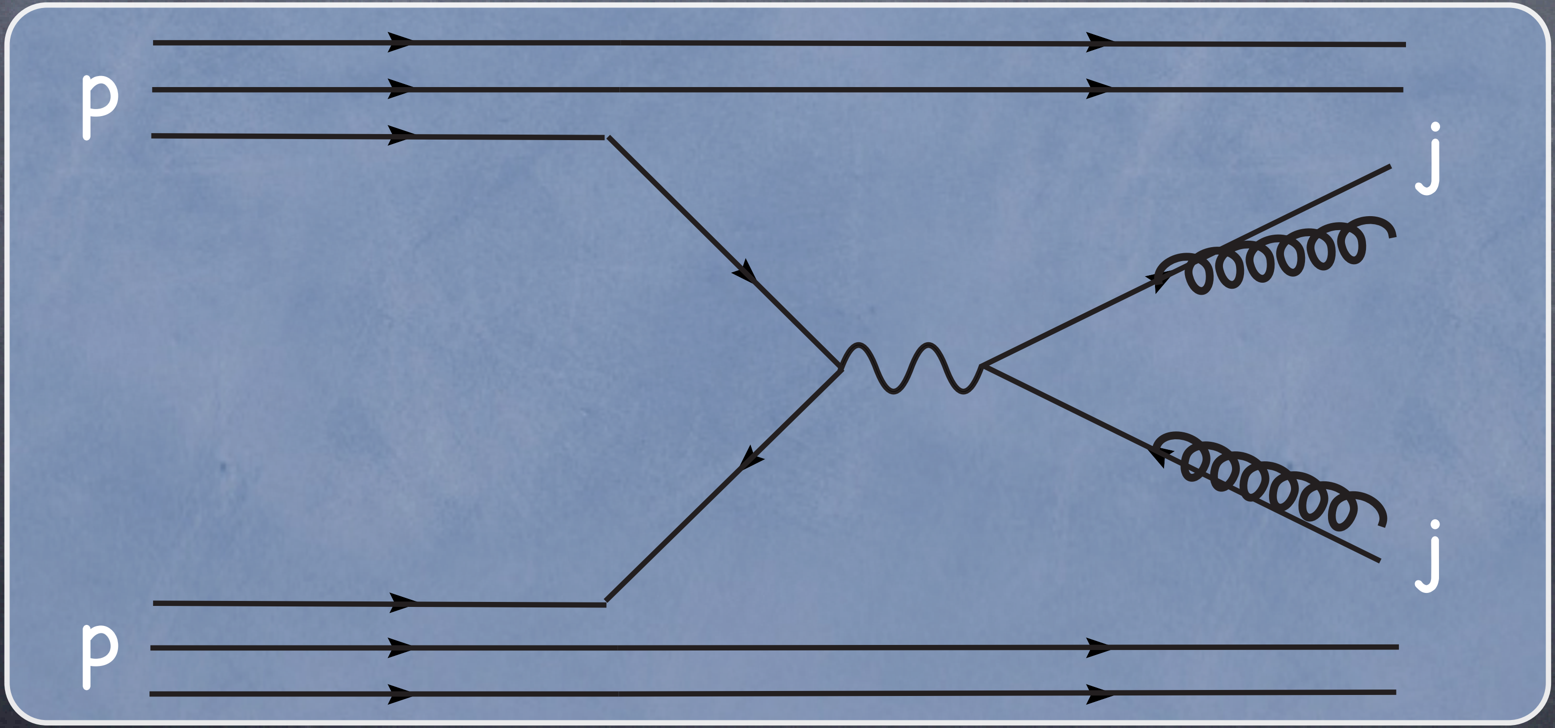

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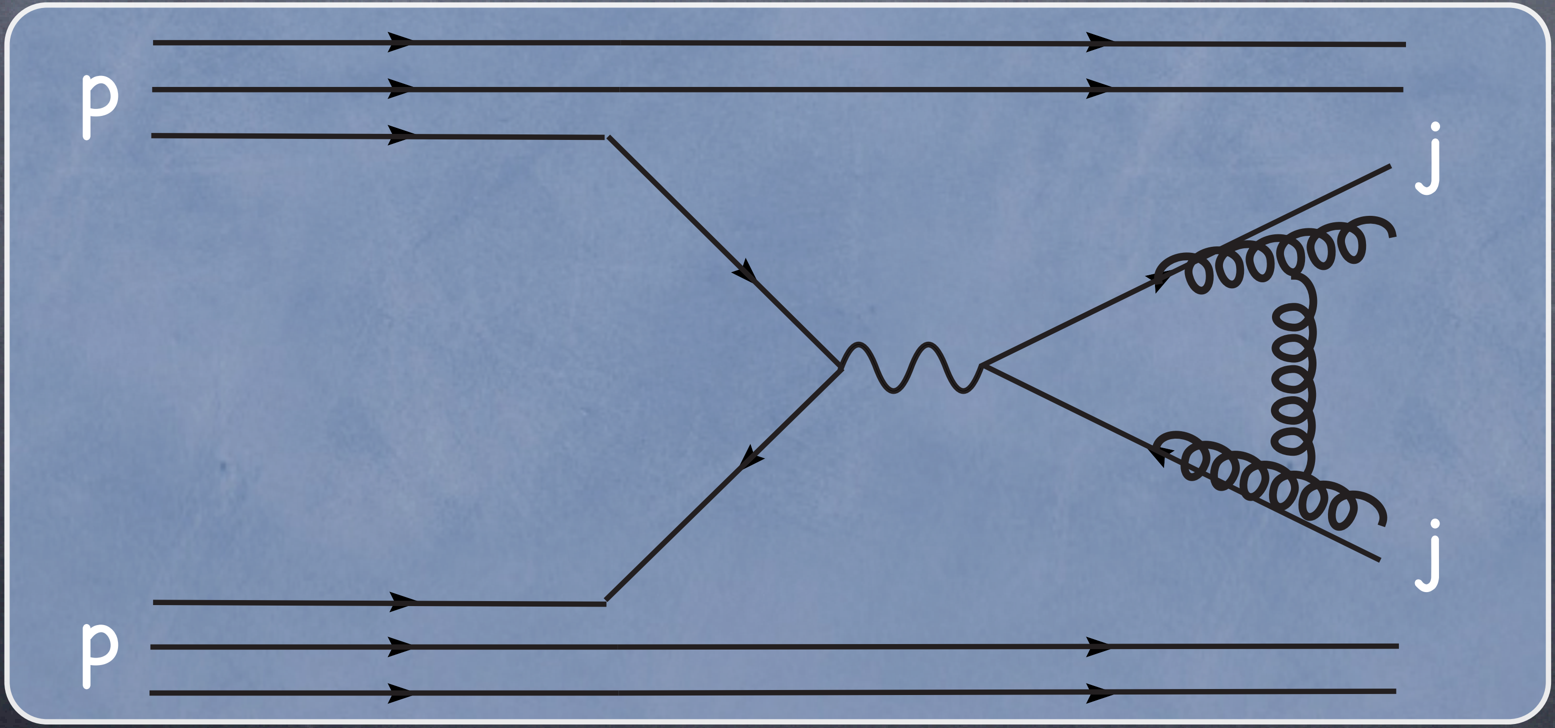

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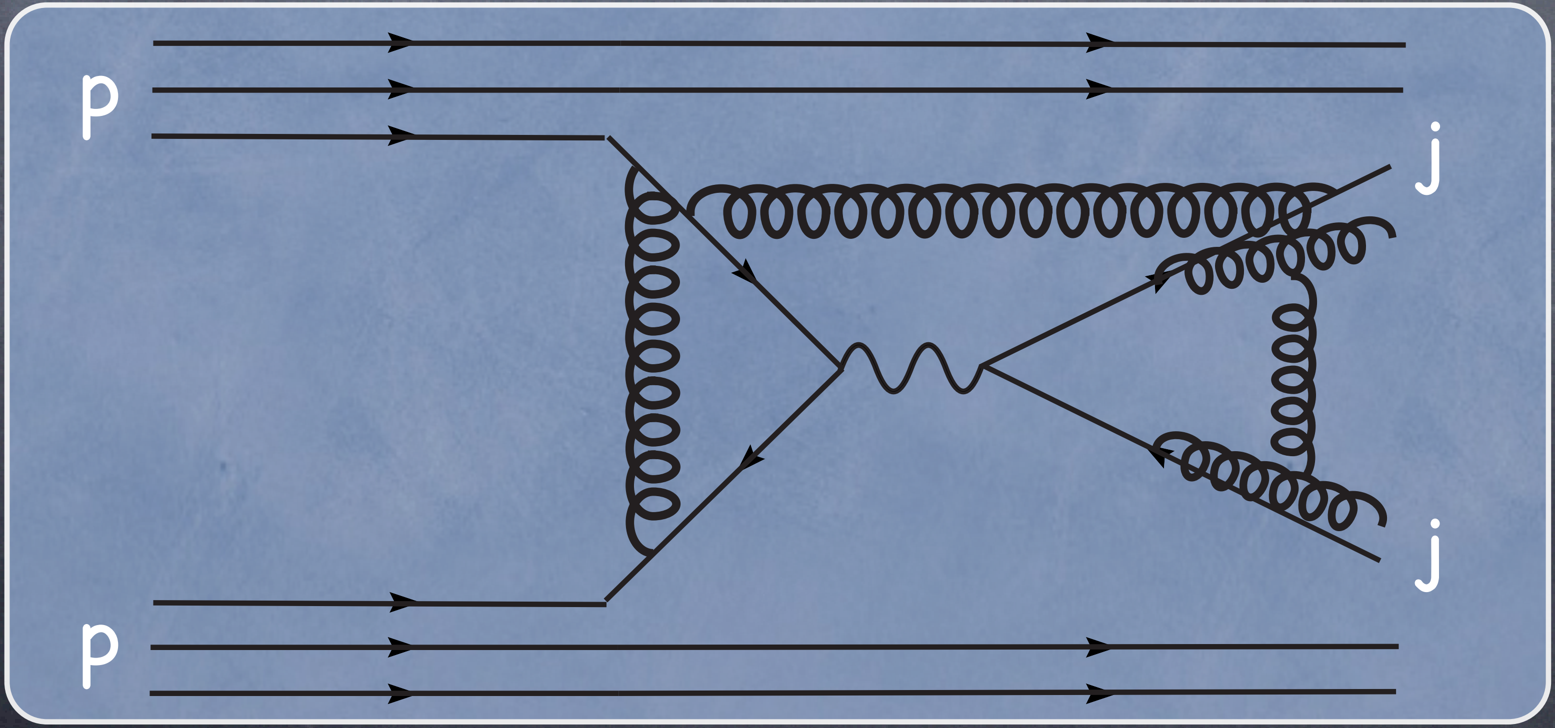


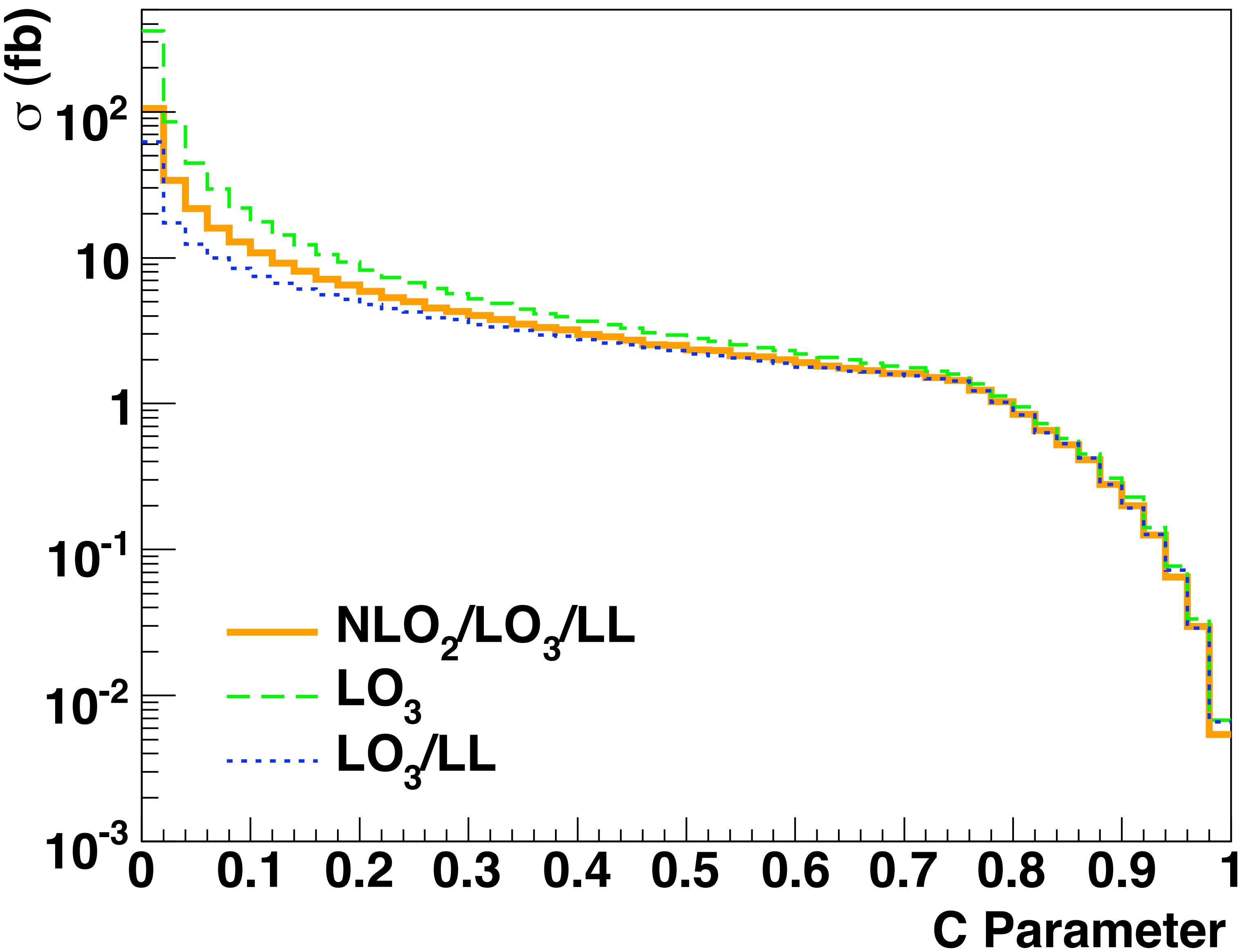
Motivation

- We usually think of QCD in terms of a perturbative expansion
- Presence of widely separated scales gives rise to logarithmic terms $\alpha_s^n \log^m(\Lambda_1/\Lambda_1)$
- Need to resum these terms to get precise theoretical prediction
- In jet physics, many different energy scales possible: m(jet), E(jet), m(jet_1,jet_2), ...

