Rutgers University – Physics Graduate Qualifying Exam
Classical Mechanics – January 13, 2010

Work problems A and B and (C1 or C2) and (D1 or D2).
Work each problem in a separate blue book.
Each problem is worth a total of 10 points.

A. A solid cylinder of mass M and radius R has a string wrapped around it. One end of the string is attached to the ceiling, and the disk is allowed to fall down, rotating as the string unwinds (like a yo-yo). The string does not slip as the cylinder unwinds.

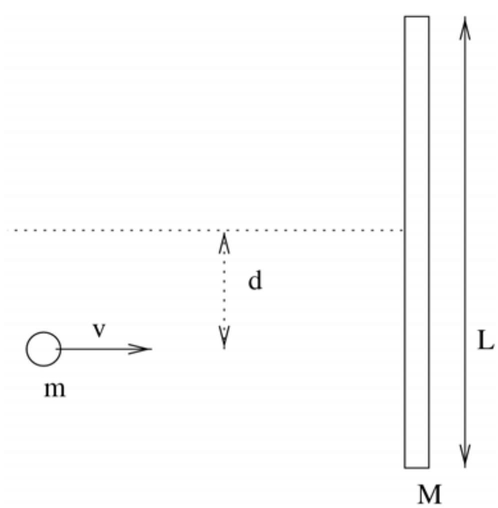
(a) Write down the Lagrangian for this system.

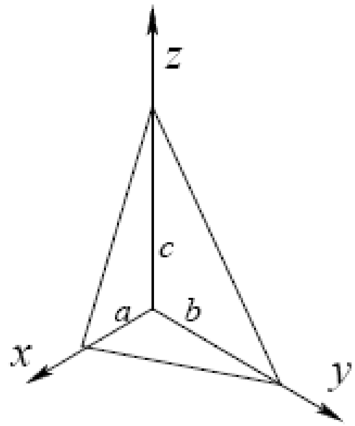
(b) Solve for the equation of motion, and find the linear acceleration of the disk.

B. A uniform stick lies on a frictionless table. It has mass M and length L and is free to move in any way on the table. A hockey puck of mass m is moving as shown in the figure below with velocity v. It collides elastically with the stick, a distance d below the center of mass of the stick.

What must the mass m of the hockey puck be so that it is at rest after the collision?

Top view of problem:





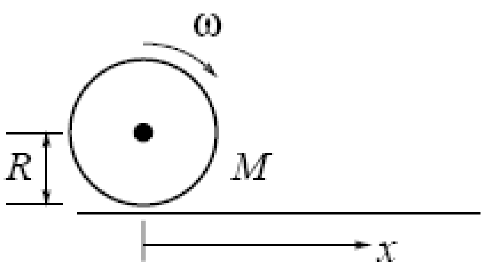
C1. A pyramid bounded by the planes $x = 0$, $y = 0$, $z = 0$ and the plane passing trough the three points (a; 0; 0), (0; b; 0), (0; 0; c) is constructed from uniform aluminum, and has a total mass M. Assume $0 < a < b < c$. Find

a) the center of mass in the above body.

b) the inertia tensor evaluated about the center of mass.

c) If there are no external forces acting on this pyramid, what can you say about the angular momentum and the angular velocity. In particular, discuss under what conditions the angular velocity can be constant, and whether such motion is stable or not.

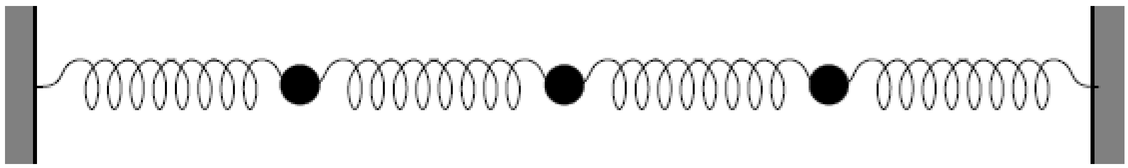
C2. A uniform disk of radius R and mass m is rotating with angular velocity ω_0 about its axis, which is horizontal, suspended infinitesimally above a horizontal plane. At time $t = 0$ the suspension disappears, so that the disk makes contact with the plane, with which it has a coefficient of friction μ (both kinetic and static coefficients $\mu_k = \mu_s = \mu$).



(a) Describe qualitatively what will happen immediately after $t = 0$ (but neglecting any bouncing), and then what proceeds to happen thereafter. Be sure to mention the qualitative change in the nature of the motion. You can assume the disk does not tip over, but remains vertical.

(b) What will the velocity and angular velocity of the disk be at the moment ($t > 0$) there is a qualitative change in the motion? How do the answers depend on the coefficient of friction μ?

D1. Three particles, each of mass *m*, are connected by four identical springs, each of spring constant *k* and equilibrium length *a*, to each other and to fixed walls a distance 4*a* apart, as shown. Assuming the system remains along the straight line.



(a) Define a set of independent coordinates to describe the state of the system.
(b) Find the normal modes and frequencies of oscillation.
(c) If the system at rest is excited by an impulse *I* delivered at time 0 to right to the middle particle, give the positions of each of the particles as a function of time.

D2. At the Sudbury mine in Canada, neutrino physics is done 2000 meters below the surface. Suppose we have a vertical shaft totally evacuated (vacuum) extending from the top. Assume we can treat the Earth as completely spherical and ignore the variation of *g* with depth.

(a) In what direction, relative to the line to the center of the Earth, is the apparent force on a particle at rest in the shaft.

(b) If the particle is dropped from rest (with respect to the shaft), show that the particle will land, 2000 m below the point from which it was dropped, at a point which is 1.35 m East of the point pointed to by the apparent gravitational force calculated in part (a).

(c) Show that the landing point is also 0.54 mm South of the point pointed to in part (a).

Some possibly useful information:

Radius of the Earth:	3.67×10^6 m
Acceleration of gravity:	$9.80\,\mathrm{m/s^2}$
Latitude of Sudbury:	$46.62°$ N
Longitude of Sudbury:	$80.80°$ W
Length of a Siderial day:	86163 s