**Rutgers University – Physics Graduate Qualifying Exam**
**QM – January 15, 2010**

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

Useful formula:

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$

$\int_0^{\frac{\pi}{2}} \sin^n x\, dx = \int_0^{\frac{\pi}{2}} \cos^n x\, dx = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (n-1)}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot n} \frac{\pi}{2}$ for even positive n .

**A.**
(a) Solve (i.e. find ALL the eigenstates and allowed energies) for the one dimensional infinite square well, i.e., $V(x) = \infty$ for $|x| \geq a$ and $V(x) = 0$ otherwise.

(b) Now turn on a perturbation in the form of $\delta V(x) = V_0$ for $|x| \leq b;\ 0 \leq b \leq a$. Calculate the change in energy levels to first order.

(c) Examine your result for as many extreme cases as you can.

(d) In particular, explain your answers for low-lying states for $b << a$.

**B.**
A free particle of energy $E$ is incident on a rectangular barrier, where $V=0$ for $x < 0$ and $x > a$, and $V=V_0$ for $0 \leq x \leq a$. For $E < V_0$, find the equations giving the probability that the particle is reflected and transmitted. You do not need to work through the algebra to simplify the equations.

**C1.** A particle $A$ with spin 1/2 decays into two particles $B$ and $C$, where $B$ has spin 1/2 and C has spin 1.

a) What are the possible values of the relative orbital angular momentum of $B$ and $C$?

b) If this relative orbital angular momentum is 0, and if $A$ is in an eigenstate of $J_z$ with eigenvalue $+\hbar/2$, find the probability that the z-component of the spin of B will also have the value $+\hbar/2$.

**C2.** Two identical spin ½ electrons are placed in an infinite square well ($V(x) = 0$ for $0 < x < a$, $V(x) = \infty$, otherwise). Their coulomb repulsion can be simulated by a simple delta function potential,

$$V(x_1, x_2) = aV_0\delta(x_1 - x_2),$$

where $V_0$ is a small positive constant with the dimension of energy and $a$ is the width of the well.

(a) First, ignoring the interacting between the electrons, write down the spatial wave function(s) and energy of the ground state(s). Also describe their corresponding spin configurations (singlet or triplet?).

(b) Again ignoring the interaction, write down spatial wave function(s) and the energy of the first excited state(s). Also describe their corresponding spin configurations (singlet or triplet?).

(c) Now using first-order perturbation theory, estimate the effect of the above delta function interaction on the energy of the first excited state(s). What are the new spin and spatial configurations of the first excited state(s)?

**D1**. An electron with the gyromagnetic ratio of $\gamma$ is at rest in an oscillating magnetic field

$$\mathbf{B} = B_0 \cos(\omega t)\, \hat{z},$$

where $B_0$ and $\omega$ are constants.

(a) Construct the Hamiltonian matrix for this system.

(b) The electron starts out at $t = 0$ in the spin-up state with respect to the $x$-axis. Determine the spinor (i.e. spin state), $\chi(t)$, at any subsequent time.

(c) Find the probability of getting $-\hbar/2$ as a function of time, if you measure $S_x$.

(d) What is the minimum field amplitude, $B_0$, that causes a complete flip of $S_x$ at a given $\omega$?

**D2.** Consider a two-dimensional harmonic oscillator

$$H^0 = \frac{1}{2} m\omega^2 \left(x^2 + y^2\right)$$

with a rotational symmetry-breaking perturbation

$$H^1 = \lambda xy$$

a) Calculate the 1st and 2nd order corrections to the ground state energy. [Hint. You will likely want to use the raising and lowering operators $a$ and $a^\dagger$ ]

$$a_x = \left(\frac{m\omega}{2\hbar}\right)^{1/2} \left(x + \frac{i}{m\omega} p_x\right) \qquad a_x^\dagger = \left(\frac{m\omega}{2\hbar}\right)^{1/2} \left(x - \frac{i}{m\omega} p_x\right)$$

$$a_y = \left(\frac{m\omega}{2\hbar}\right)^{1/2} \left(y + \frac{i}{m\omega} p_y\right) \qquad a_y^\dagger = \left(\frac{m\omega}{2\hbar}\right)^{1/2} \left(y - \frac{i}{m\omega} p_y\right)$$

b) Calculate the 1st order correction to the ground-state eigenstate.

c) Calculate the 1st order correction to the energy of the lowest excited states and find their eigenstates to 0th order.