No known way to sum these logarithms without factorization of process

Some output from GenEvA

CWB, Tackmann, Thaler ('08)



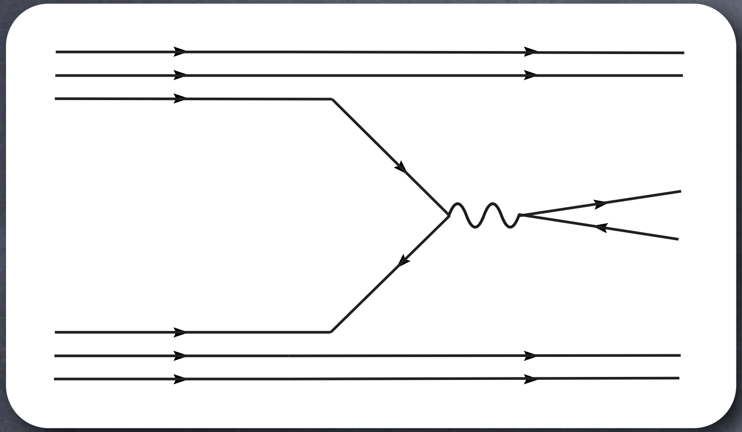
Previous work

- First proofs of factorization based on pioneering work of Collins, Soper and Sterman
  Collins, Soper, Sterman (80s)
- Study properties of Feynman diagrams to separate long and short distance physics
- Very well understood for bread&butter physics (DIS, DY, ...)
- Much work for more general processes
  For a review, see Sterman's TASI lectures
- Effective field theory treatment possible since invention of SCET
  CWB, Fleming, Pirjol, Rothstein, Stewart ('02)

Introduction to Factorization

Introduction to Factorization

Drell Yan: $p + p \rightarrow X + e^- + e^+$



$$\sigma(p+p\rightarrow X+e^-+e^+)$$
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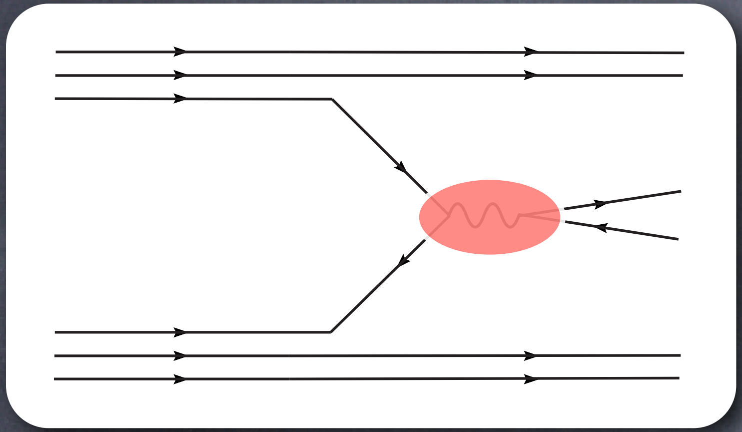

$\sigma(p+p \to X+e^-+e^+)$

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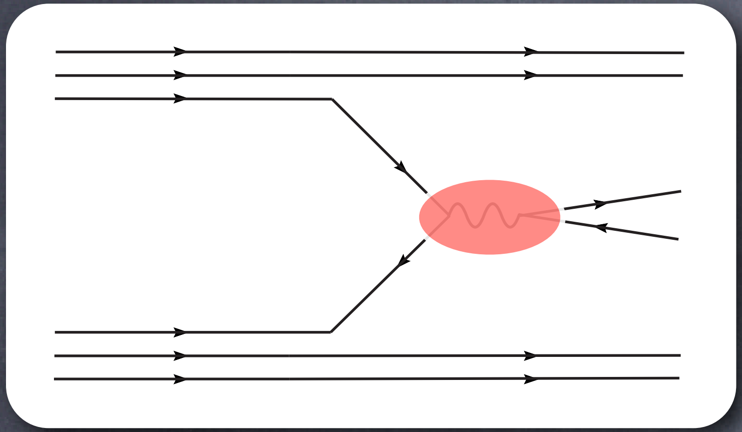

$\sigma(p+p \to X+e^-+e^+)$

$= \sigma(q+q \to e^-+e^+)$

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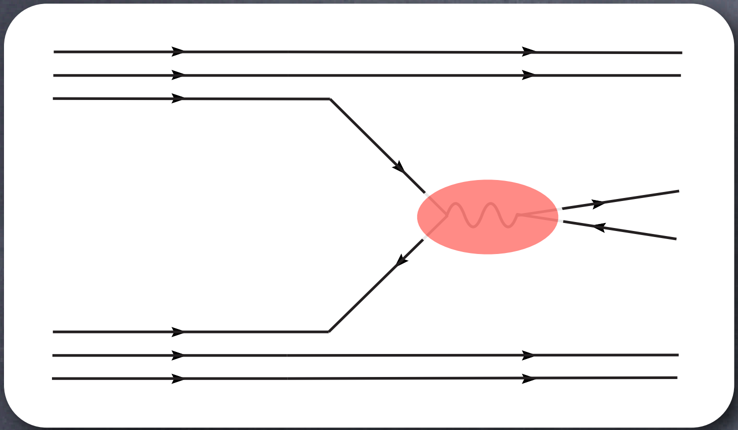

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- Partonic cross section
- Short distance
- Perturbative

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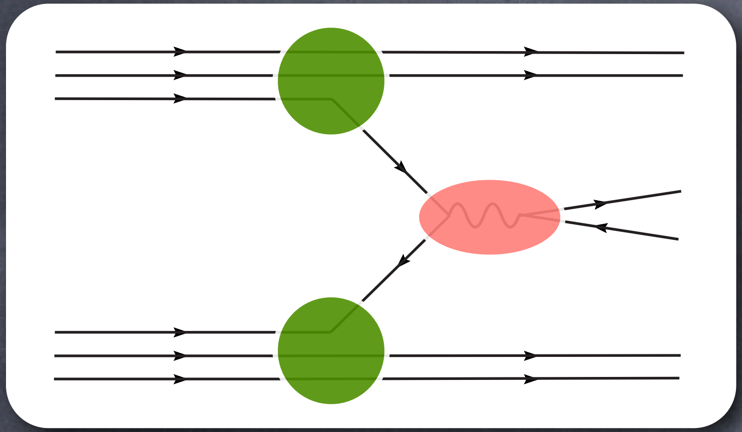

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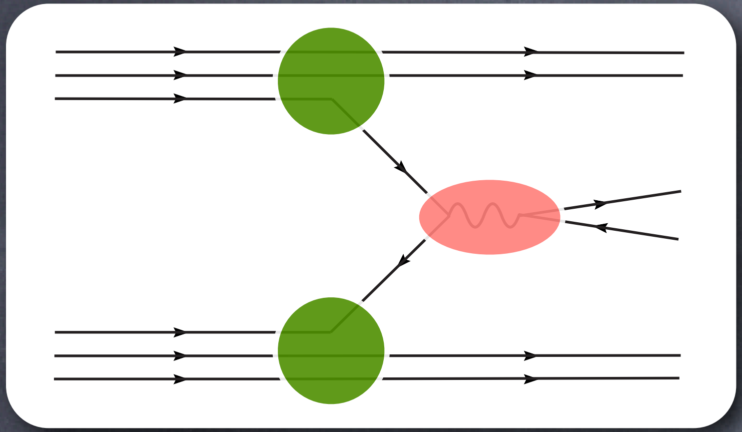

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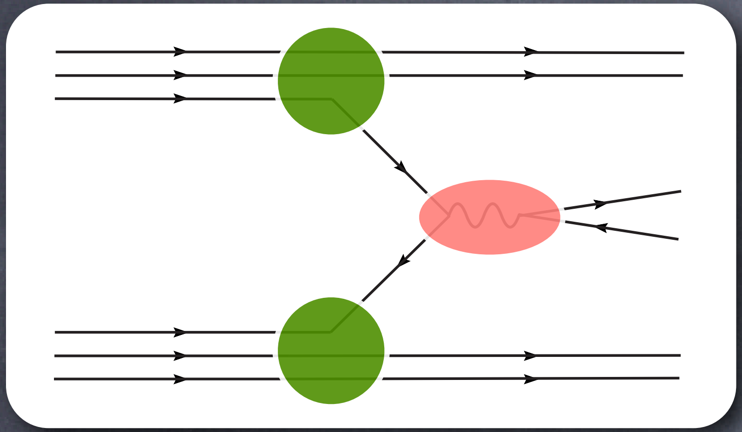

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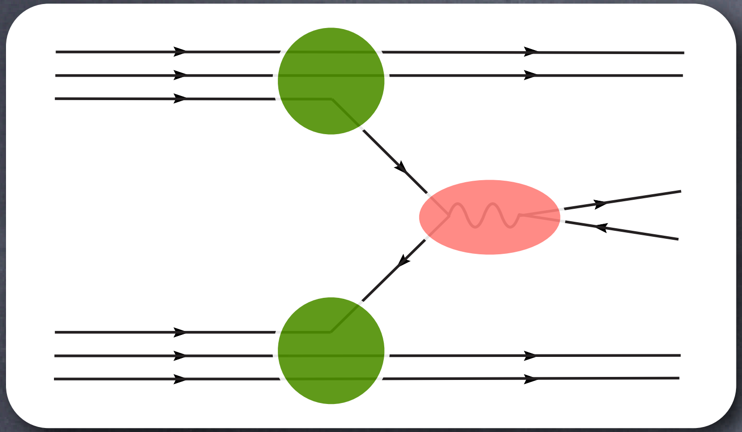

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How do we get non-perturbative information?

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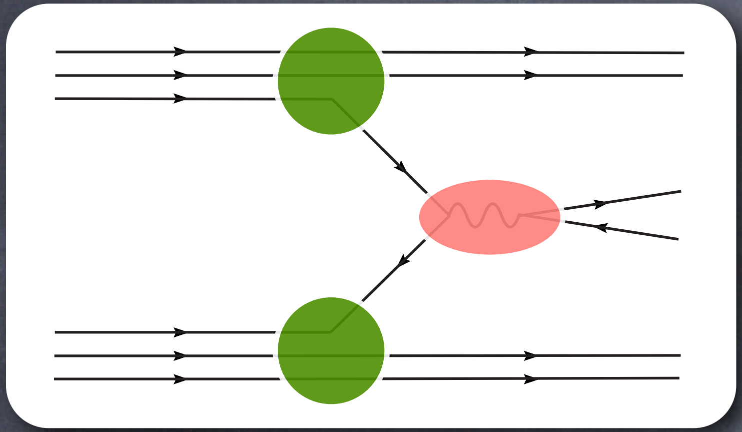

$$\sigma(p+p\to X+e^-+e^+)$$
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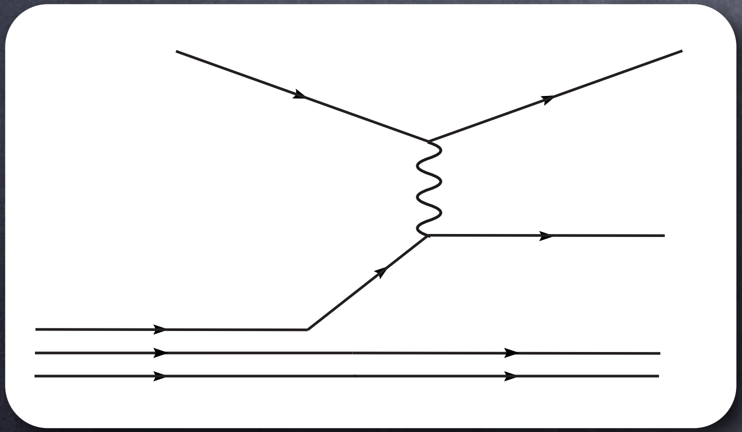
DIS: $p + e^- \to X + e^-$



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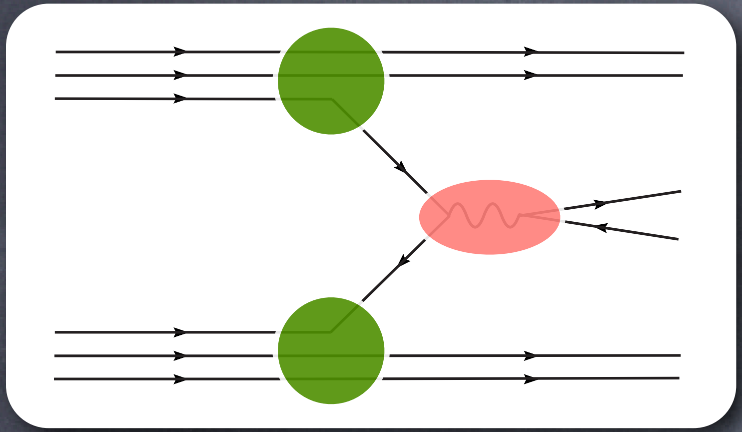

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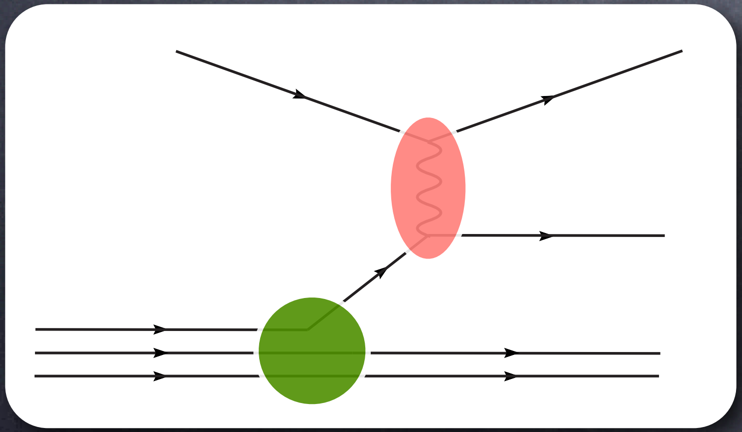

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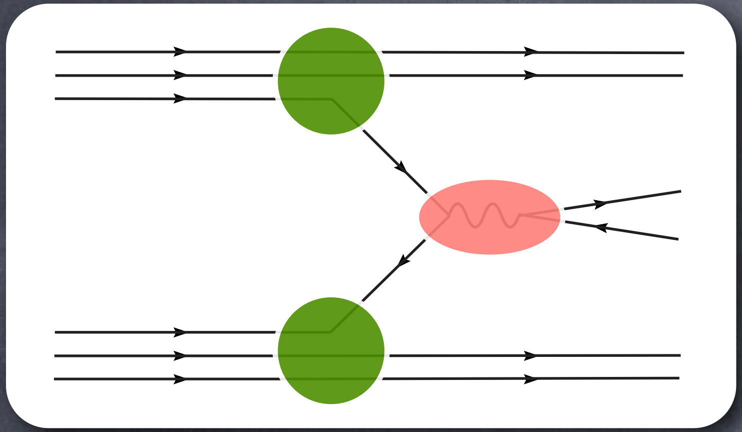

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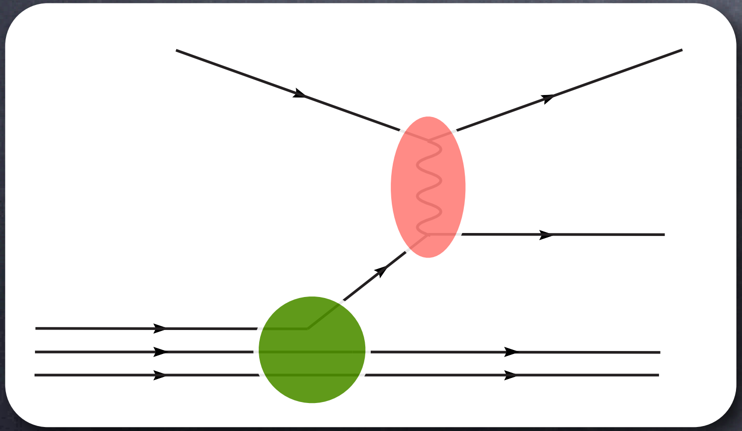
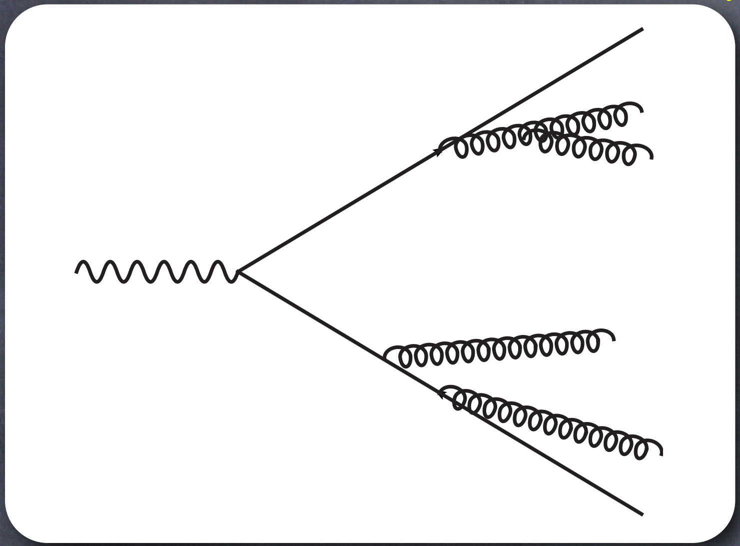
DIS: $p + e^- \to X + e^-$



$$\sigma(p+e^-\to X+e^-) = f_q \otimes \sigma(q+e^-\to q+e^-)$$

Introduction to Factorization

Event shapes: $e^+ + e^- \to$ hadrons



$\sigma(e^+ + e^- \to \text{hadrons})$

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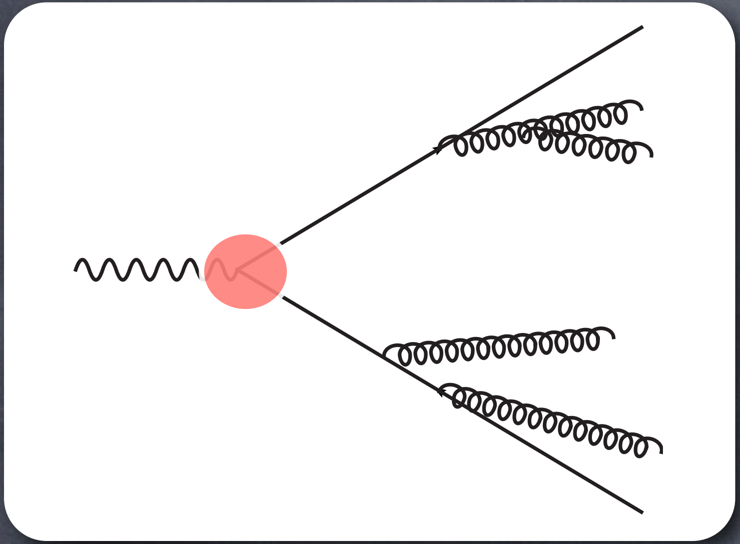

