Rutgers University – Physics Graduate Qualifying Exam
Thermal Physics – January 15, 2010

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

A. 0.5 kg of a hot metal at 80°C is dropped into a large pool of water at 20°C. The specific heat of the metal at constant pressure is independent of temperature, with a value of C_V = 100 J/(kg-K).

(a) How much does the entropy of the metal change? How much does the total entropy (of both the metal and water) change? Does it increase or decrease?

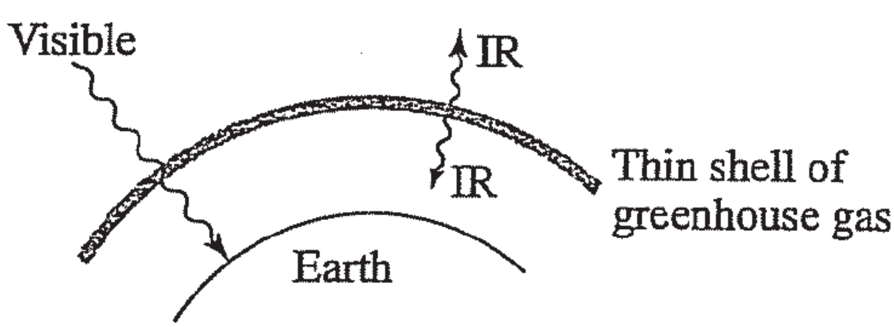
(b) Calculate the ratio $\Omega_{final}/\Omega_{initial}$, where Ω_{final} and $\Omega_{initial}$ are the multiplicities of the final equilibrium macropartition and the initial non-equilibrium macropartition, respectively.

B.

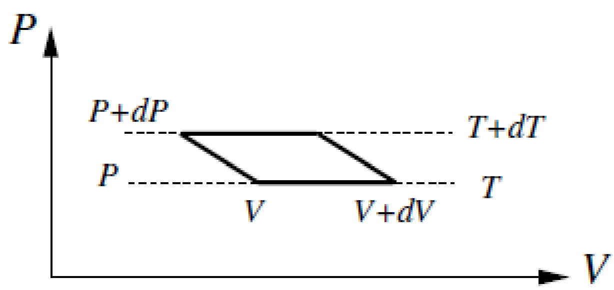
(a) Estimate the temperature of the Earth assuming the Sun and Earth are blackbodies (α=1). Take the surface temperature of the Sun to be T_{Sun}=5800K, the radius of the Sun to be $R_{Sun}=7\times10^5$ km, and the Sun-Earth distance to be $D=1.5\times10^5$ km. Does your answer depend on the size of the Earth?

(b) Actually the Earth reflects 30% of the visible light incident upon it (α=0.7). How does this fact alter your estimate of the Earth's temperature?

(c) Finally, assume the Earth is surrounded by a thin spherical shell of gas that is transparent to the Sun's energy but opaque to infrared radiation. What then is the temperature of the Earth's surface? Note: the shell radiates energy both in space and back toward the surface of the Earth (see the Figure).



C1. The aim of this problem is to obtain the blackbody radiation relation, $E(T,V) \propto VT^4$ starting from the equation of state, by performing an infinitesimal Carnot cycle on the photon gas.



(a) Express the work done, W in the above cycle, in terms of dV and dP.

(b) Express the heat absorbed, Q, in expanding the gas along an isotherm, in terms of P, dV, and an appropriate derivative of $E(T,V)$.

(c) Using the efficiency of the Carnot cycle, relate the above expressions for W and Q to T and dT.

(d) Observations indicate that the pressure of the photon gas is given by $P = A\,T^4$, where $A = \pi^2 k_B^4 / 45(\hbar c)^3$ is a constant. Use this information to obtain $E(T,V)$, assuming $E(0,V) = 0$.

(e) Find the relation describing the *adiabatic paths* in the above cycle.

C2. Consider a non-relativistic Fermi gas in two dimensions: N electrons are confined to a square with area $A = L^2$

(a) Derive the density of electronic states in two dimensions per unit area, $g^{2D}(\varepsilon) = m/\pi\hbar^2$.

(b) Find the Fermi energy (in terms of N and A).

(c) Find the average energy per electron at $T = 0$ (in terms of E_F).

(d) Calculate E_F for the electron density $N/A = 10^{16}$ m^{-2} - a typical density of electrons in a field-effect transistor (FET), assume that the effective mass is 0.2 m_0. Is the electron gas in a FET degenerate at room temperature?

D1. By shining an intense laser beam on a semiconductor, one can create a metastable collection of electrons (charge $-e$, and effective mass m_e) and holes (charge $+e$, and effective mass m_h) in the bulk. The oppositely charged particles may pair up (as in a hydrogen atom) to form a gas of *excitons*, or they may dissociate into a plasma. We shall examine a much simplified model of this process.

(a) Calculate the free energy of a gas composed of N_e electrons and N_h holes, at temperature T, treating them as classical non-interacting particles of masses m_e and m_h.

(b) By pairing into an exciton, the electron hole pair lowers its energy by ΔE. [The binding energy of a hydrogen-like exciton is $\varepsilon \approx me^4/(2\hbar^2\varepsilon^2)$ where ε is the dielectric constant, and $m^{-1} = m_e^{-1} + m_h^{-1}$.] Calculate the free energy of a gas of N_p excitons, treating them as classical non-interacting particles of mass $m = m_e + m_h$..

(c) Calculate the chemical potentials μ_e, μ_h, and μ_p of the electron, hole, and exciton states, respectively.

(d) Express the equilibrium condition between excitons and electron/holes in terms of their chemical potentials.

(e) At a high temperature T, find the density n_p of excitons, as a function of the total density of excitations $n \approx n_e + n_h$.