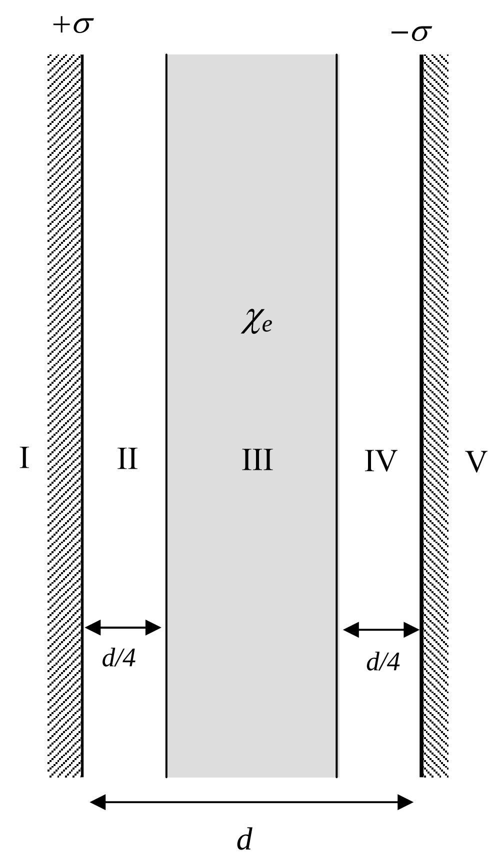
Rutgers University – Physics Graduate Qualifying Exam
E&M – January 13, 2010

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

A. A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder (radius a) and a uniform surface charge density on the outer cylindrical shell (radius b). Find the potential difference between a point on the axis and a point on the outer cylinder.

B. Two infinite planar conductors have free surface charge densities $+\sigma$ and $-\sigma$, respectively. The planes are parallel and separated by a distance *d*. The volume between the planes contains an infinite slab of dielectric with susceptibility χ_e and thickness *d*/2. The slab is parallel to the conductors and its surfaces are a distance *d*/4 from the nearest conductor as shown in the figure. Space is therefore divided into five regions, I, II, III, IV, and V.



a) What is ***E*** in each region of space?
b) What is ***D*** in each region?
c) Find an expression for the bound surface charge density on each surface, and the volume charge density in, the dielectric.
d) What is the potential difference between the conductors?
e) Recall that the capacitance of a configuration of conductors is C = Q/V. What is the capacitance per unit area of this arrangement? What would it be if the gap between the conductors were empty? Which situation enables one to store more charge for a given potential difference?

C1. A point charge q moves along the x-axis at a constant velocity v through a region with a uniform electric field and magnetic field given by $\boldsymbol{E} = E_o\hat{\boldsymbol{z}}$ and $\boldsymbol{B} = -B_o\hat{\boldsymbol{y}}$ and experiences no force. What are the electric and magnetic fields, $\boldsymbol{E}^*$ and $\boldsymbol{B}^*$, in the rest frame of the charge? Does the charge experience a force in this frame? Why or why not?

C2. The electrostatic and vector potentials, Φ and **A**, in a cylindrical coordinate system **x** = ([illegible]) are

$$\Phi=0$$

and

$$A = \frac{\xi\hat{\theta} - \lambda t\hat{\rho}}{2\pi\rho} \quad ,$$

where ξ and λ are constants and t is time.

i) Find the electric and magnetic fields that correspond to these potentials.

ii) Describe physically what the electric and magnetic field correspond to, or equivalently what gives rise to these fields.

D1. Consider a two-dimensional circular region of radius a. The electrostatic potential on the boundary of the region in circular coordinates $\boldsymbol{x} = (\rho,\theta)$ is given by

$$\Phi(a,\theta) = \Phi_0 \cos(3\theta).$$

i) Calculate the electrostatic potential everywhere inside the circular region.

ii) Calculate the electrostatic energy inside the circular region.

D2. A small linear antenna of length ℓ has a current that is approximately constant along its length and varies harmonically in time as

$$I(t) = I_0 \sin(\omega t)$$

The length of the antenna is small compared with the wavelength of radiated electromagnetic waves, $\ell \ll \lambda = 2\pi c/\omega$, where c is the speed of light.

i) Calculate the radiation zone electric and magnetic fields generated by this antenna.

ii) Calculate the time averaged power per unit solid angle emitted by the antenna.