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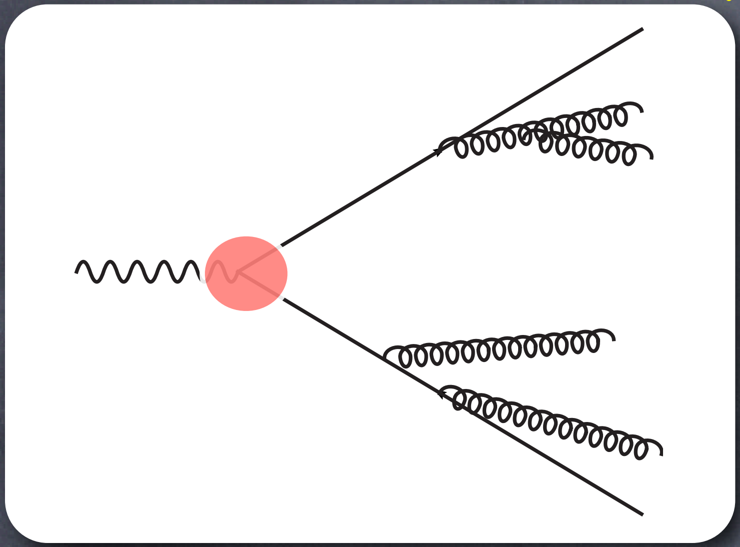

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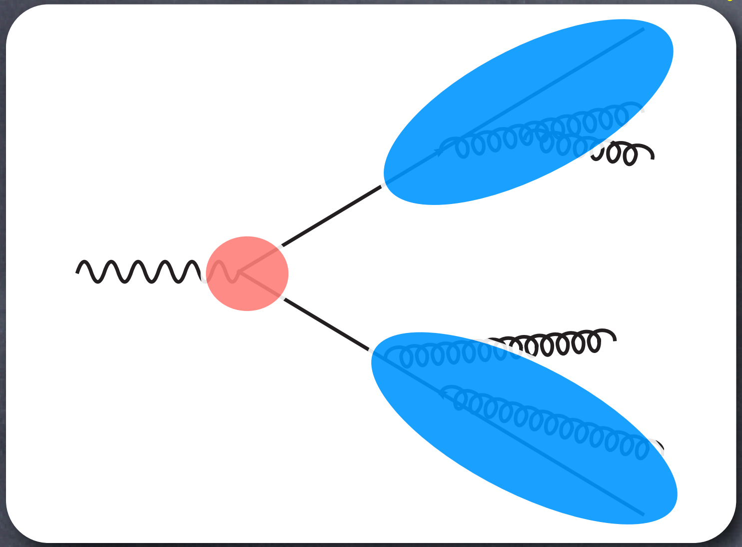

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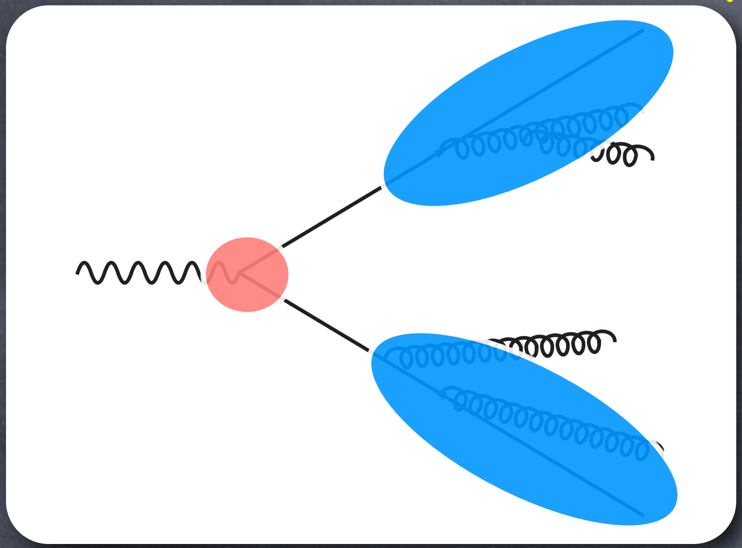

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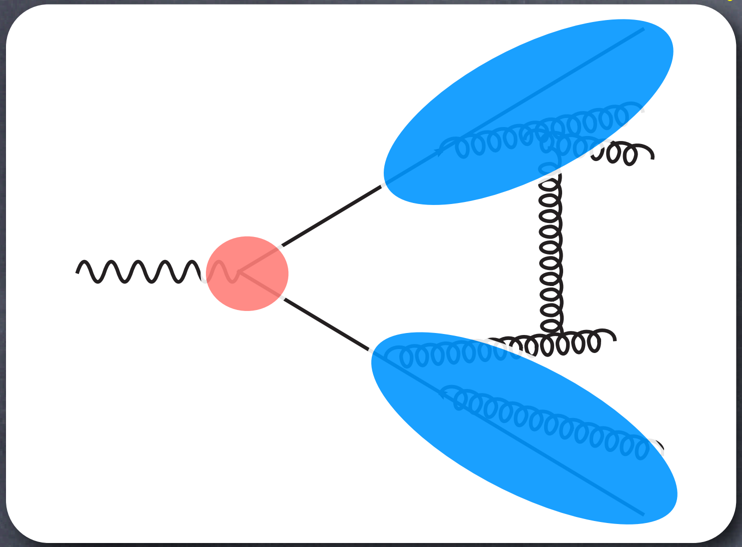

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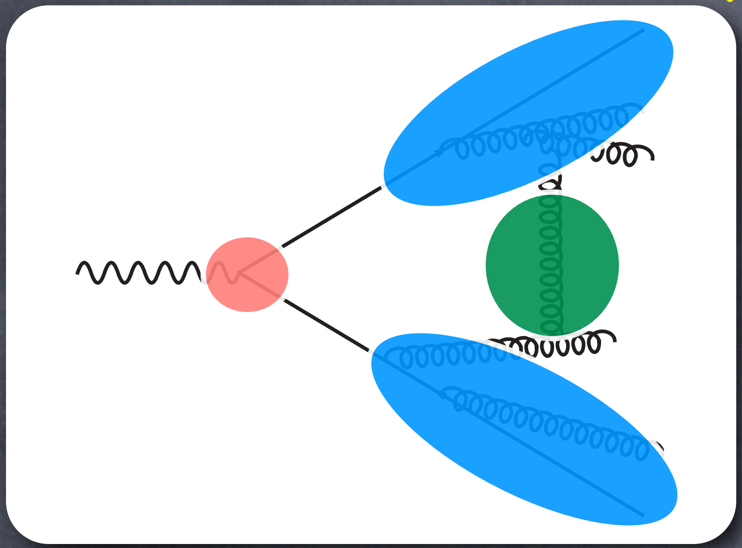

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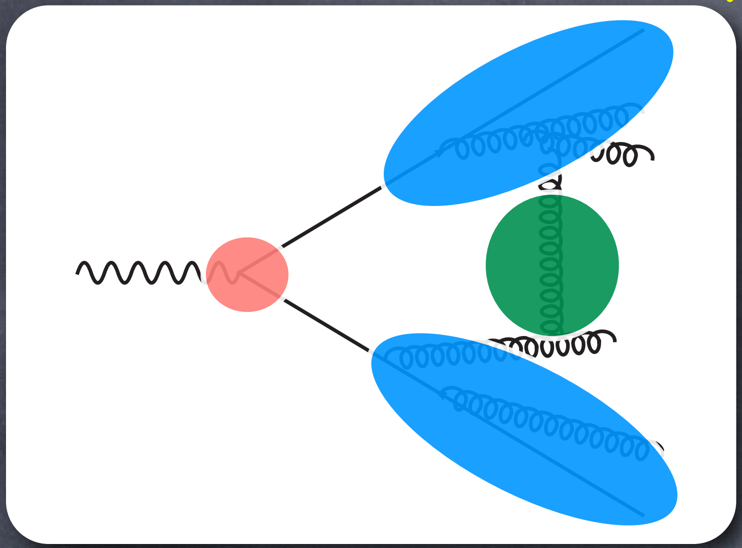

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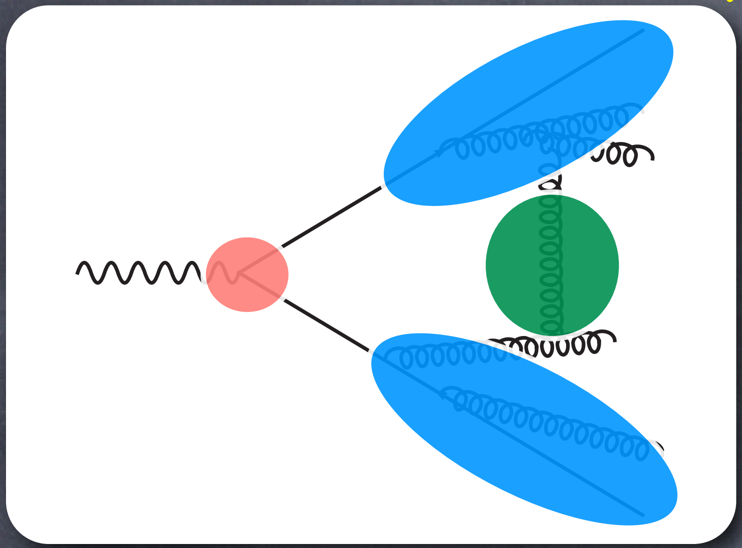

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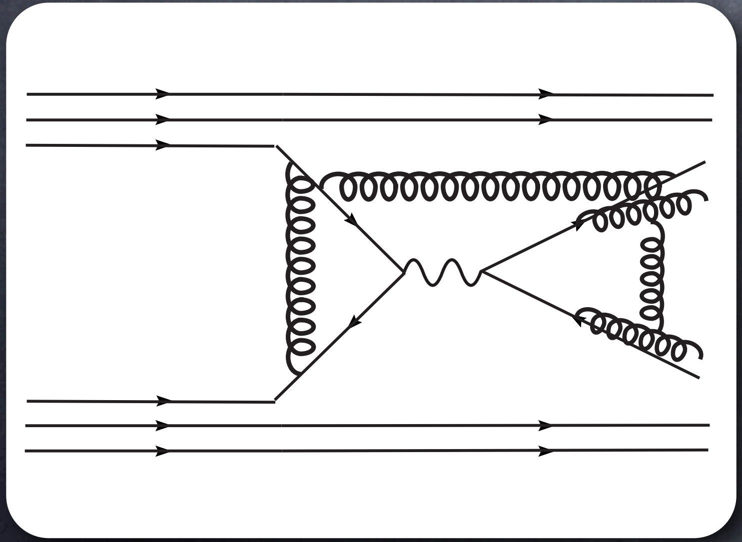
Jet production: p + p → jets



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Introduction to Factorization

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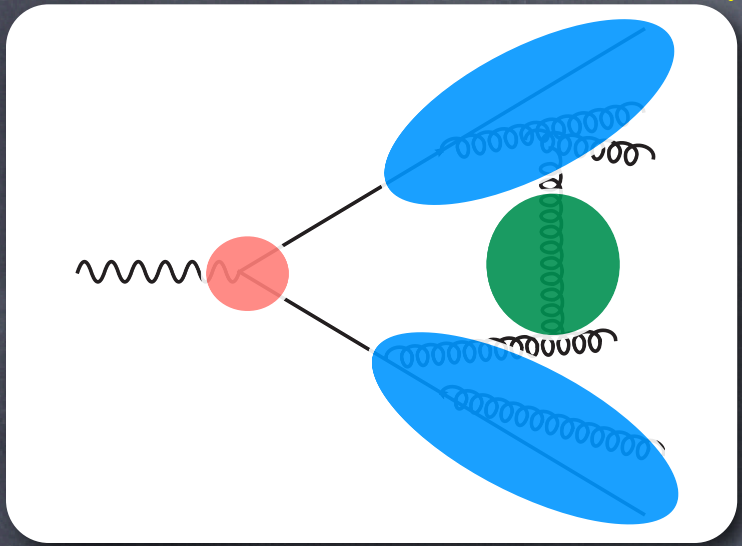

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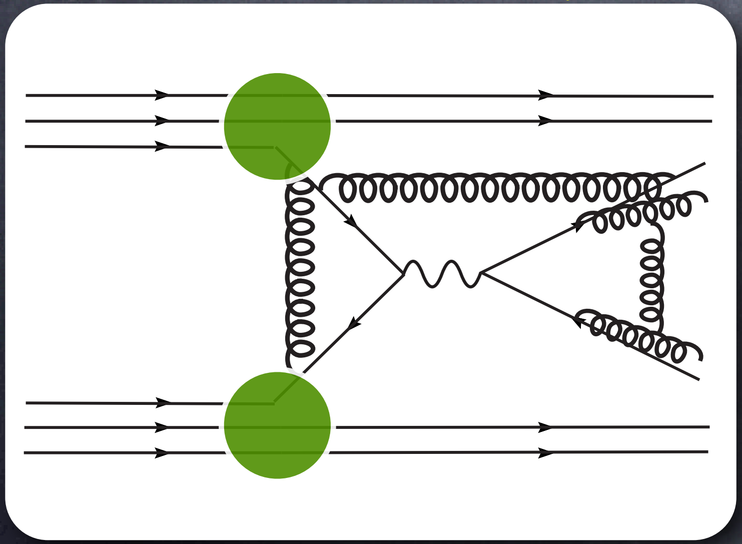

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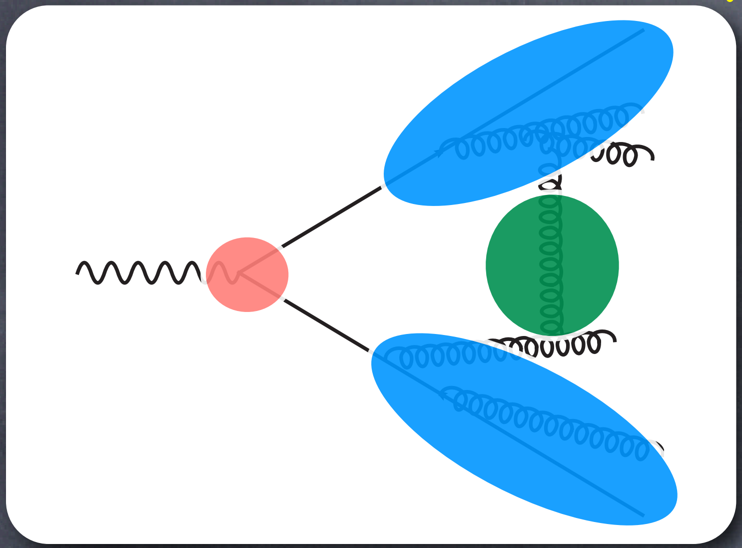

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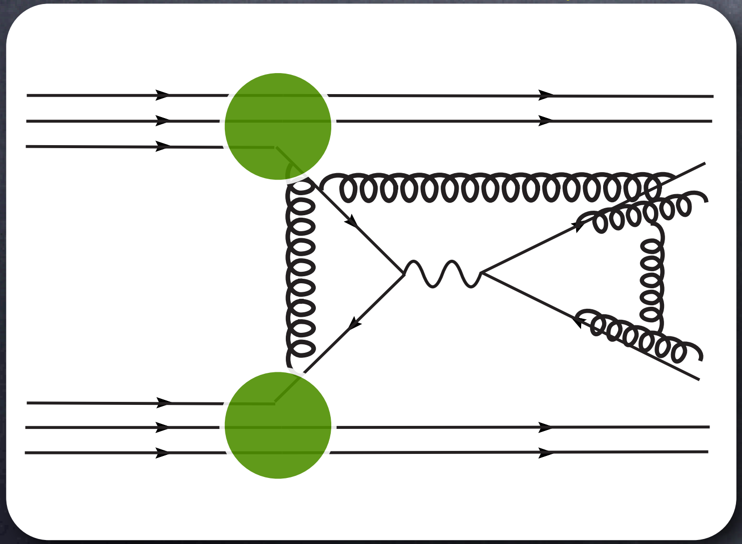

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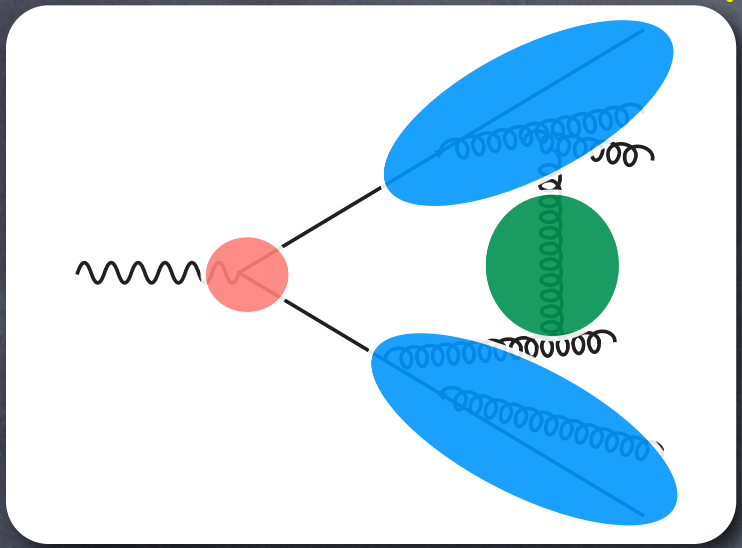

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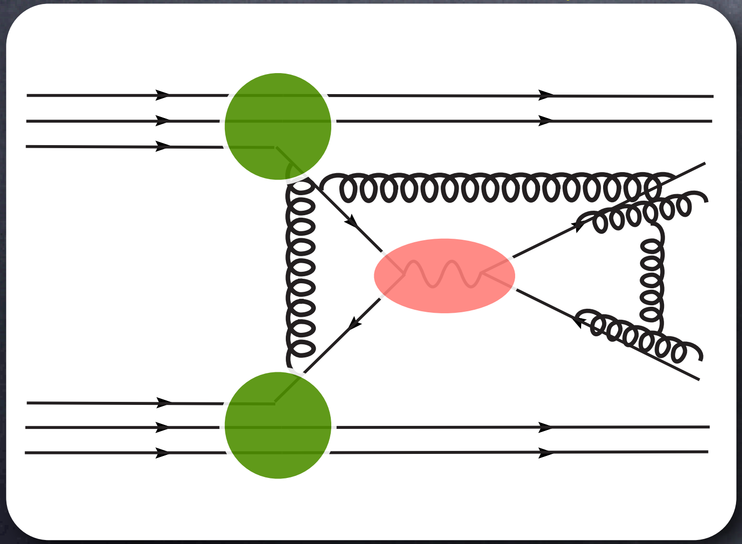

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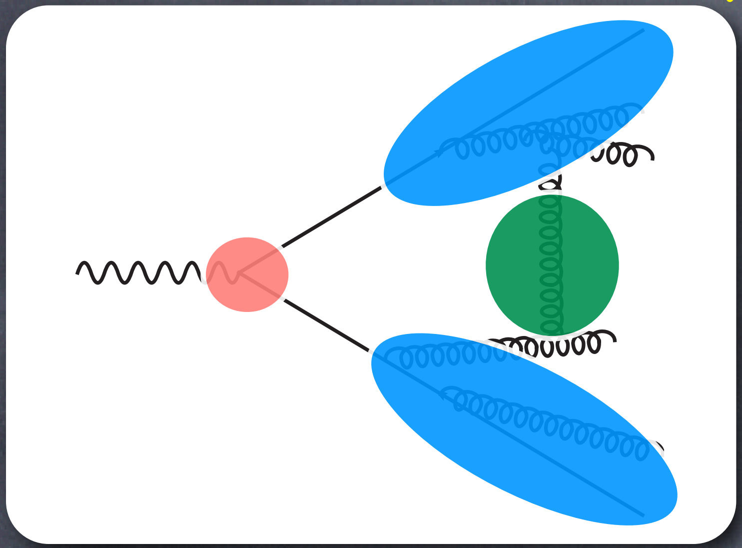

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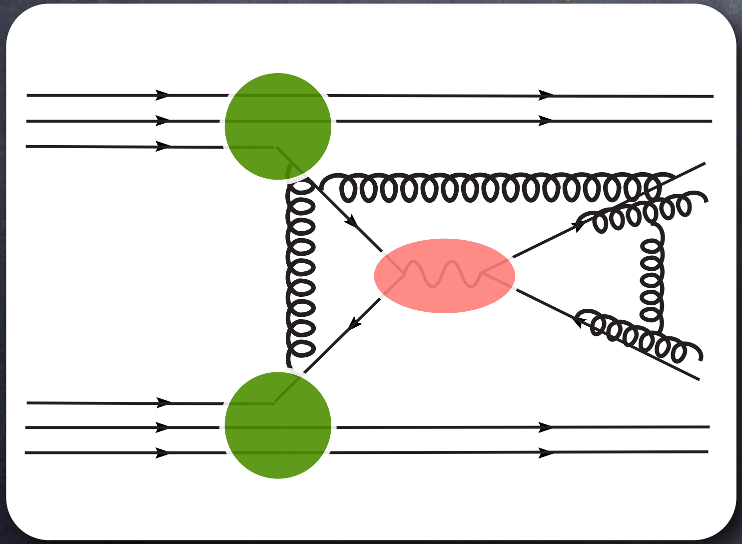

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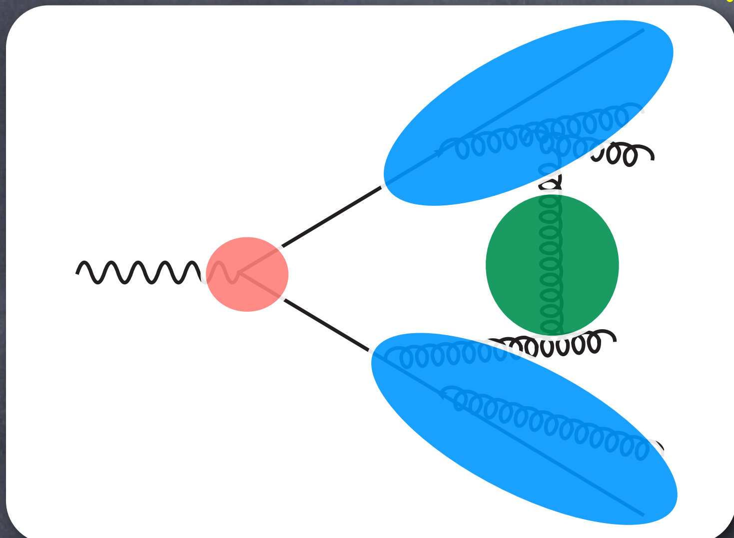

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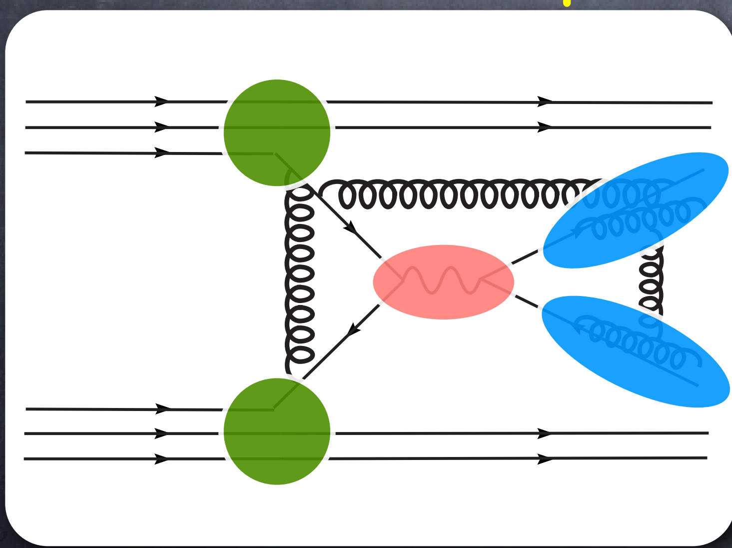

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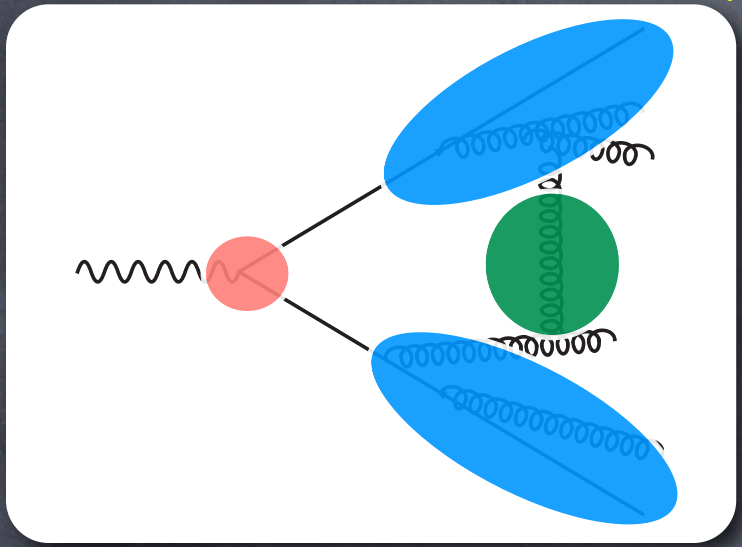

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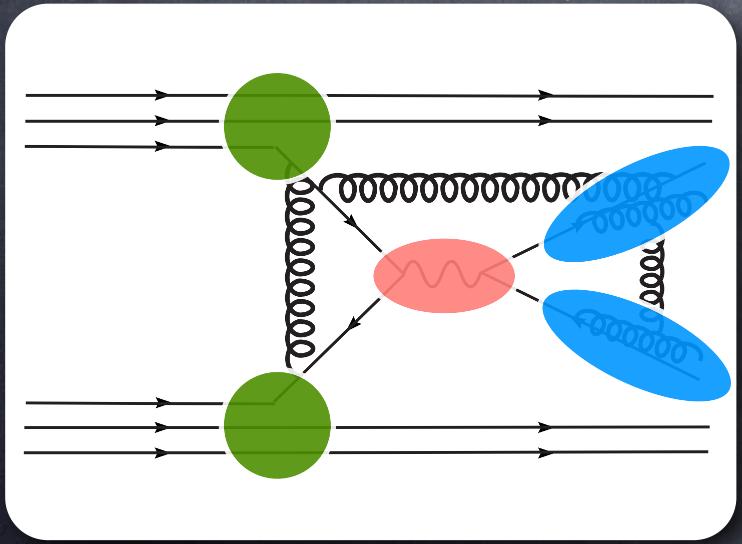

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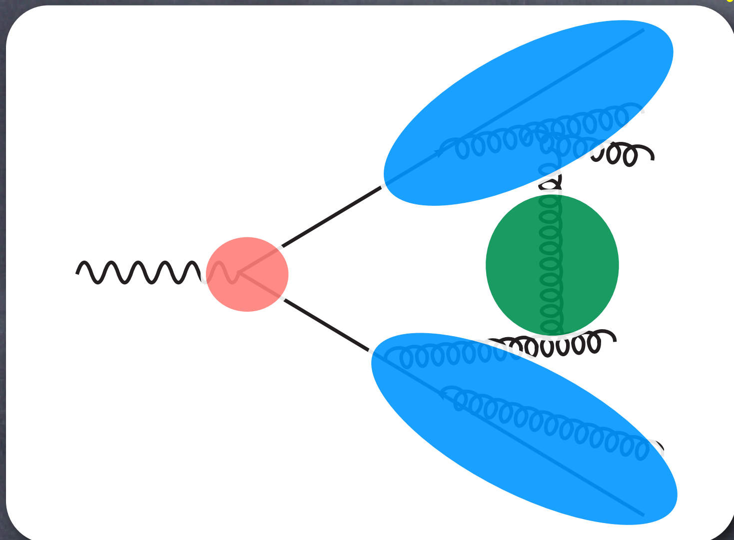

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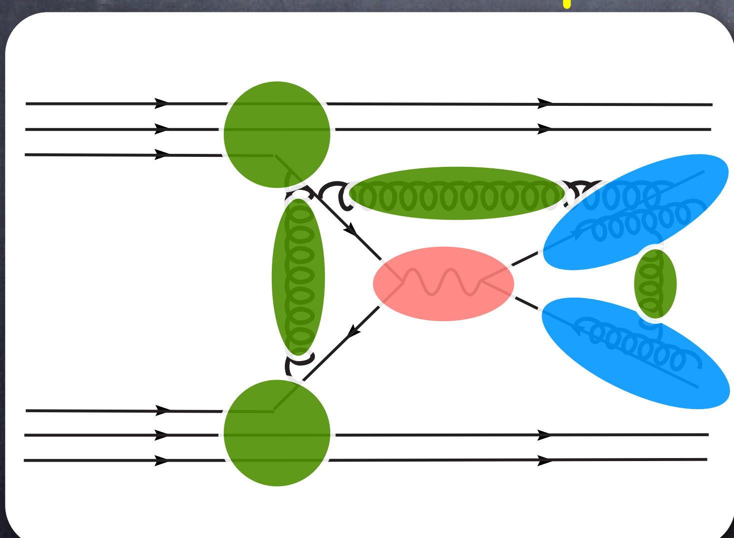

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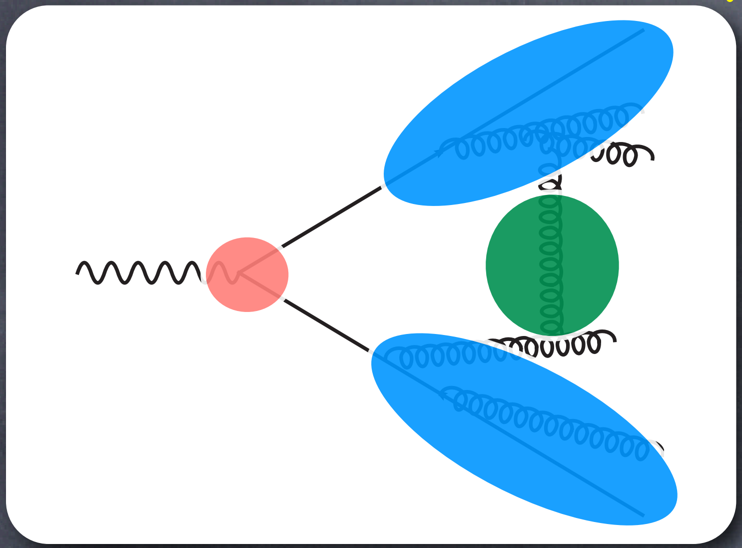

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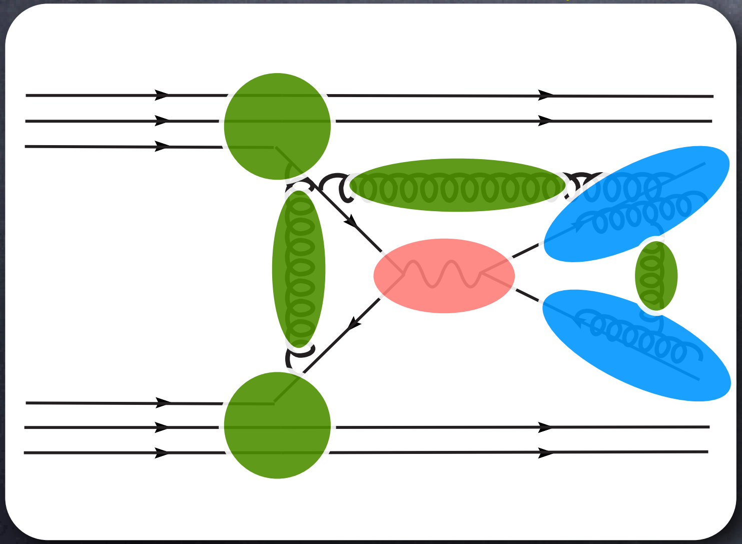

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Quick introduction to SCET

Field content of SCET

CWB, Fleming, Luke ('00)
CWB, Fleming, Pirjol, Stewart ('00)

Light cone coordinates: $p^\mu = (n \cdot p, \bar{n} \cdot p, p^\perp)$

$n \cdot p = \frac{1}{2}(p_0 - p_3)$, $\bar{n} \cdot p = \frac{1}{2}(p_0 + p_3)$, $p^\perp = p_i$

Degrees of freedom

Type	$(p^+, p^-, p^\perp)$	Fields
collinear	$(\lambda^2, 1, \lambda)$	χ_n, A_n
soft	$(\lambda^2, \lambda^2, \lambda^2)$	q_s, A_s

Construct the most general operators with required field content to given order in λ

Interactions in SCET

Leading order collinear Lagrangian

$$\mathcal{L} = \sum_n \bar{\chi}_n \left[in\cdot D_n + gn\cdot A_s + i\not{D}_n^\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^\perp \right] \frac{\not{\bar{n}}}{2} \chi_n$$

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Collinear fields

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Collinear fields

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Collinear fields

Soft gluon

Interactions in SCET

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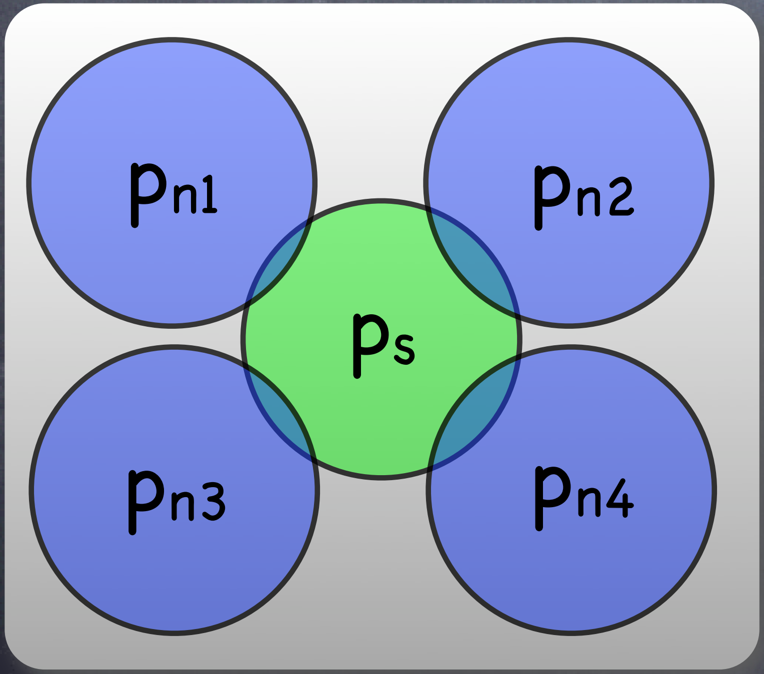
Collinear fields   Soft gluon

- No interactions between collinear fields of different directions
- Interaction between collinear and soft fields only via one single term

Soft/collinear decoupling

CWB, Pirjol, Stewart ('00)

$$\mathcal{L} = \sum_n \bar{\chi}_n \left[in\cdot D_n + g n\cdot A_s + i\not{D}_n^{\perp} \frac{1}{i\bar{n}\cdot D_n} i\not{D}_n^{\perp} \right] \frac{\not{\bar{n}}}{2} \chi_n$$



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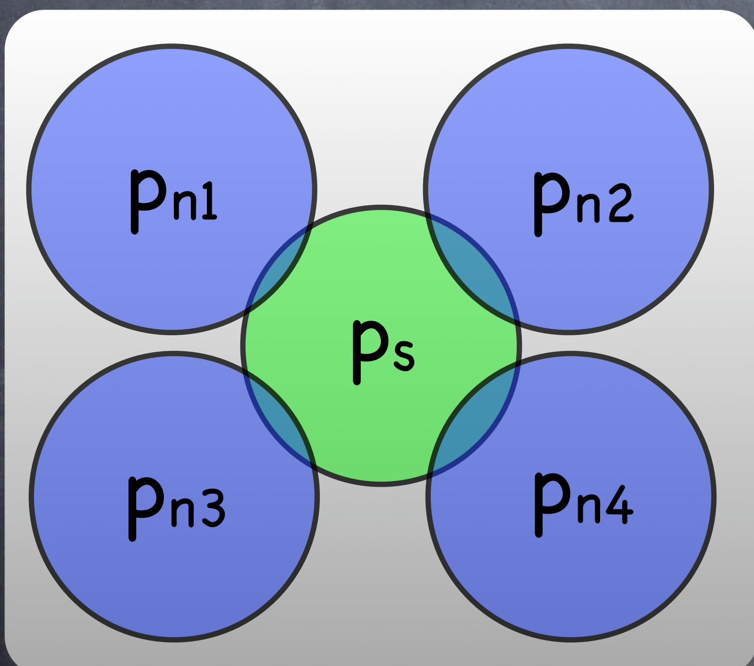

Perform field redefinition

$$\chi_n = Y_n \chi_n^{(0)} \qquad A_n = Y_n A_n^{(0)} Y_n^\dagger$$

$$Y_n = \mathrm{P}\exp\left[ig\int_0^\infty ds\; n\cdot A_s(ns)\right]$$

$Y_n Y_n^\dagger = 1$ $\qquad$ $inDY_n = Y_n in\partial$

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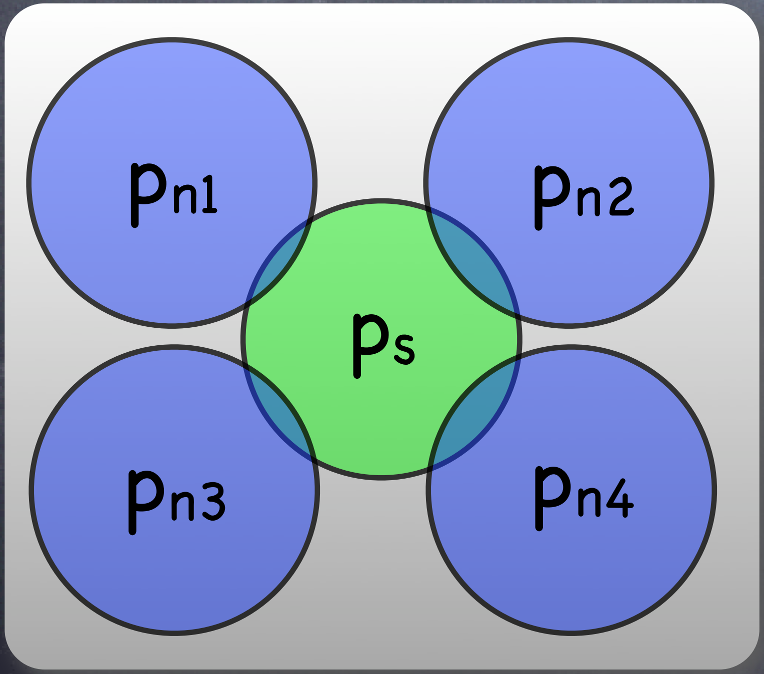

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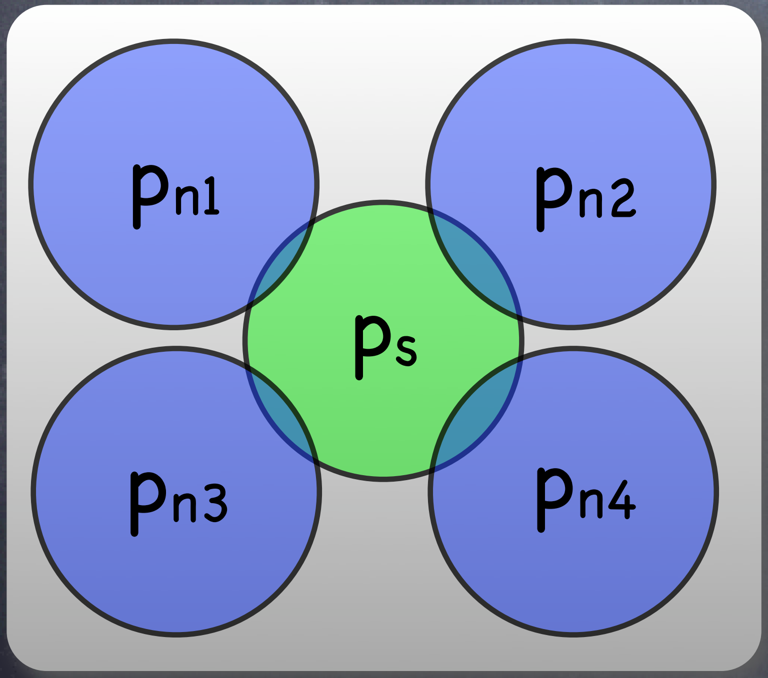
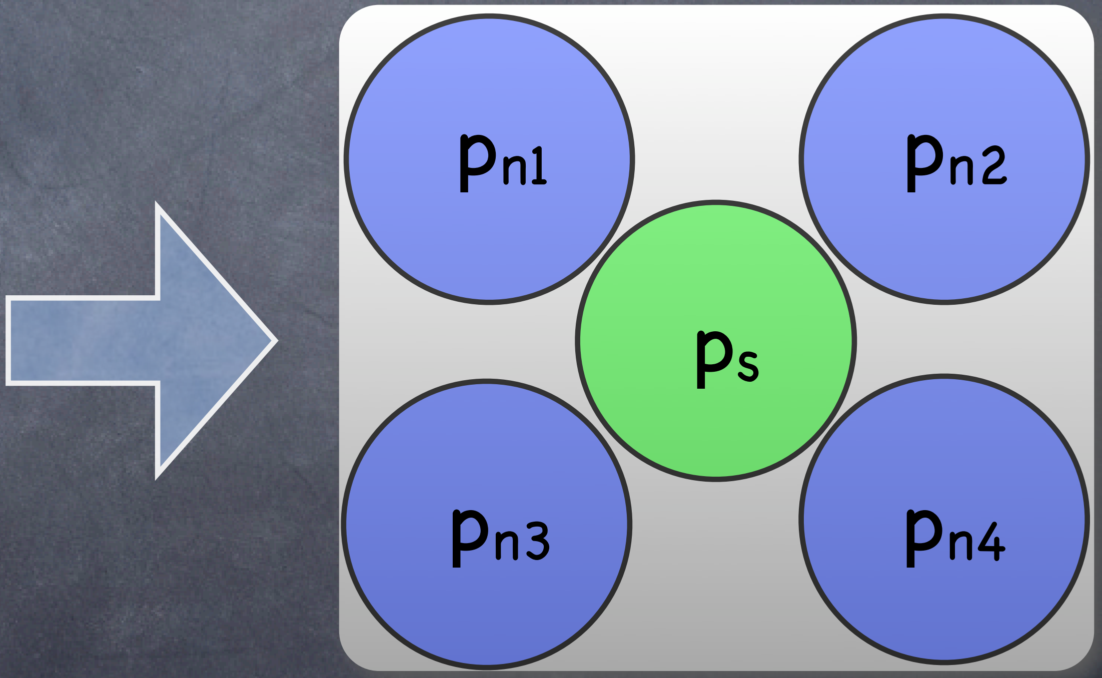
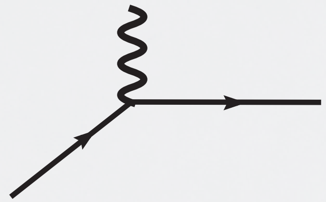
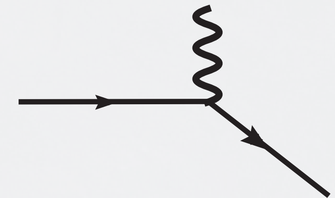




Deep inelastic scattering

Very simple proof

$$\sigma = \sum_X \langle p|$$  $$|X\rangle\langle X|$$  $$|p\rangle$$

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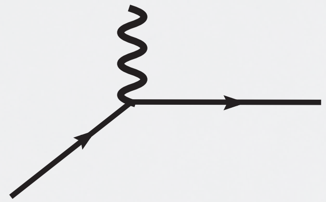
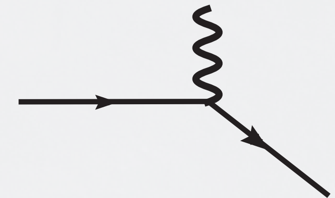
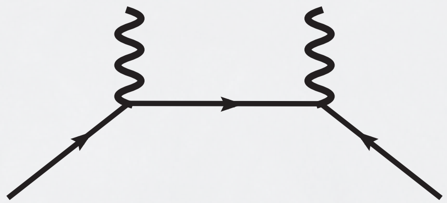
$\sigma = \sum_X \langle p|$  $|X\rangle\langle X|$  $|p\rangle$

$= \langle p|$  $|p\rangle$

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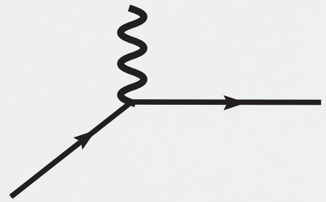
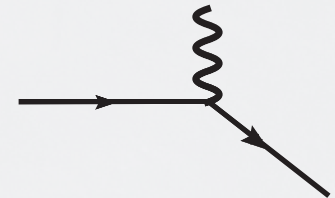
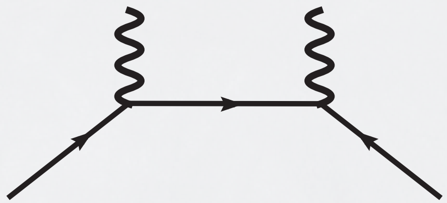
$\sigma = \sum_X \langle p|$  $|X\rangle\langle X|$  $|p\rangle$

$= \langle p|$  $|p\rangle$

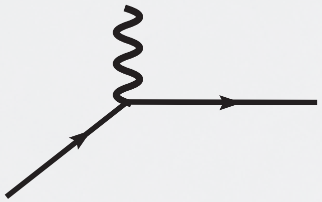
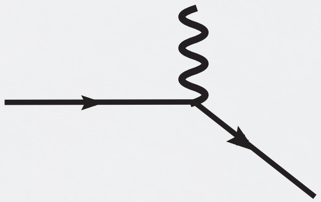
Partonic kinematics

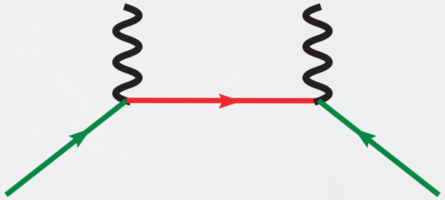
$$p_\gamma = Q\,(\,-1\,,\,1\,,\,0\,)$$

$$p_p = Q\,(\,x m_p^2/Q^2\,,\,1/x\,,\,0\,) \Rightarrow p_p^2 = m_p^2 \approx \Lambda^2$$

$$p_X = Q\,(\,-1-x m_p^2/Q^2\,,\,(1-x)/x\,,\,0\,) \Rightarrow p_X^2 \approx Q^2$$

Very simple proof

$\sigma = \sum_X \langle p|$  $|X\rangle\langle X|$  $|p\rangle$

$= \langle p|$  $|p\rangle$

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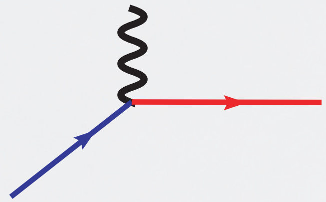
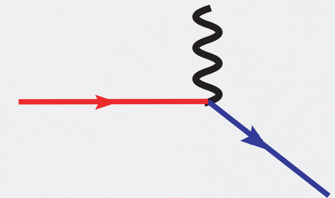
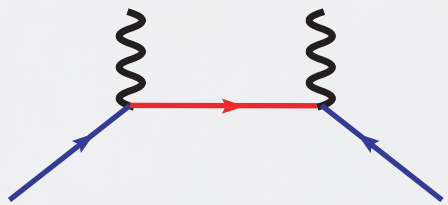
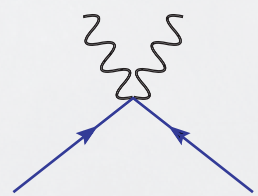
$\sigma = \sum_X \langle p|$  $|X\rangle\langle X|$  $|p\rangle$

$= \langle p|$  $|p\rangle$

$= H \otimes \langle p|$  $|p\rangle$

$$\sigma = H \otimes f_q$$

Event shapes near endpoint

What are event shapes?

Simple example is thrust T

$$T = \frac{1}{Q} \max \sum_{i \in X} |\mathbf{t} \cdot \mathbf{p_i}|$$

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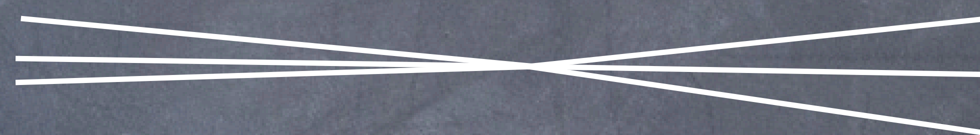
For 2-jet events T=1



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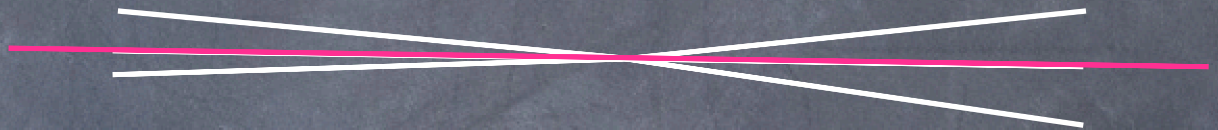

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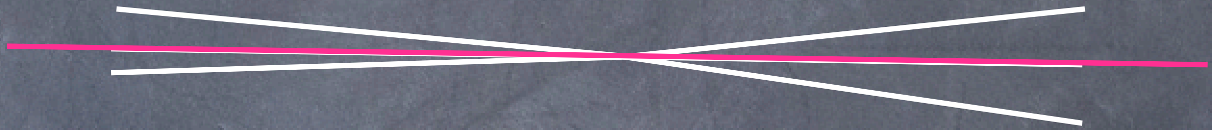

More generally, define event shape e

$$e = \frac{1}{Q} \sum_{i \in X} |\mathbf{p_i^T}| f_e(\eta_i)$$

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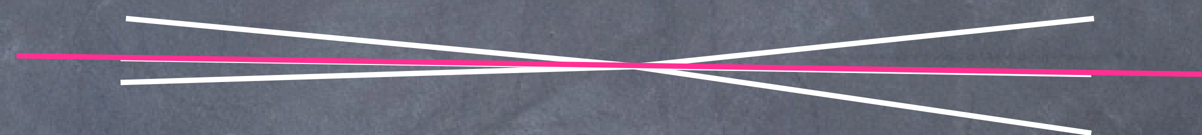


More generally, define event shape e

$$e = \frac{1}{Q} \sum_{i \in X} |\mathbf{p_i^T}| f_e(\eta_i)$$

$$f_T = \exp(-|\eta|)$$

$$f_C = 3/\cosh(\eta)$$

Differences from DIS

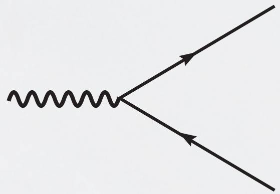
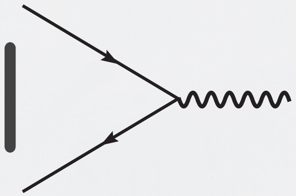
- DIS is completely inclusive process
  - Observables formed from leptonic variables
  - Every final state contributes same to final observable

Allows to perform the sum over final states

- Event shape is weighted cross section
  - Observables formed from hadronic variables
  - Different final states contribute with different weight to final observable

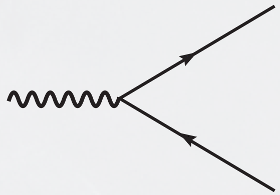
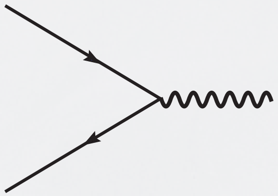
Sum over final states not possible

4 steps to the factorization

$$\sigma(e) = \sum_X \langle 0|$$  $$|X\rangle\langle X|$$  $$|0\rangle\delta(e - e(X))$$

1. Write $e(X)|X\rangle = \hat{e}|X\rangle$ and sum over states
2. Match onto operators in SCET
3. Use decoupling of Lagrangian to factorize operator
4. Factorize the matrix element and obtain final result

1: Define ê and sum over states

$\sigma(e)=\sum_X \langle 0|$  $|X\rangle\langle X|$  $|0\rangle\delta(e-e(X))$

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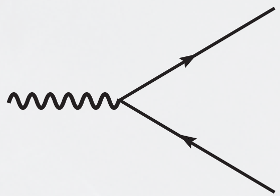
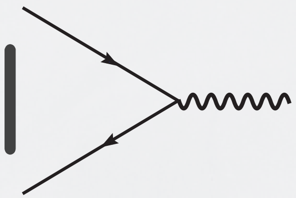
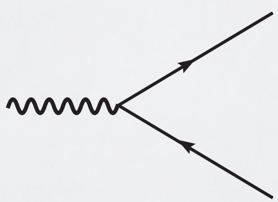
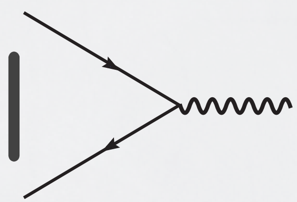
$\sigma(e)=\sum_X \langle 0|$  $|X\rangle\langle X|$  $|0\rangle\,\delta(e-e(X))$

$= \sum_X \langle 0|$  $\delta(e-\hat{e})\,|X\rangle\langle X|$  $|0\rangle$

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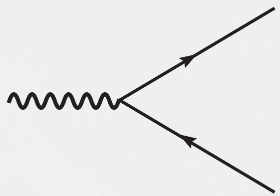
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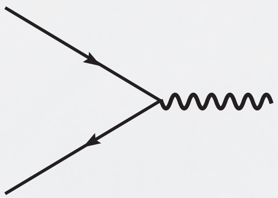
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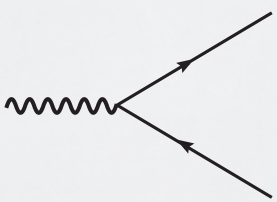
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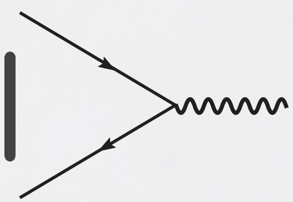
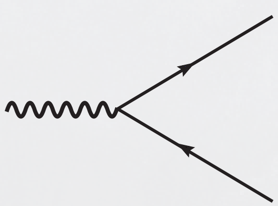
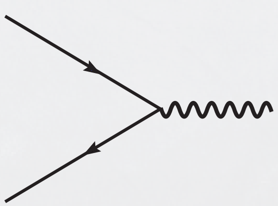

$$|0\rangle$$

$$= \langle 0|$$

$$\delta(e-\hat{e})$$

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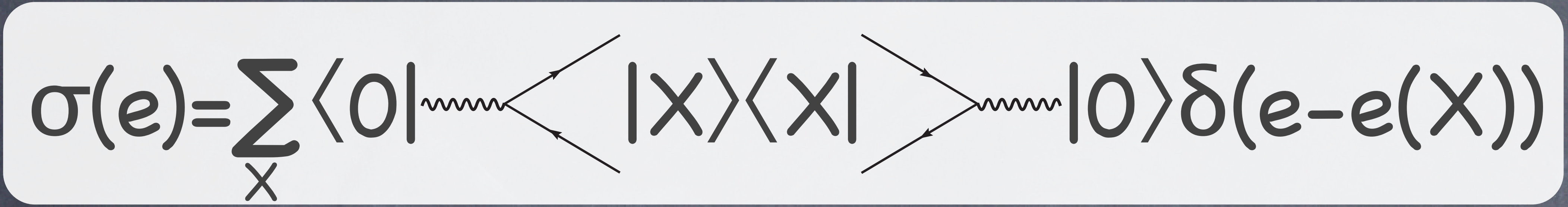
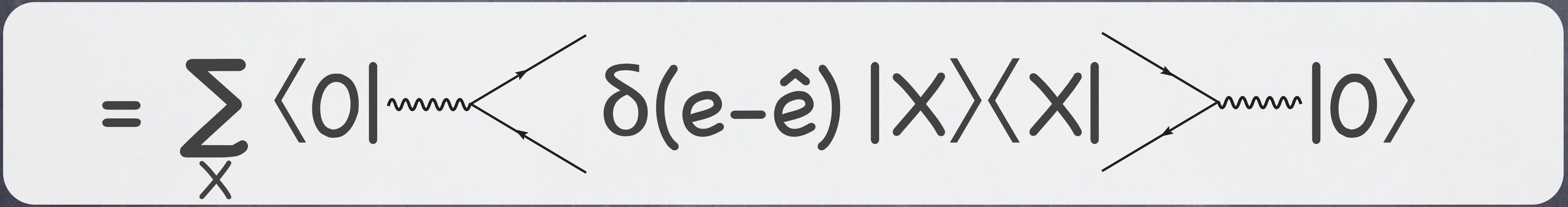
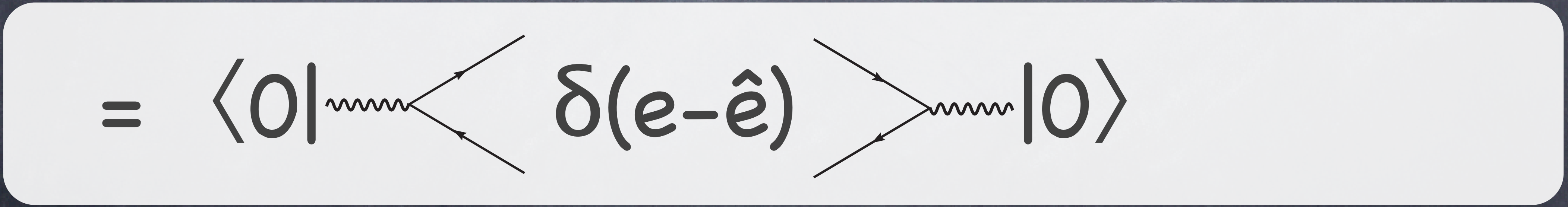
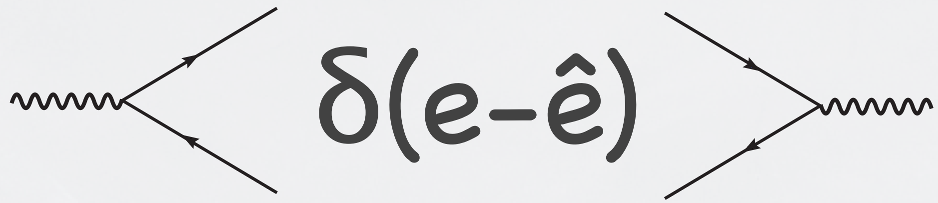






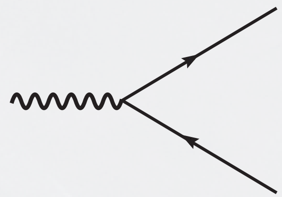
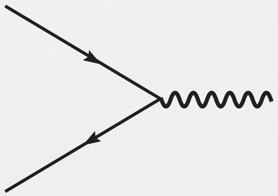
Next step: match onto SCET

2: Match onto SCET

$\sigma(e) = \langle 0|$  $|0\rangle$

Match full QCD current onto SCET

2: Match onto SCET

$\sigma(e) = \langle 0|$  $\delta(e-\hat{e})$  $|0\rangle$

Match full QCD current onto SCET

$[\bar{q}\ \Gamma^\mu\ q]$

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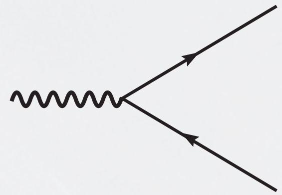
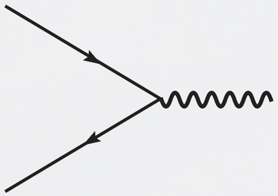
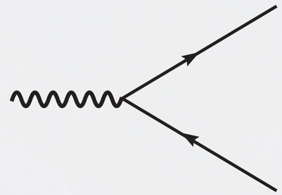
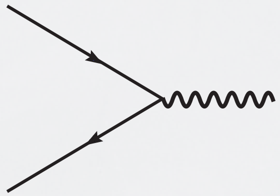
$\sigma(e) = \langle 0|$  $\delta(e-\hat{e})$  $|0\rangle$

Match full QCD current onto SCET

$$[\bar{q}\ \Gamma^\mu\ q] = C\ [\bar{\chi}_n\ \Gamma^\mu\ \chi_{\bar{n}}]$$

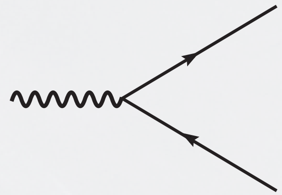
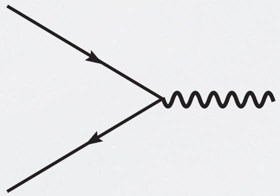
2: Match onto SCET

$\sigma(e) = \langle 0|$  $\delta(e-\hat{e})$  $|0\rangle$

Match full QCD current onto SCET

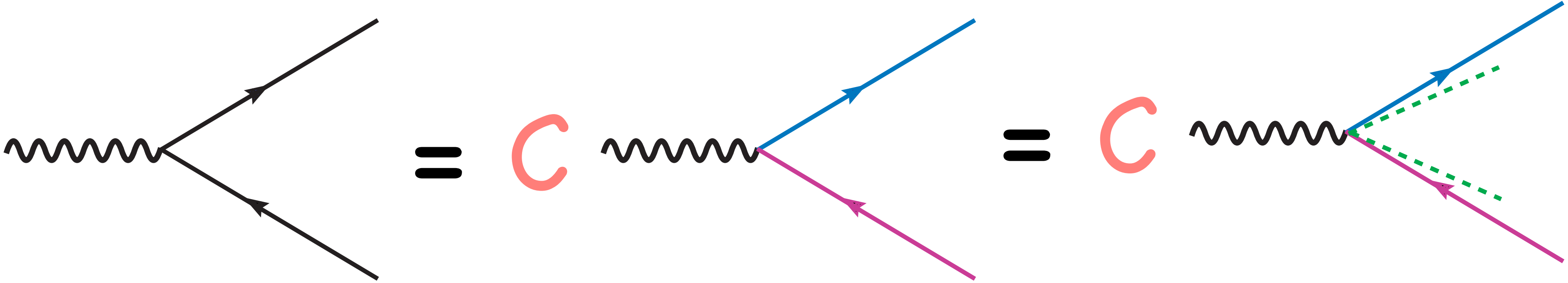
$$[\bar{q}\, \Gamma^\mu\, q] = C\, [\bar{\chi}_n\, \Gamma^\mu\, \chi_{\bar{n}}] = C\, [\bar{\chi}_n^{(0)} Y_n^\dagger\, \Gamma^\mu\, Y_{\bar{n}} \chi_{\bar{n}}^{(0)}]$$

2: Match onto SCET

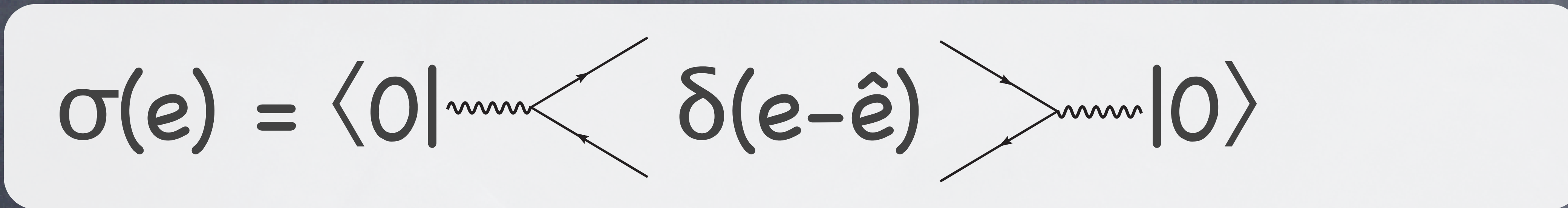
$\sigma(e) = \langle 0|$  $\delta(e-\hat{e})$  $|0\rangle$

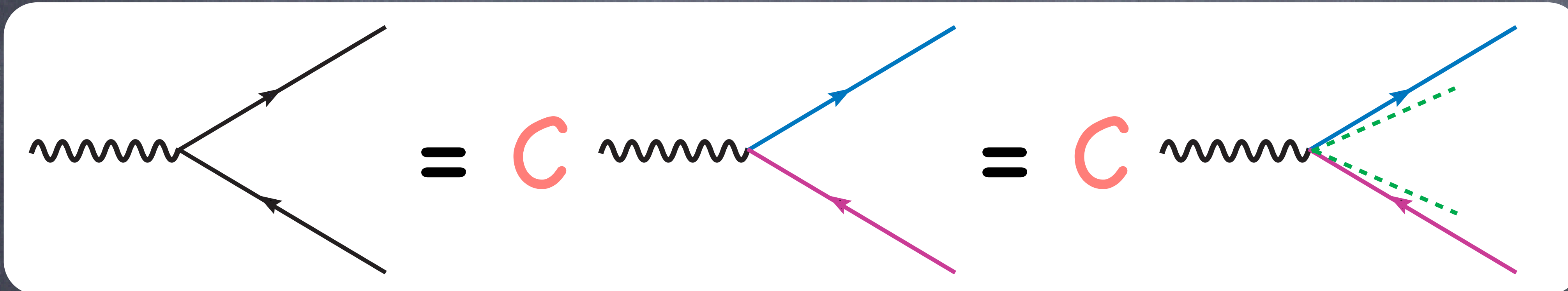
Match full QCD current onto SCET

$$[\bar{q}\,\Gamma^\mu\, q] = C\,[\bar{\chi}_n\,\Gamma^\mu\,\chi_{\bar{n}}] = C\,[\bar{\chi}_n^{(0)} Y_n^\dagger\,\Gamma^\mu\,Y_{\bar{n}}\chi_{\bar{n}}^{(0)}]$$

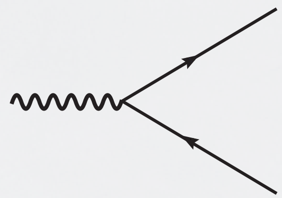
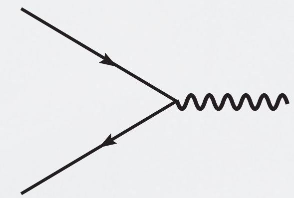


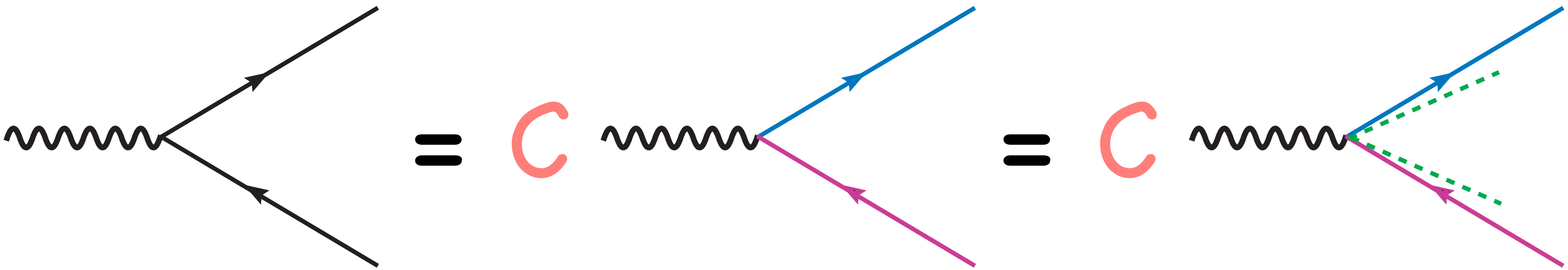
3: Match onto SCET



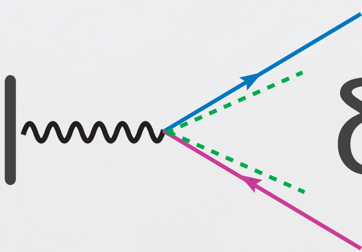
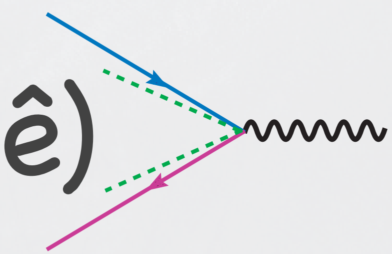


3: Match onto SCET

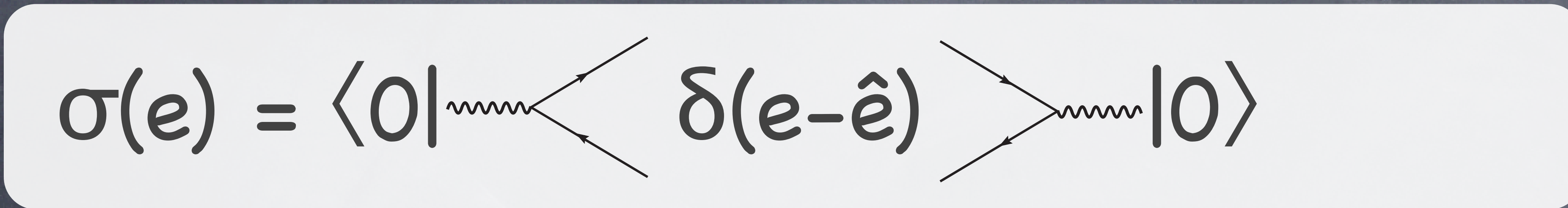
$\sigma(e) = \langle 0|$  $\delta(e-\hat{e})$  $|0\rangle$

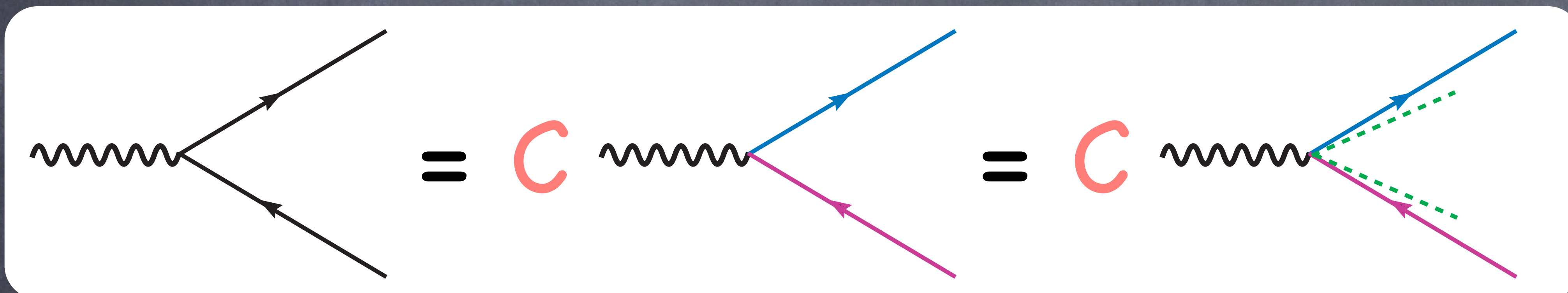


This gives for the cross section

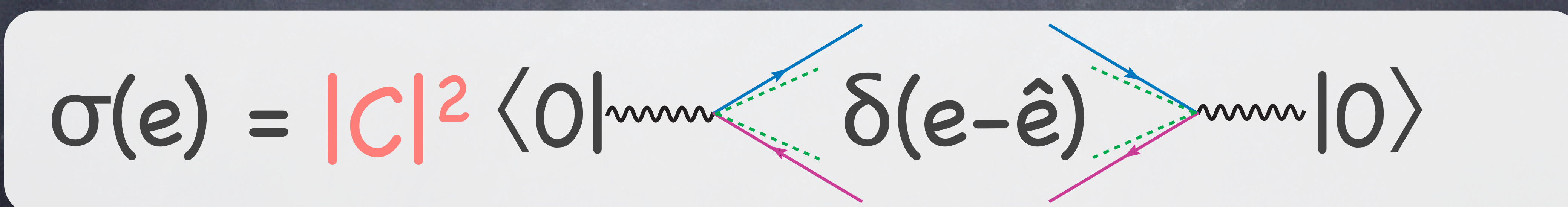
$\sigma(e) = |C|^2 \langle 0|$  $\delta(e-\hat{e})$  $|0\rangle$

3: Match onto SCET





This gives for the cross section



What is ê?

4. Factorize ê

$$\hat{e} = \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, f_e(\eta)\mathcal{E}_T(\eta;\hat{t})$$

Transverse energy flow operator defined as

$$\mathcal{E}_T(\eta) = \frac{1}{\cosh^3\eta}\int_0^{2\pi} d\phi \lim_{R\to\infty} R^2 \int_0^{\infty} dt\, \hat{n}_i T_{0i}(t, R\hat{n})$$

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Lagrangian completely decoupled

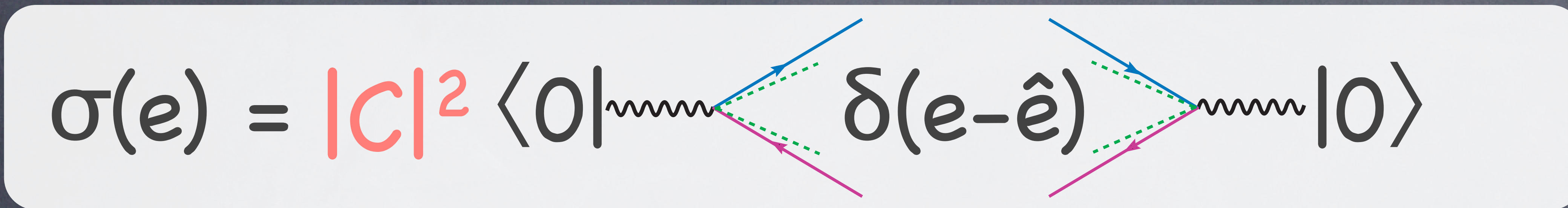
$L = L_n + L_{\bar{n}} + L_s \Rightarrow e = \hat{e}_n + \hat{e}_{\bar{n}} + \hat{e}_s$

Allows to write

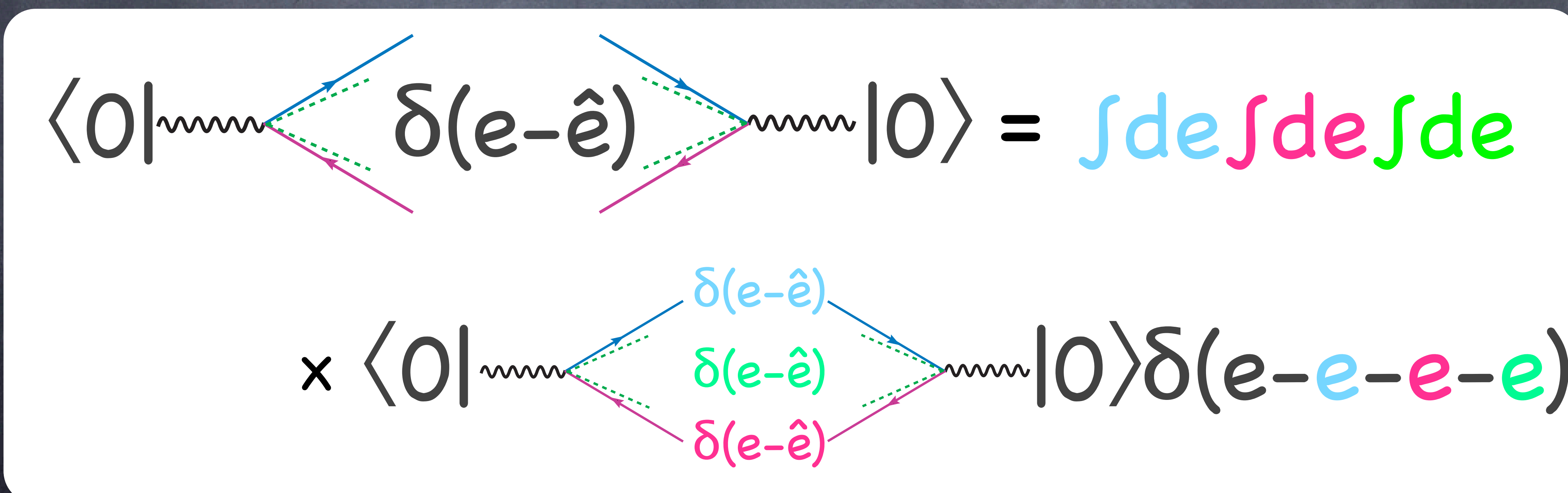
δ(e-ê) = ∫de∫de∫de δ(e-e-e-e)

δ(e-ê) δ(e-ê) δ(e-ê)

5: Factorize matrix element

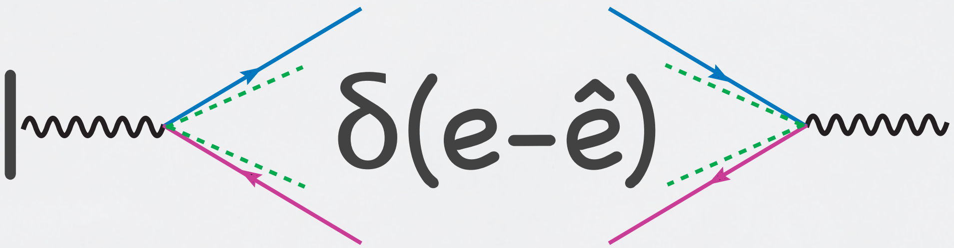


Using operator identity from previous slide



5: Factorize matrix element

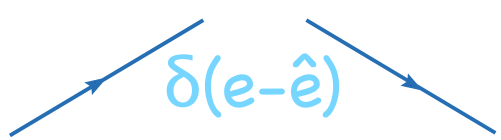
$$\sigma(e) = |C|^2 \langle 0|$$



$$|0\rangle$$

$$\sigma(e) = |C|^2 \int de \int de \int de \, \delta(e - e - e - e)$$

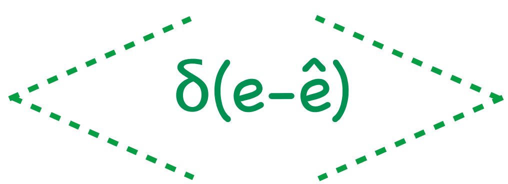
$$\langle 0|$$



$$|0\rangle \langle 0|$$

$$|0\rangle$$

$$\langle 0|$$



$$|0\rangle$$

$$\sigma(e) = H \otimes J_1 \otimes J_2 \otimes S$$

The same in equations

$$\sigma(e) = H \otimes J_1 \otimes J_2 \otimes S$$

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = |C_2(Q;\mu)|^2 \int de_n\, de_{\bar{n}}\, de_s\, \delta(e - e_n - e_{\bar{n}} - e_s) J_n(e_n;\mu) J_{\bar{n}}(e_{\bar{n}};\mu) S(e_s;\mu)$$

with

The same in equations

σ(e) = H ⊗ J$_1$ ⊗ J$_2$ ⊗ S

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = |C_2(Q;\mu)|^2 \int de_n\, de_{\bar n}\, de_s\, \delta(e - e_n - e_{\bar n} - e_s) J_n(e_n;\mu) J_{\bar n}(e_{\bar n};\mu) S(e_s;\mu)$$

with

$$J_n(e_n;\mu) \equiv \int \frac{dk^+}{2\pi} \mathcal{J}_n(e_n, k^+;\mu)$$

$$J_{\bar n}(e_{\bar n};\mu) \equiv \int \frac{dl^-}{2\pi} \mathcal{J}_{\bar n}(e_{\bar n}, l^-;\mu)$$

$$S(e_s;\mu) \equiv \int \frac{d^4 r}{(2\pi)^4} S(e_s, r;\mu)$$

The same in equations

σ(e) = H ⊗ J1 ⊗ J2 ⊗ S

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = |C_2(Q;\mu)|^2 \int de_n\, de_{\bar n}\, de_s\, \delta(e - e_n - e_{\bar n} - e_s) J_n(e_n;\mu) J_{\bar n}(e_{\bar n};\mu) S(e_s;\mu)$$

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$$S(e_s;\mu) \equiv \int \frac{d^4 r}{(2\pi)^4} S(e_s, r;\mu)$$

$$\langle 0|\, \chi_n(x)_\beta \delta(e_n - \hat e_n) \bar\chi_n(0)_\gamma\, |0\rangle \equiv \int \frac{dk^+ dk^- d^2 k_\perp}{2(2\pi)^4} e^{-ik\cdot x} \mathcal{J}_n(e_n, k^+;\mu) \left(\frac{\not n}{2}\right)_{\beta\gamma}$$

$$\langle 0|\, \bar\chi_{\bar n}(x)_\alpha \delta(e_{\bar n} - \hat e_{\bar n}) \chi_{\bar n}(0)_\delta\, |0\rangle \equiv \int \frac{dl^+ dl^- d^2 l_\perp}{2(2\pi)^4} e^{-il\cdot x} \mathcal{J}_{\bar n}(e_{\bar n}, l^-;\mu) \left(\frac{\not{\bar n}}{2}\right)_{\delta\alpha}$$

$$\frac{1}{N_C} \mathrm{Tr}\, \langle 0|\, \overline{Y}_{\bar n}(x) Y_n^\dagger(x) \delta(e_s - \hat e_s) Y_n(0) \overline{Y}_{\bar n}(0)\, |0\rangle \equiv \int \frac{d^4 r}{(2\pi)^4} e^{-ir\cdot x} S(e_s, r;\mu)$$

What can we do with this?

Lee, Sterman ('06)

- Results splits up into several simpler functions
- Operator definition of non-perturbative contributions

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^{\dagger} Y_n \delta\left(e - \frac{1}{Q}\int d\eta f_e(\eta)\mathcal{E}(\eta)\right) Y_n^{\dagger} Y_{\bar{n}} \right| 0 \right\rangle$$

Using boost along **n** direction with rapidity η', show

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^{\dagger} Y_n \delta\left(e - \frac{1}{Q}\int d\eta f_e(\eta)\mathcal{E}(\eta+\eta')\right) Y_n^{\dagger} Y_{\bar{n}} \right| 0 \right\rangle$$

Choose $\eta' = -\eta$ and define $F_e = \int d\eta\, f_e(\eta)$

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^{\dagger} Y_n \delta\left(e - \frac{F_e}{Q}\mathcal{E}(0)\right) Y_n^{\dagger} Y_{\bar{n}} \right| 0 \right\rangle$$

What can we do with this?

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \delta\left(e - \frac{F_e}{Q}\, \mathcal{E}(0) \right) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle$$

What can we do with this?

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^{\dagger} Y_n \delta \left(e - \frac{F_e}{Q} \mathcal{E}(0) \right) Y_n^{\dagger} Y_{\bar{n}} \right| 0 \right\rangle$$

Calculable parameter

F_T=2, F_C=3π

What can we do with this?

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^{\dagger} Y_n \delta \left(e - \frac{F_e}{Q} \mathcal{E}(0) \right) Y_n^{\dagger} Y_{\bar{n}} \right| 0 \right\rangle$$

Calculable parameter

$F_T=2$, $F_C=3\pi$

Universal operator

What can we do with this?

$$S(e) = \left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \delta\left(e - \frac{F_e}{Q}\mathcal{E}(0)\right) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle$$

Calculable parameter
$F_T=2$, $F_C=3\pi$

Universal operator

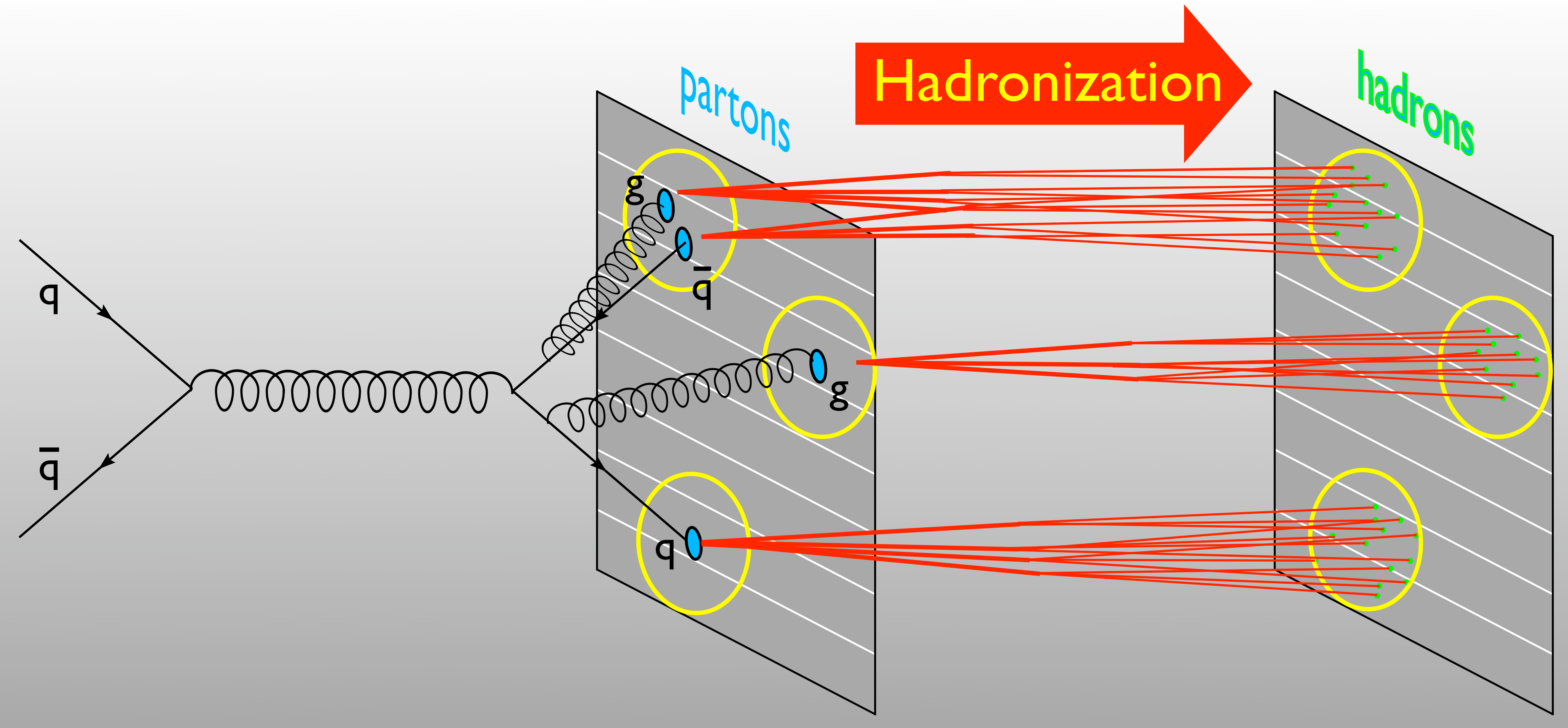
For example, expand in 1/Q to find

$$S(e) = \delta(e) + \delta'(e)\frac{F_e}{Q}\left\langle 0 \left| Y_{\bar{n}}^\dagger Y_n \mathcal{E}(0) Y_n^\dagger Y_{\bar{n}} \right| 0 \right\rangle + \ldots$$

Non-perturbative matrix element independent on which event shape is considered

Towards factorization for jet production

How to deal with jets



How to deal with jets

- Jets: collection of hadrons which are “close together”
- How can we quantify this statement?
- Need jet algorithms, many possibilities

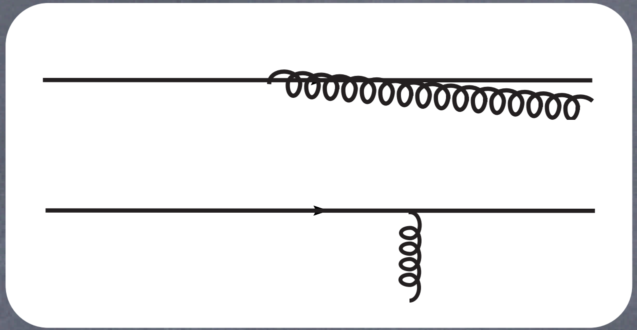
Jet algorithms groups all particles into jets, and returns the four-momentum of every jet

Requirements on jet algorithms

- Has to be efficient
- Needs to be IR safe

Requirement for IR safety

QCD has collinear and soft divergences



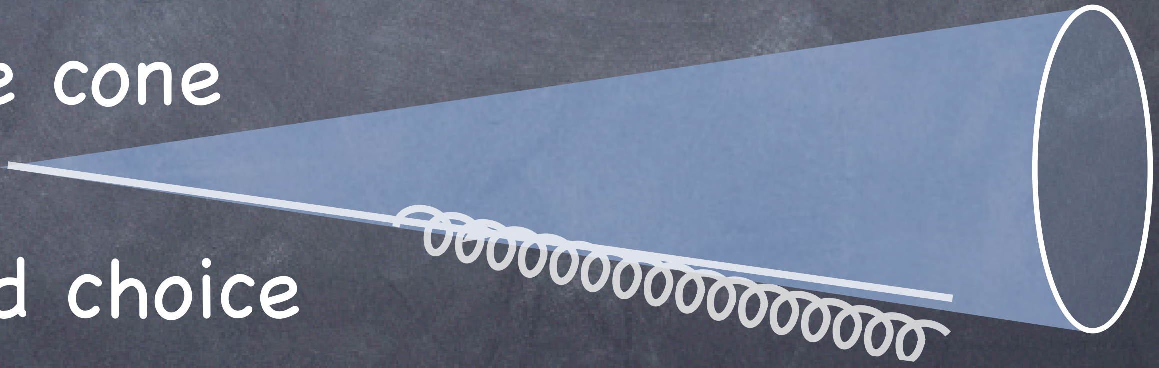
Collinear gluon

Soft gluon

Both emissions have large couplings in QCD

Jet algorithm should not be sensitive to either

For example, a naive cone based algorithm would not be a good choice



Total momentum of jet is unaffected by soft or collinear emission

Jet observables

1. Jet algorithm groups particles into jets...

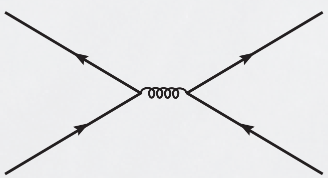
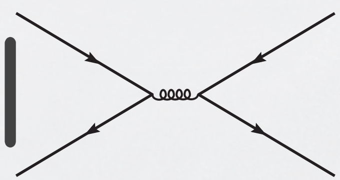
$$\{p_1, p_2, \ldots , p_N\}=\{\{p_1,\ldots, p_{n1}\}, \{p_1,\ldots, p_{n2}\}, \ldots \}$$

2. ...and returns total four momentum of each jet

$$J(\{p_1, p_2, \ldots , p_N\}) = \{P_1 , P_2 , \ldots \}$$

3. Observables are formed out of the jet momenta

$$o = O(\{P_1 , P_2 , \ldots \}) = O[J](\{p_1, p_2, \ldots , p_N\})$$

$\sigma(o)=\sum_X \langle pp|$  $|X\rangle\langle X|$  $|pp\rangle$

$$\times\ \delta(o-O[J](\{p_1(X),\ldots,p_N(X)\}))$$

Need operator that picks out momenta of particles

Using the operator idea again

A simple operator that is available

$$\mathcal{E}(\hat{n})\,|X\rangle = \sum_{i\in X} \omega_i \delta^3(\hat{n} - \hat{n}_i)\,|X\rangle$$

$$\mathcal{E}(\hat{n}) = \lim_{R\to\infty} R^2 \int_0^\infty dt\ \hat{n}_i\, T_{0i}\,(t, R\hat{n})$$

As before

$\hat{\varepsilon} = \hat{\varepsilon} + \hat{\varepsilon} + \hat{\varepsilon}$

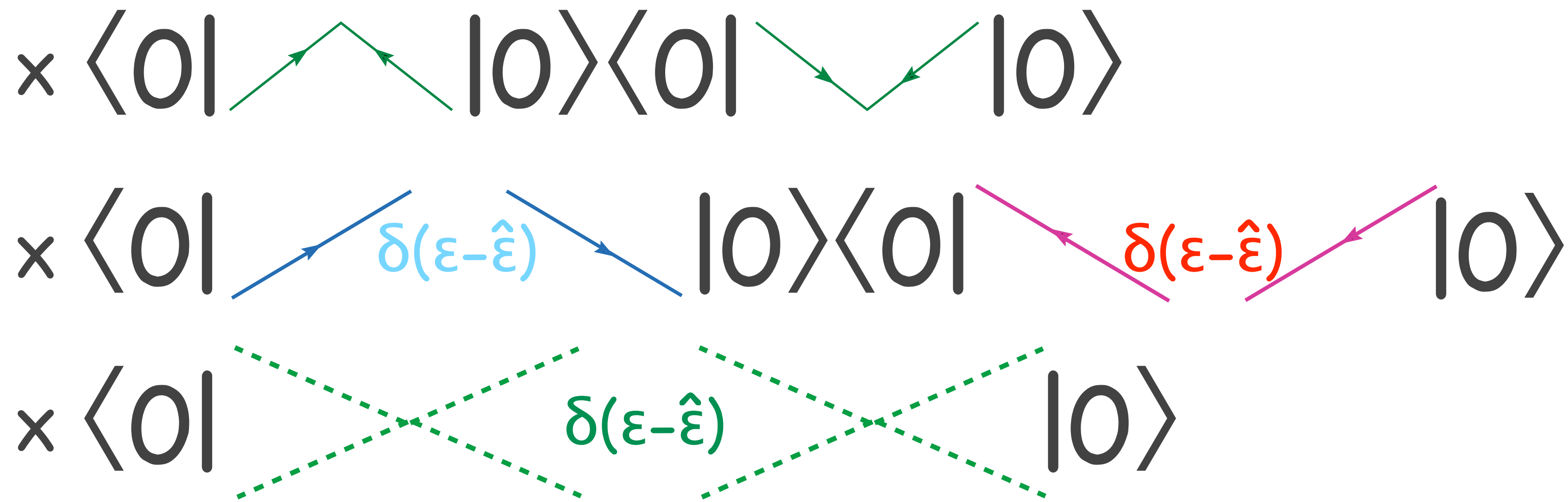
Allows to write

$\delta(\varepsilon-\hat{\varepsilon}) = \int d\varepsilon \int d\varepsilon \int d\varepsilon\ \delta(\varepsilon-\varepsilon-\varepsilon-\varepsilon)$

$\delta(\varepsilon-\hat{\varepsilon})\ \delta(\varepsilon-\hat{\varepsilon})\ \delta(\varepsilon-\hat{\varepsilon})$

Factorize matrix element

$$\sigma(o) = |H|^2 \int d\varepsilon \int d\varepsilon \int d\varepsilon \, O[J][\varepsilon+\varepsilon+\varepsilon]$$



Need to study behavior of O[J]. For reasonable jet def's, depends at leading order either on ε or ε

$$\sigma(o) = O[J] \otimes H \otimes f_q \otimes f_q \otimes J_1 \otimes J_2 \otimes S$$

Is the soft function relevant?

$$\sigma(o) = |H|^2 \int d\varepsilon \int d\varepsilon \int d\varepsilon \, O[J][\varepsilon+\varepsilon+\varepsilon]$$

$$\times \; fq \otimes fq \otimes J_1(\varepsilon) \otimes J_1(\varepsilon) \otimes S(\varepsilon)$$

If

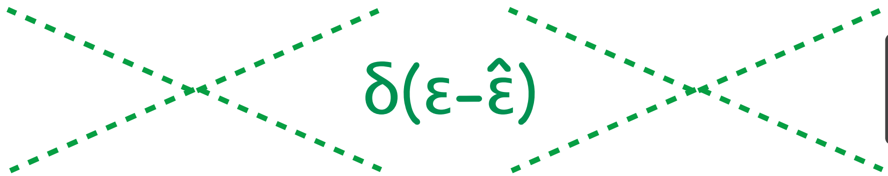
$$O[J](\varepsilon+\varepsilon+\varepsilon) = O[J](\varepsilon+\varepsilon)$$

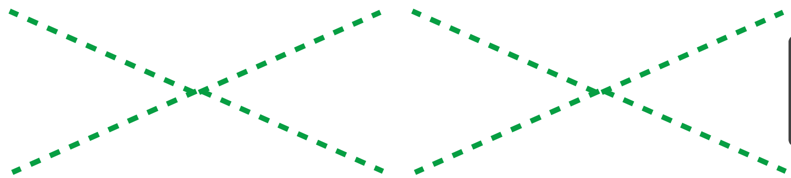
Example is m(j1,j2)

$$\sigma(o) = |H|^2 \int d\varepsilon \int d\varepsilon \, O[J][\varepsilon+\varepsilon]$$

$$\times \; fq \otimes fq \otimes J_1(\varepsilon) \otimes J_1(\varepsilon) \otimes \int d\varepsilon S(\varepsilon)$$

Is the soft function relevant?

$\int d\varepsilon \langle 0|$  $|0\rangle$

$= \langle 0|$  $|0\rangle$

$$= \langle 0| \, [Y_1 Y_2^\dagger Y_3 Y_4^\dagger] \, [Y_4 Y_3^\dagger Y_2 Y_1^\dagger] \, |0\rangle$$

$$= \; 1$$

Soft function only important if observable sensitive to soft momenta

Conclusions

- Understanding factorization is crucial to make theoretical predictions for experimental observables
- Factorization can be understood using SCET
- Factorization simple for totally inclusive processes
- For weighted cross sections, need operator statement about restricted final states
- Allows to understand factorization in event shapes without assumption about parton-hadron duality
- Similar methods applicable to jet production at hadron colliders, but some more work required